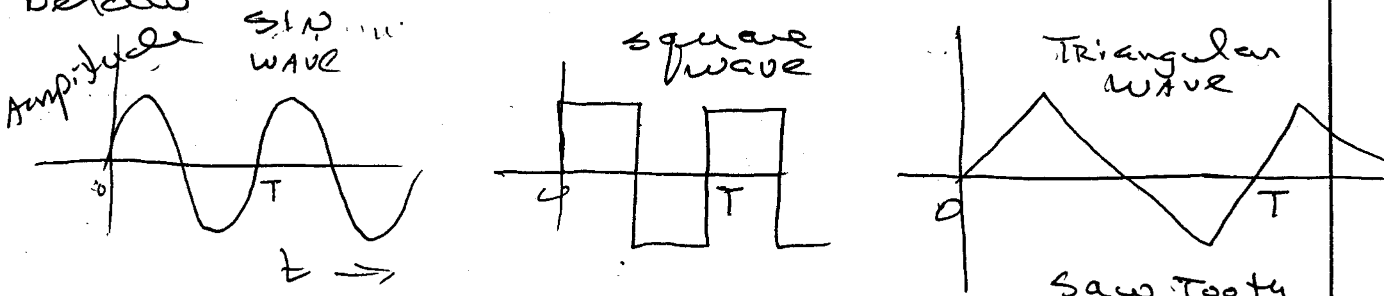
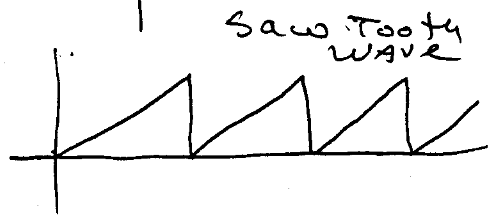
Time-Dependent Signal Sources:

Nominal waveforms encountered in the laboratory are sketched below

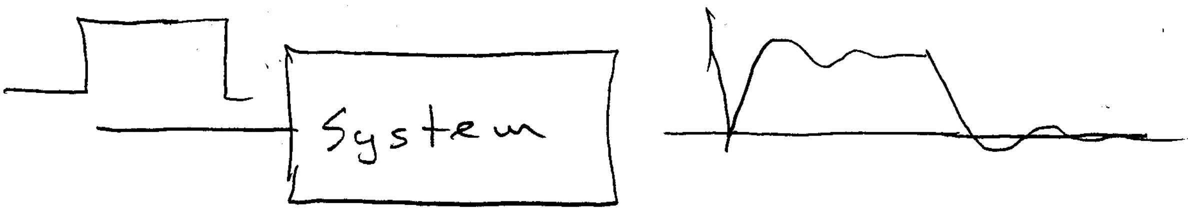


And of course DC.



Why these waveforms? To start with, the sine wave is the fundamental waveform used in the power industry, worldwide. It is 60 Hertz (cycles/sec) in the US. In parts of Europe, maybe Japan it is 50 Hertz.

The square wave is used a lot for testing systems. We often specify system performance in terms of rise time, overshoot and settling time of a system with a step input applied;



In the following we consider the properties of signals such as

- how to write math expressions for the signals.
- how to determine average values.
- how to determine rms values.

Average Value

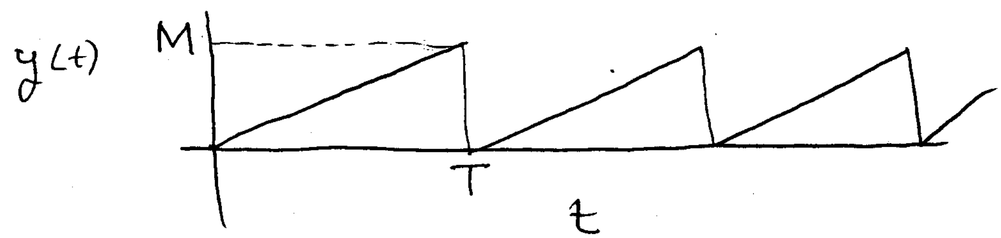
The average value of a waveform expressed as $x(t)$ is

$$x(t)_{Avg} = \frac{1}{T}\int_0^T x(t)\,dt$$

We deal with average with many things – in school, particularly the average of exam scores.

EXAMPLE 11.1

Find the average value of the periodic waveform shown below



We can easily see that over on period the waveform can be expressed as;

$$y(t) = \frac{M}{T} t \qquad \text{(one period)}$$

$$y(t)_{AVG} = \frac{1}{T} \int_0^T \frac{M}{T} t \, dt$$

$$= \frac{M}{2T^2} t^2 \Big|_0^T = \frac{M}{2T^2} \left[T^2 \right]$$

$$y(t)_{AVG} = \frac{M}{2}$$

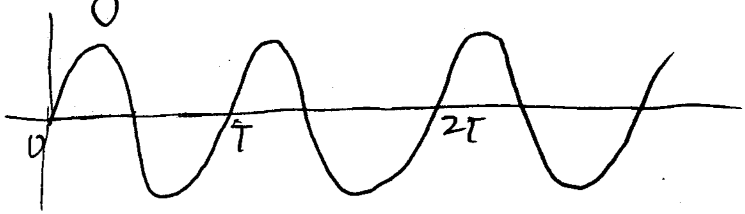
There are some waveforms that you can tell by inspection, the average value. The sinusoid is one of these waveforms.

EXAMPLE 11.2

Find the average value of

$$x(t) = A\sin(\omega t)$$

The general waveform is



$$\omega = 2\pi f = \frac{2\pi}{T}$$

f is the frequency in hertz (H)

T is the period which is $\frac{1}{f}$.

Obviously, the average over T is zero.
We show this by math as follows.

$$x(t)_{AVG} = \frac{1}{T}\int_0^T A\sin\left(\frac{2\pi}{T}t\right)dt$$

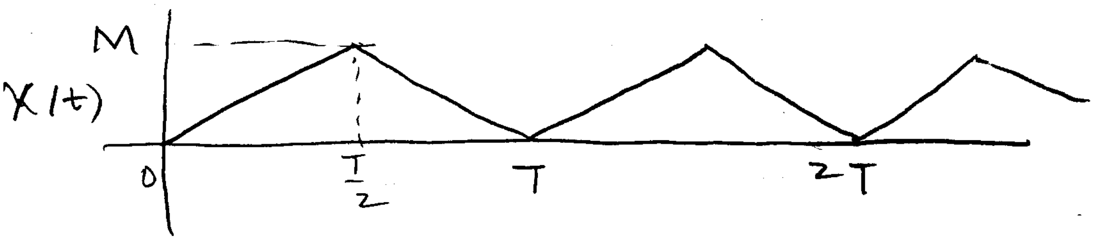
$$= -\left(\frac{1}{T}\right)\frac{T}{2\pi}\cos\left(\frac{2\pi}{T}t\right)\Bigg|_0^T$$

$$x(t)_{AVG} = -\frac{1}{2\pi}\left[\cos(2\pi) - \cos 0\right] = 0$$

As expected.

EXAMPLE 11.3

Find the average value of the following triangular waveform.



We can find the average as follows;

$$X(t)_{Avg} = \frac{1}{T/2}\int_0^{T/2} \frac{M}{T/2}\, t\, dt$$

$$= \frac{2}{T} \times \frac{2M}{T}\, \frac{1}{2} t^2 \Big|_0^{\frac{T}{2}}$$

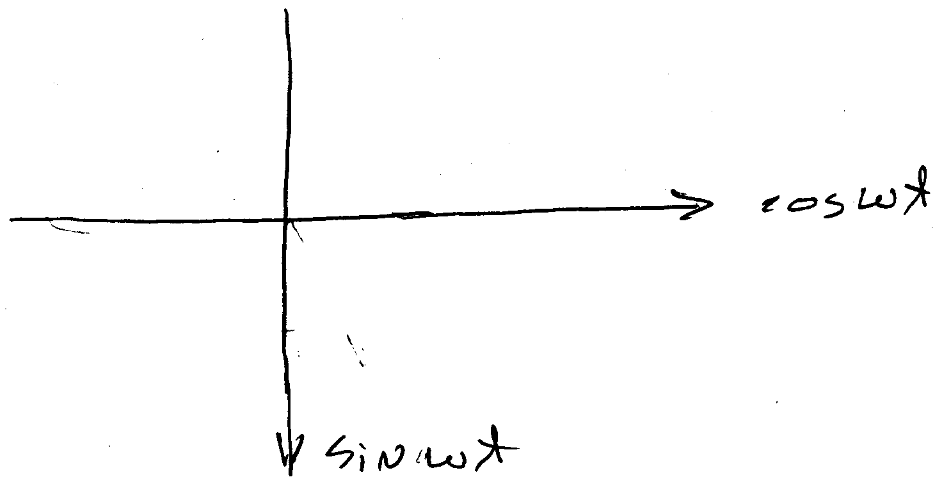
$$X(t)_{Avg} = \frac{2M}{T^2} \times \frac{T^2}{4} = \frac{M}{2}$$

Observing EXAMPLE 11.1 would have told us this.

Later on when we are dealing with AC circuits we find it necessary to refer our input signals to cosine$(\omega t + \theta_1)$. So if we are given $\sin(\omega t + \theta_2)$ we will want to convert these cosine. We can use trig functions such as

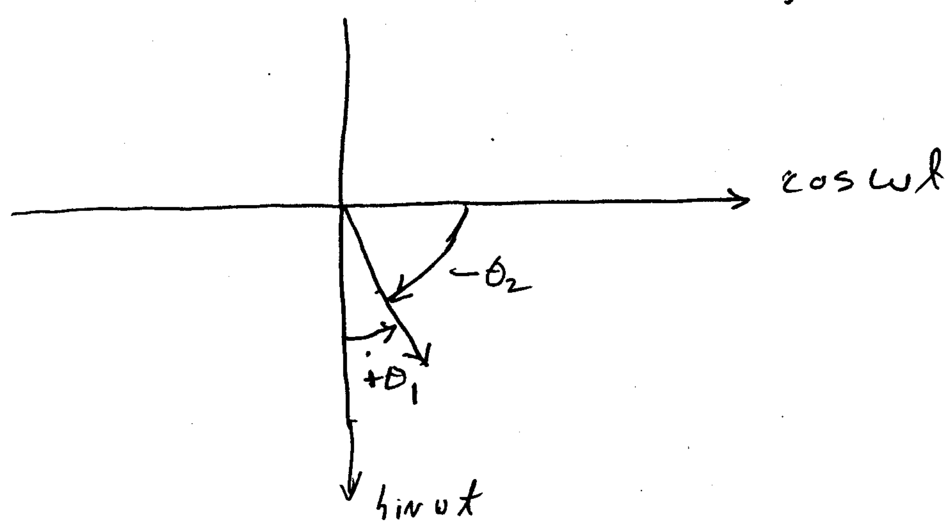
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

An easy way to keep us with changing from $\sin \longleftrightarrow \cos$ is as follows. Prepar a set of axis as below



Angles are measured as positive in the counterclockwise direction as normal with polar coordinates.

Suppose we want to relate $\sin(\omega t+\theta_1)$ to a cosine. Then $\sin(\omega t+\theta_2)=\cos(\omega t-\theta_2)$



EXAMPLE 11.4

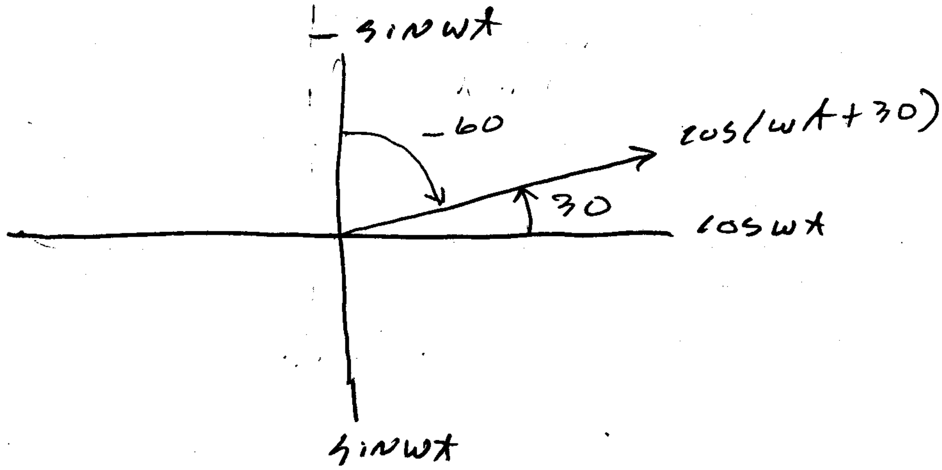
Change the following as indicated

(a) $\cos(\omega t + 30^\circ) \longrightarrow \sin(\omega t + \text{angle})$

(b) $-\sin(\omega t - 50^\circ) \longrightarrow \cos(\omega t + \theta)$

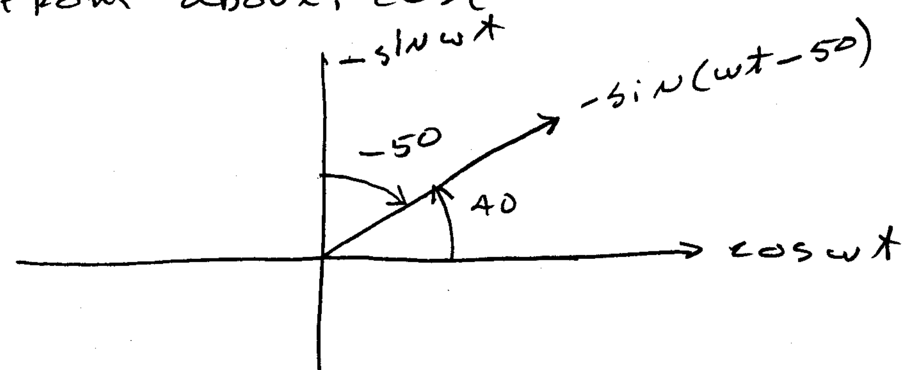
(c) $\sin(377t + \pi/6) \longrightarrow \cos(377t + \theta)$

(a)



From above: $\cos(\omega t + 30) = -\sin(\omega t - 60)$

(b)

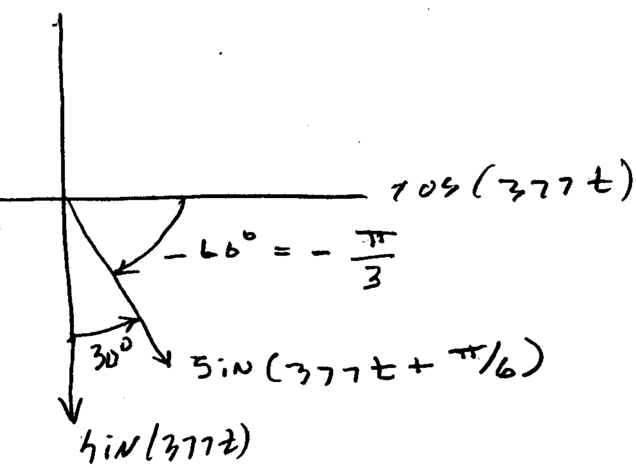


$-\sin(\omega t - 50^\circ) = \cos(\omega t + 40)$

(c)

$\sin(377t + \pi/6)$

$= \cos(377t - \frac{\pi}{3})$



Answer in the text, page 158, Check Your Understanding, is incorrect.

RMS Value

RMS stands for root means square and we see why later. RMS value is also called Effective Value.

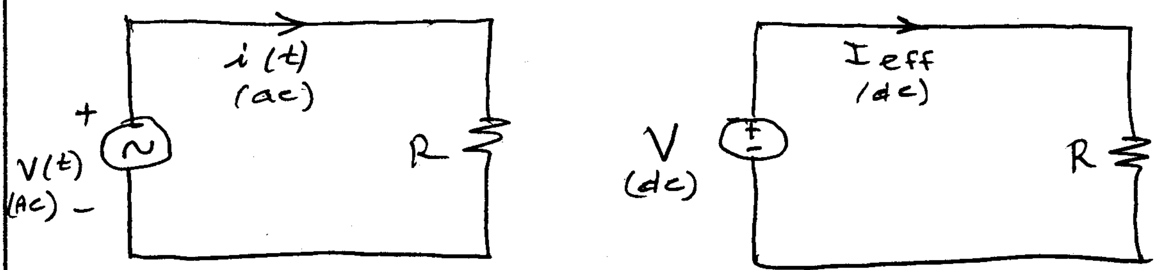
The idea of RMS or Effect Value precipitated from the need to measure the effectiveness of a current or voltage source in delivering power to a resistance.

Definition

The effective value of a periodic current (voltage) is the dc current (voltage) that delivers the same average power to a resistor as the periodic current (voltage).

We need to express this with mathematics:

We do this by considering two circuits. One circuit uses an AC current to deliver power to a resistor R. The second circuit uses a constant current, I_{eff}, to deliver power to the same resistor. This is shown below



Average Power delivered

$$P = \frac{1}{T}\int_0^T i^2(t)\, R\, dt$$

Average Power delivered

$$P = I_{eff}^2 R$$

We set these two powers equal

$$I_{eff}^2 R = \frac{1}{T}\int_0^T i^2(t)\, R\, dt$$

so

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\, dt}$$

Since the $\sqrt{\ }$, the average, the square are involved, we call this the

Root, Means, Square or just

rms ⟷ RMS

It has become common to talk about the rms value of any signal and express this as

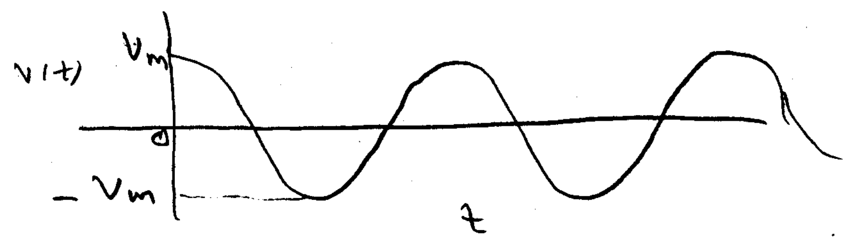
$$x_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$$

So that is the story behind rms. We need to see how to use this.

EXAMPLE 11.5

The signal that is generally present in our homes for power delivery is a sinusoid. Suppose we express this as

$$v(t) = V_m \cos(\omega t)$$



What is the effective value of this voltage?

We have

$$V_{eff} = \sqrt{\frac{V_m^2}{T}\int_0^T \cos^2(\omega t)\,dt}$$

OR

$$V_{eff} = \sqrt{\frac{V_m^2}{T}\int_0^T \frac{1}{2}(1+\cos 2\omega t)\,dt}$$

$$V_{eff} = \sqrt{\frac{V_m^2}{2}}$$

so

$$V_{eff} = \frac{V_m}{\sqrt{2}}$$

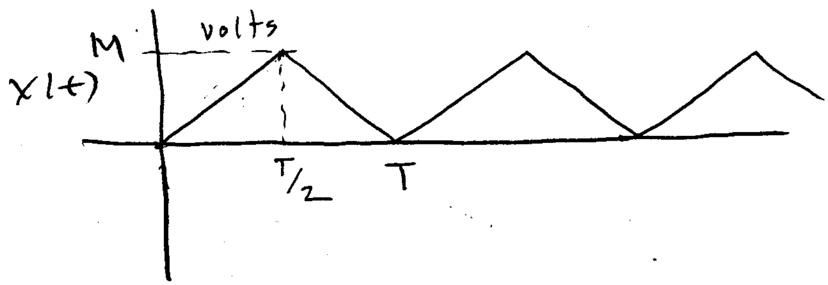
We normally say that the voltage at an outlet in our homes is 110 volts. This is the rms value of the voltage. The peak value is

$$V_m = \sqrt{2} \times 110 = 155.6\ V$$

We consider find the rms values of some signals given in the text.

EXAMPLE 11.6

Find the RMS value of the following sawtooth.



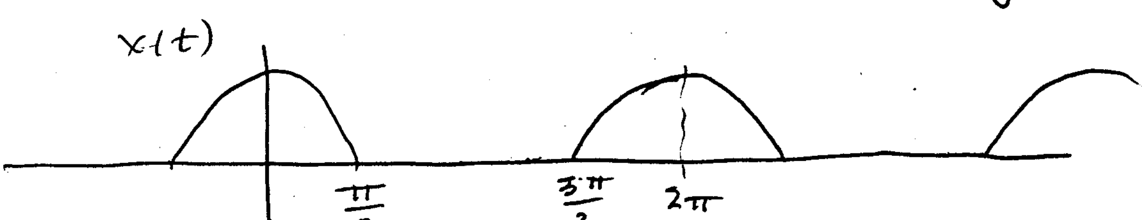
It is sufficient to find the RMS of the signal from 0 to T/2. This will be the same RMS as from 0 to T because of symmetry (and you are squaring x(t)).

$$X_{rms} = \sqrt{\frac{2}{T}\int_0^{T/2}\left(\frac{2M}{T}t\right)^2 dt} = \frac{M}{\sqrt{3}}$$

If $M = 3$, then $X_{rms} = 1.732\text{ V}$ (book gives 2.89 V)

EXAMPLE 11.7

Find the RMS of the following



Because of symmetry

11.13

$$x(t) = \sqrt{\frac{2}{2\pi}\int_0^{\frac{\pi}{2}} \cos^2 t \, dt}$$

$$x(t) = \sqrt{\frac{2}{2\pi}\left[\int_0^{\pi/2} \frac{1}{2}(1+\cos 2t)\,dt\right.}$$

$$x(t) = \sqrt{\frac{1}{2\pi}\left[t + \frac{1}{2}\sin t\right]_0^{\pi/2}}$$

$$x(t) = \sqrt{\frac{1}{2\pi} \times \frac{\pi}{2}} = \frac{1}{\sqrt{4}} = \frac{1}{2} = 0.5$$

checks with text, p 161