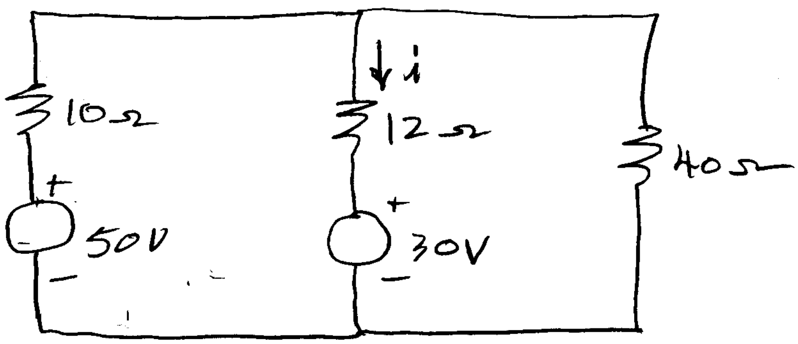
Further work with Thevenin

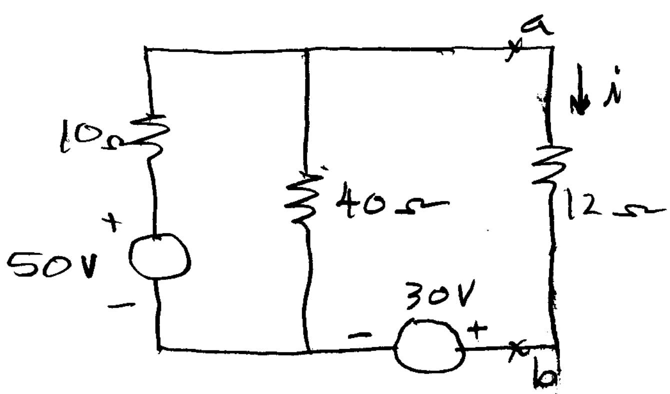
Examples best illustrate Thevenin

Consider the following:

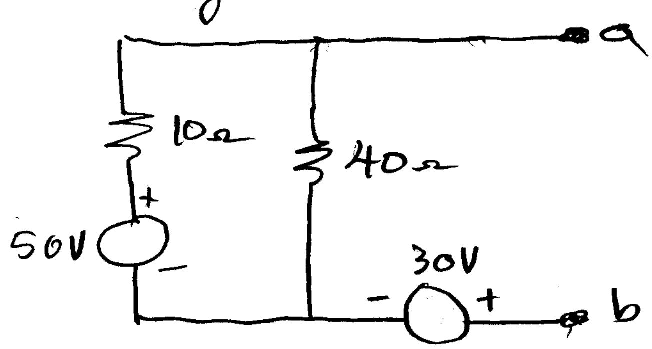
Use Thevenin's theorem to find the current i in the following circuit.



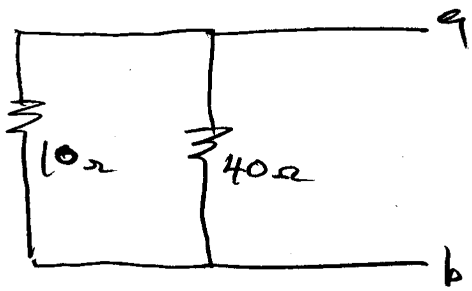
Re-draw the ckt.



Breaking the ckt at a-b gives

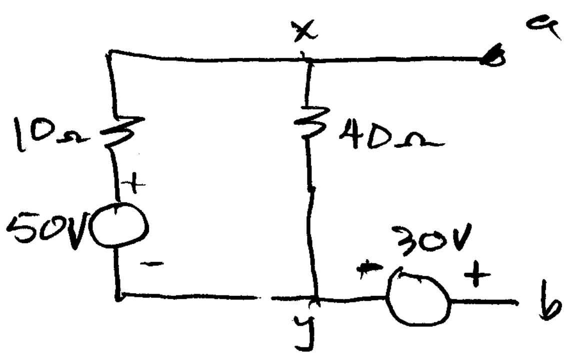


We want to find R_{TH}. We use the following with the voltage sources (independent ones) disabled.



$$R_{TH} = 10 \| 40 = 8\Omega$$

To find the open circuit voltage we work with



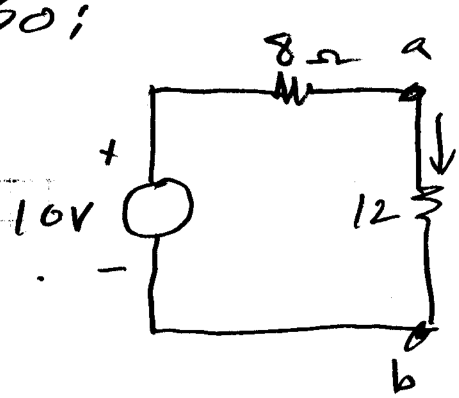
$$V_{xy} = \frac{50 \times 40}{40 + 10} = 40V$$ (voltage divider)

Then $V_{TH} = V_{ab}$;

KVL gives $V_{ab} + V_{yx} + 30 = 0$

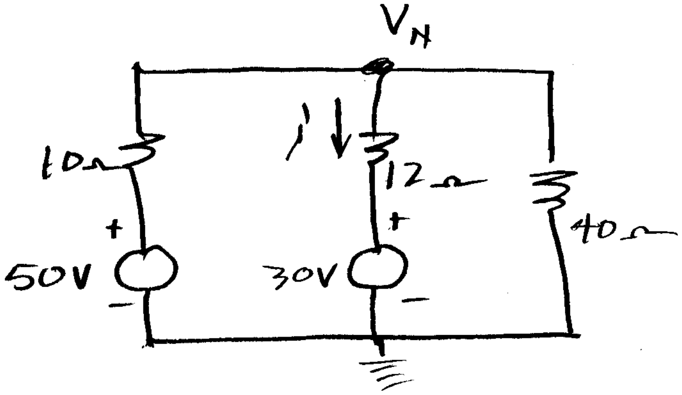
$$V_{ab} = -V_{yx} - 30 = V_{xy} - 30 = 10V$$

So;



$$\boxed{i = \frac{10}{20} = 0.5A}$$

A node voltage solution to this problem follows:



$$\frac{V_N - 50}{10} + \frac{V_N - 30}{12} + \frac{V_N}{40} = 0$$

$$12V_N - 600 + 10V_N - 300 + 3V_N = 0$$

$$25V_N = 900$$

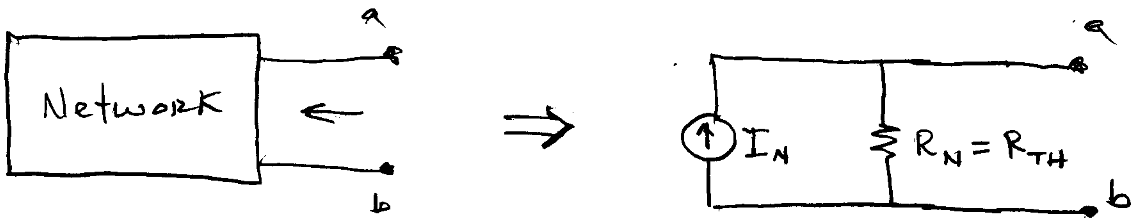
$$V_N = 36$$

$$i = \frac{V_N - 30}{12} = \frac{6}{12} = 0.5A \qquad \text{QED}$$

We could have worked the problem by (a) mesh analysis, (b) superposition. Which analysis method is <u>best?</u>
This question is sort of like, which golf club is best to use: depends on what the shot call for – answer lies with having experience. Circuit theory gives us a bag of clubs. Experience tells us which one to use.

Norton's Theorem

Nortons theorem states that as far as the terminals of the one port network shown below is concerned, we can replace the network with a current source shunted by a resistor, as shown.



The current, I_N, is the current that would flow from a to b, if a-b were shorted. The resistance R_N is found exactly as we found R_{TH}.

Restrictions

- The network must be LTI (linear, time invariant)
- Source can be voltage or current sources, both independent and dependent.

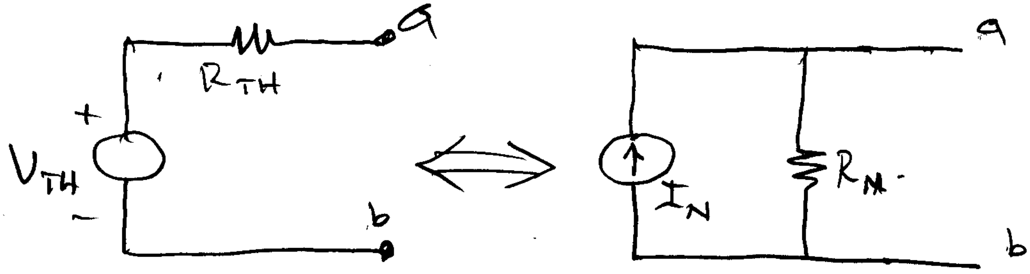
It is of interest to note that if we have the Thevenin equivalent circuit we can change it to a Norton equivalent by

$$I_N = \frac{V_{TH}}{R_{TH}}; \qquad R_N = R_{TH}$$

We can also change a Norton to a Thevenin, by going the other way;

$$V_{TH} = I_N \times R_N \; ; \; R_{TH} = R_N$$

Keep the model form in mind;



$V_{TH} = I_N \times R_N$

$R_{TH} = R_N$

$I_N = \dfrac{V_{TH}}{R_{TH}}$

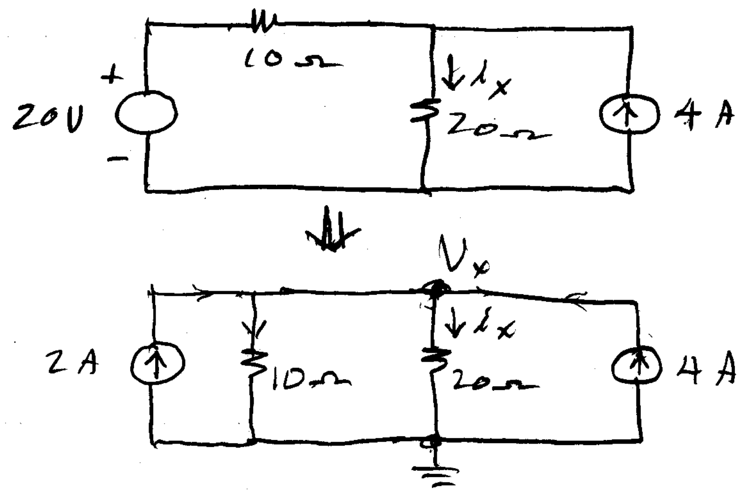
Before going further, let us discuss source transformation.

Source Transformation

We can always go from a voltage source in series with a resistor to a current source shunted by a resistor; or we can go from a current source shunted by a resistor to a voltage source in series with a resistor. We use Thevenin to Norton; Norton to Thevenin to do this; Simple example below

Example

Use source transformation in the problem below to find i_x. Use Node analysis:



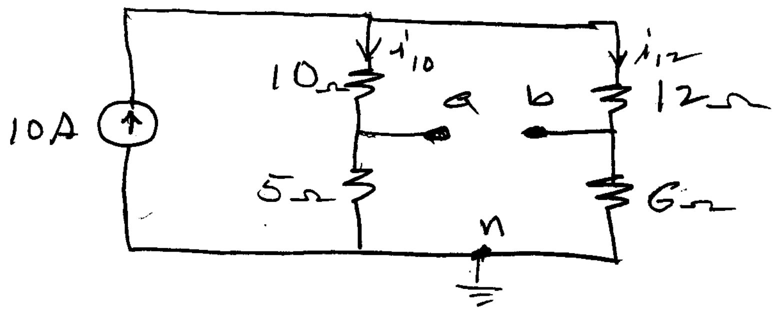
$$\frac{V_x}{10} + \frac{V_x}{20} = 6 \Rightarrow 2X + V_x = 120.$$

$$V_x = 40\,V$$

$$i_x = \frac{40}{20} = 2A$$

Example

Use Thevenin and Norton
Find the Thevenin and Norton circuit at terminals a-b for the following network,



The Thevenin ckt:

$$i_{10} = \frac{10 \times 18}{18 + 15} = \frac{180}{33}$$

$$i_{12} = \frac{10 \times 15}{18 + 15} = \frac{150}{33}$$

$$V_{ab} + V_{na} + V_{bn} = 0$$

$$V_{ab} = V_{TH} = -V_{na} - V_{bn} = V_{an} - V_{bn}$$

$$V_{ab} = i_{10} \times 5 - i_{12} \times 6$$

$$V_{ab} = \frac{180}{33} \times 5 - \frac{150}{33} \times 6 = 0\,V$$

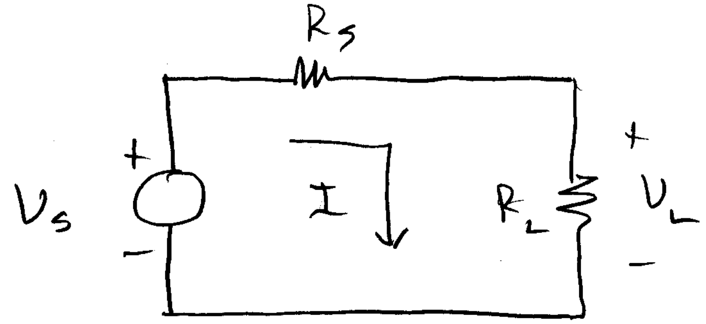
$$R_{TH} = 15 \parallel 18 = \frac{15 \times 18}{15 + 18} = 8.182\,\Omega$$

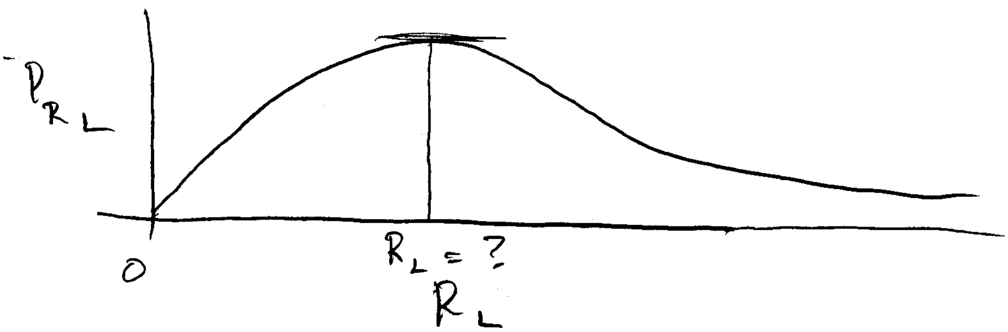
$$I_N = \frac{V_{TH}}{R_{TH}} = \frac{0}{8.182} = 0\,A$$

Use another circuit method to show this is true.

Maximum Power Transfer

Given the following. Determine R_L so that maximum power is transferred to R_L





The power delivered to R_L is

$$P_{R_L} = I^2 R_L = \frac{V_s^2 \times R_L}{(R_s + R_L)^2}$$

Now

$$\frac{\partial P_{R_L}}{\partial R_L} = \frac{(R_s+R_L)^2 V_s^2 - V_s^2 R_L \times 2(R_s+R_L)}{(R_s+R_L)^4}$$

set this derivative to zero to find the maximum.

$$\frac{(R_s+R_L)^2 V_s^2 - V_s^2 R_L \times 2(R_s+R_L)}{(R_s+R_L)^4} = 0$$

$$\Downarrow$$

$$(R_s+R_L)^{\cancel{2}} \cancel{V_s^2} - \cancel{V_s^2} R_L \times 2 (R_s \cancel{+R_L}) = 0$$

$$\Downarrow$$

$$(R_s+R_L) - 2R_L = 0$$

$$2R_L = R_s + R_L$$

$$\therefore \boxed{R_L = R_s}$$