Superposition:

A concept applied to linear systems:

Example, not from circuits:

$$\frac{d^2 y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = At + Be^{-j4t}$$

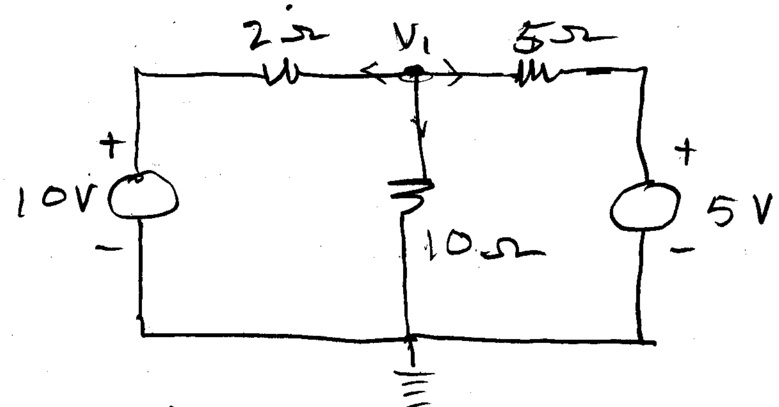
$y(t) = y_1(t) \rightarrow$ solution for At

$y(t) = y_2(t) \rightarrow$ solution for Be^{-j4t}

$y(t) = y_1(t) + y_2(t) \rightarrow$ solution for $At + Be^{-j4t}$

System must be linear & time invariant

Illustration for a simple circuit



Find V_1 using superposition;

First find the total solution by node analysis.

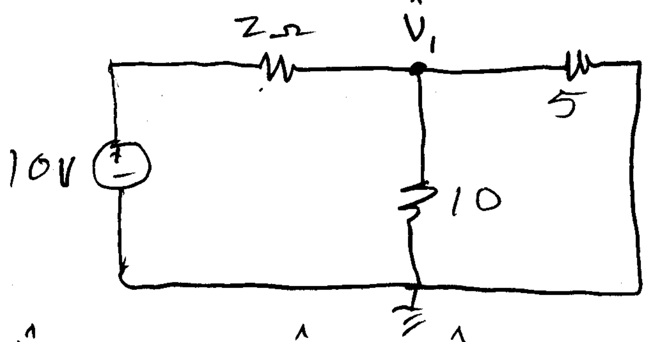
$$\frac{V_1 - 10}{2} + \frac{V_1}{10} + \frac{V_1 - 5}{5} = 0$$

$$5V_1 - 50 + V_1 + 2V_1 - 10 = 0$$

$$8V_1 = 60$$

$$\boxed{V_1 = 7.5\ \text{V}}$$

With 10V present

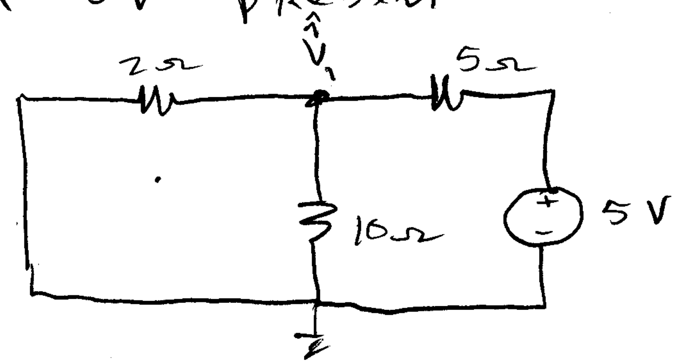


$$\frac{\hat{V}_1 - 10}{2} + \frac{\hat{V}_1}{10} + \frac{\hat{V}_1}{5} = 0$$

$$5\hat{V}_1 - 50 + \hat{V}_1 + 2\hat{V}_1 = 0$$

$$8\hat{V}_1 = 50 \longrightarrow \hat{V}_1 = \frac{50}{8}$$

With 5V present



$$\frac{\hat{\hat{V}}_1}{2} + \frac{\hat{\hat{V}}_1}{10} + \frac{\hat{\hat{V}}_1 - 5}{5} = 0$$

$$5\hat{\hat{V}}_1 + \hat{\hat{V}}_1 + 2\hat{\hat{V}}_1 - 10 = 0$$

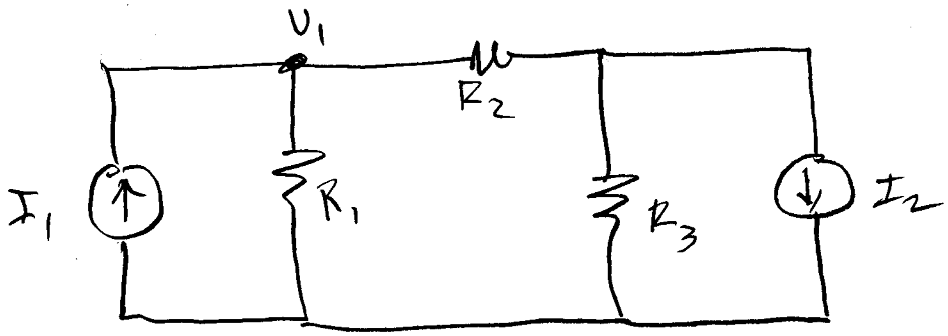
$$8\hat{\hat{V}}_1 = 10$$

$$\hat{\hat{V}}_1 = \frac{10}{8}$$

$$V_1 = \hat{V}_1 + \hat{\hat{V}}_1 = \frac{50}{8} + \frac{10}{8} = \frac{60}{8} = 7.5V$$

<u>checks</u>

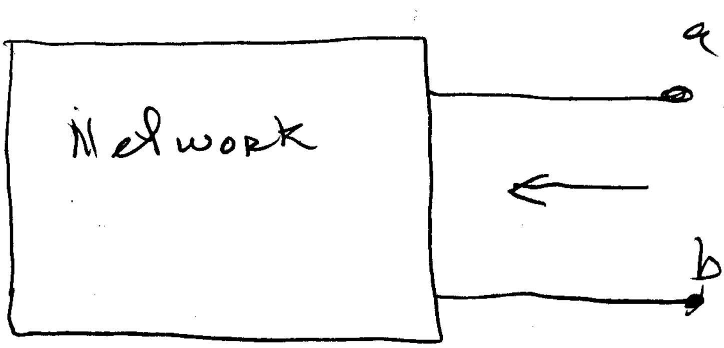
Would apply also to



Active I_1, get $\tilde{V}_1$; I_2 opened
Active I_2, get $\tilde{\tilde{V}}_1$; I_1 opened

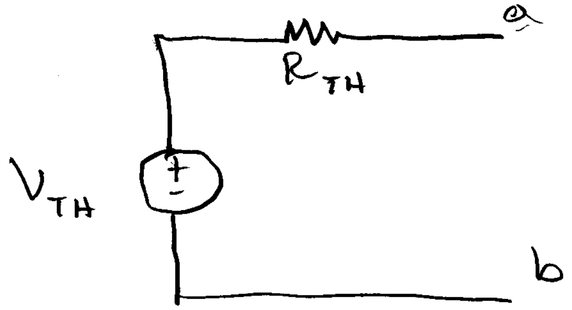
Thevenin's Theorem

Given a network in the box. We allow independent & dependent sources. We focus on independent sources.



Suppose we have only resistors and sources (voltage and/or current sources).

If you think about this for a while, you will reconcile that this network can be replaced by:

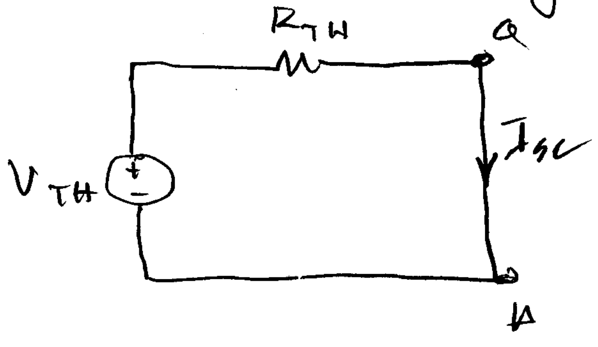


After all, if you placed a voltmeter across a-b you would read a voltage: This is the "open-circuit" voltage.

$$\therefore V_{TH} = V_{ab} \text{ (opened)}$$

Similarly, if we disable all sources and place an ohmmeter across a-b (opened to the right) we read R_{TH}. R_{TH} = Resistance at a-b with all sources disabled.

You can also say the following

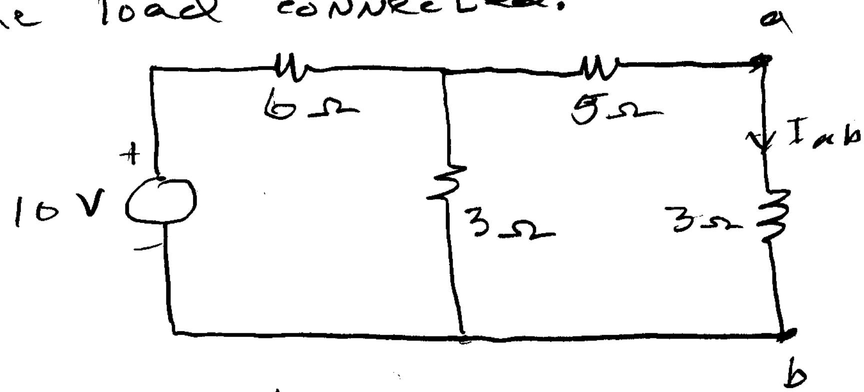


$$R_{TH} = \frac{V_{TH}}{I_{sc}}$$

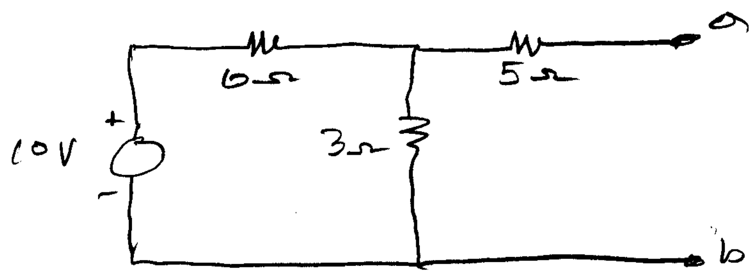
We illustrate this with an example.

Example 1

Find the Theven equivalent circuit to the left of a-b and then find V_{ab} with the load connected. Also find I_{ab} with the load connected.



Remove the load. Find V_{os}.

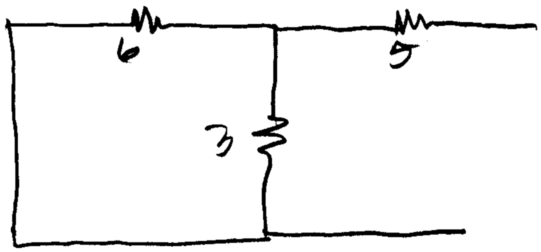


$$V_{ab} = V_{os} = V_{TH} = \frac{10 \times 3}{9} = \frac{30}{9} = \frac{10}{3} \text{ V}$$

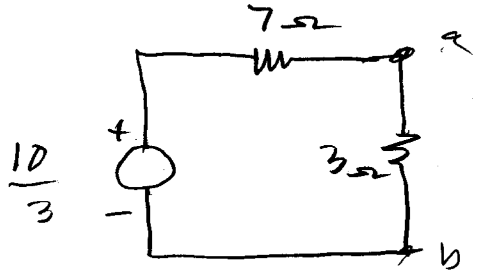
(No current flows through the 5Ω resistor so the voltage across the 5Ω resistor is zero.

$\therefore V_{TH} = \frac{10}{3}$

For R_{TH}



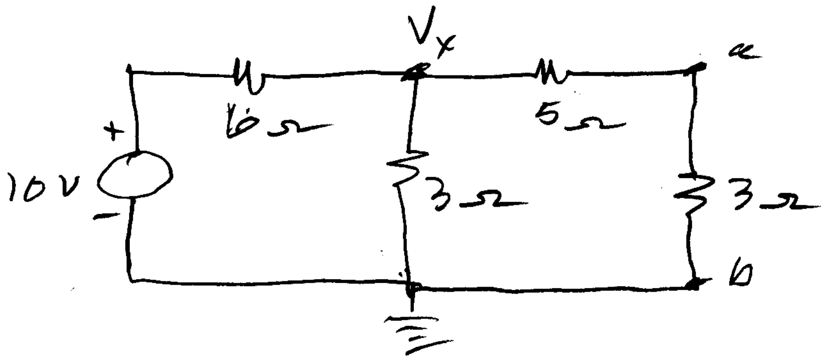
$\Rightarrow$ $5 + 3 \| 6 = 7\Omega$



$$V_{ab} = \frac{\left(\frac{10}{3}\right) \times 3}{7+3} = 1\,V$$

$$I_{ab} = \frac{10/3}{10} = \frac{1}{3}\,A$$

Suppose we work this by Node analysis:



24 $$\frac{V_x - 10}{6} + \frac{V_x}{3} + \frac{V_x}{8} = 0$$

$$4V_x - 40 + 8V_x + 3V_x = 0$$

$$15 V_x = 40$$

$$V_x = \frac{40}{15} = \frac{8}{3}\,V$$

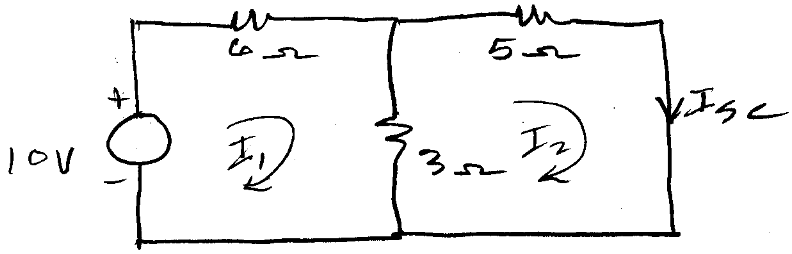
$$I_{ab} = \frac{8/3}{8} = \frac{1}{3}\,A$$

$$V_{ab} = 3 \times I_{ab} = 3 \times \frac{1}{3} = 1\,V$$

Checks with Thevenin solution.

Now suppose, for verification, we want to find I_{sc}. Then determine

$$R_{TH} = \frac{V_{TH}}{I_{sc}}$$



$$9I_1 - 3I_2 = 10$$

$$-3I_1 + 8I_2 = 0$$

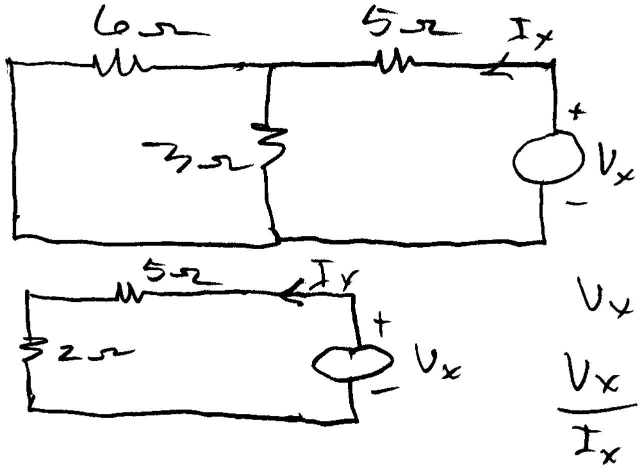
$$\therefore I_2 = 0.47619 = \frac{10}{21} = I_{sc}$$

$$R_{TH} = \frac{V_{os}}{I_{sc}} = \frac{10/3}{10/21} = \frac{21}{3} = 7\,\Omega$$

<u>Checks</u>

Also, we can place a voltage or current source at the output with all independent sources deactivated and find $R_{TH} = \frac{V_x}{I_x}$.

For this example



$$V_x = 7I_x$$

$$\frac{V_x}{I_x} = R_{TH} = 7\,\Omega$$