Lesson 13

Solving AC Circuit Problems

This lesson is dedicated to solving AC circuit problems.

Example 13.1

For the circuit shown in Figure 13.1, solve for $\bar{V}_R$, $\bar{V}_C$ and $\bar{V}_L$. Prepare the phasor diagram for the above phasor voltages. Finally, write the time domain equation for the steady state current in the circuit.

Figure 13.1: Circuit for Example 13.1

$V(t) = 150 \sin (377t + 120^\circ) \ V$

$\bar{V}(t) = 150 \cos (377t + 30^\circ) \ V$

$\bar{V} = 150 \angle 30^\circ \ V$
We can write:

\[ \mathbf{I} = \frac{150 \angle 30^\circ}{30 - j15 + j40} = 3.84 \angle -9.8^\circ \text{ A} \quad \text{Eq. 13.1} \]

Now calculate the voltages:

\[ \mathbf{V}_R = 30 \times \mathbf{I} = 115.2 \angle -9.8^\circ \text{ V} \]
\[ \mathbf{V}_e = (151 - 90) (3.84 \angle -9.8^\circ) = 57.6 \angle -97.8^\circ \text{ V} \]
\[ \mathbf{V}_L = (401 - 90) (3.84 \angle -9.8^\circ) = 153.6 \angle 80.2^\circ \text{ V} \]

By KVL:

\[ \mathbf{V} = \mathbf{V}_R + \mathbf{V}_e + \mathbf{V}_L \quad \text{Eq. 13.2} \]

Putting in the above values gives:

\[ 150 \angle 30^\circ = 115.2 \angle -9.8^\circ + 57.6 \angle -97.8^\circ + 153.6 \angle 80.2^\circ \]

According to my calculator, the right hand side of the above equation is: 149.757\angle 30^\circ \text{ V}.

This is a good check since we have rounded numbers along the way.

What the phasor diagram shows is that \( \mathbf{V}_R + \mathbf{V}_e + \mathbf{V}_L \) will be equal to \( \mathbf{V} \). We now show this, roughly.
Figure 13.2: Phasor Diagram for Voltages for Example 13.1

We note the following:

- $V_L$ and $V_c$ are in opposite directions. This will always happen (+j for inductor, -j for capacitor).

- The graphical sum of $V_R + V_L + V_c = \overline{V}$, verification of KVL.

Finally, in the time domain, $i(t)$ is

$$i(t) = 3.84 \cos(377t - 9.8) \text{ A} \quad \text{Eq (13,3)}$$
Example 13.2

Find the mesh currents $\bar{I}_1$ and $\bar{I}_2$ in the following circuit.

![Circuit Diagram]

Figure 13.3: Circuit for Example 13.2.

This is about as tough as an AC circuit problems get to be. We write the following two mesh equations, using $\sum V = 0$.

$$50 \angle 0 \, - \, (20 + j30) \bar{I}_1 - (j10)(\bar{I}_1 - \bar{I}_2) + 70 \angle 20^\circ = 0$$

Simplifying gives:

$$(20 + j20) \bar{I}_1 + (j10) \bar{I}_2 = 50 \angle 0 + 70 \angle 20 \quad \text{Eq} \, 13.4$$

Around mesh 2, we have

$$-70 \angle 20 \, - \, (\bar{I}_2 - \bar{I}_1)(-j10) - (25 - j14) \bar{I}_2 - 100 \angle 60 = 0$$

or

$$+j10 \bar{I}_1 + (25 - j10 - j14) \bar{I}_2 = -70 \angle 20 - 100 \angle 60 \quad \text{Eq} \, 13.6$$
Eqs. 13.4 & 13.5 can be put in the form

\[(20+j20)I_1 + j10I_2 = 118.23/111.7^\circ\]
\[j10I_1 + (25-j24)I_2 = 160/136.3^\circ\]

OR

\[
\begin{bmatrix}
20+j20 & j10 \\
-j10 & 25-j24
\end{bmatrix}
\begin{bmatrix}
\bar{I}_1 \\
\bar{I}_2
\end{bmatrix}
=
\begin{bmatrix}
118.23/111.7 \\
160/136.3
\end{bmatrix}
\]

From this we find:

\[\bar{I}_1 = 2.385/24.45^\circ \ A\]
\[\bar{I}_2 = 5.26/89.7^\circ \ A\]

If we want to express these currents in the time domain we have

\[i_1(t) = 2.385 \cos (\omega t - 24.45) \ A\]
\[i_2(t) = 5.26 \cos (\omega t - 89.7) \ A\]

We would need to know if we wanted to plot the above equations as time functions.
Example 13.3

We make this a 3 mesh problem and write the equations by inspection. Given the circuit in Figure 13.4.
Assume \( w = 377 \text{ rad/sec} \).

![Circuit Diagram]

**Figure 13.4: Circuit for Example 13.2.**

The equations of this circuit are of the form:

\[
\begin{bmatrix}
Z_{11} & Z_{12} & Z_{13} \\
Z_{21} & Z_{22} & Z_{23} \\
Z_{31} & Z_{32} & Z_{33}
\end{bmatrix}
\begin{bmatrix}
\vec{I}_1 \\
\vec{I}_2 \\
\vec{I}_3
\end{bmatrix}
=
\begin{bmatrix}
\Xi V_1 \\
\Xi V_2 \\
\Xi V_3
\end{bmatrix}
\quad \text{Eq. 13.6}
\]

\( \Xi V_i = \text{sum of the rises around mesh } i \)

For this case:

\( \Xi V_1 = 50 \angle 0^\circ - 100 \angle 30^\circ = 61.97 \angle -126.2^\circ \text{ V} \)

\( \Xi V_2 = 100 \angle 30^\circ \text{ V} \)

\( \Xi V_3 = 60 \angle -30^\circ \text{ V} \)
\[ Z_{12} = \text{impedance shared between mesh one and mesh two. There is a negative sign placed before the impedance if mesh currents } I_1 \text{ and } I_2 \text{ are in opposition. They usually are in opposition if one follows the establish method for mesh analysis.} \]

\[ Z_{12} = -[20] \Omega \]

\[ Z_{13} = \text{impedance shared between mesh one and mesh three. Again, there is a negative sign placed before the impedance if } E_1 \text{ and } E_3 \text{ are in opposition.} \]

\[ Z_{13} = 0 \]

\[ Z_{21} = Z_{12}. \text{ True for RLC linear circuits. May not be true when we have transformers.} \]

\[ Z_{21} = -20 \Omega \]
\[ Z_{22} = \text{impedance of mesh two, common to mesh current } I_2. \]

\[ Z_{22} = 20 - j15 + 20 + j20 \]

\[ Z_{22} = (40 + j15) \Omega \]

\[ Z_{23} = \text{impedance shared between mesh two and mesh three. Again, there is a negative sign placed before the impedance when } I_2 \text{ and } I_3 \text{ are opposition through this impedance.} \]

\[ Z_{23} = -([170] \Omega) \]

\[ Z_{31} = Z_{13} \text{ for the same reason as stated for } Z_{12}, Z_{21}. \]

\[ Z_{31} = 0 \]

\[ Z_{32} = Z_{23} \text{ for the reason stated above.} \]

\[ Z_{33} = \text{impedance of mesh three, common to mesh current } I_3. \]

\[ Z_{33} = j20 + 40 - j15 + j10 \]

\[ Z_{33} = (40 + j15) \Omega \]
Substituting into eq 13.6 gives

\[
\begin{bmatrix}
20 + j30 & -20 & 0 \\
-20 & 40 + j15 & -j20 \\
0 & -j20 & 40 + j15
\end{bmatrix}
\begin{bmatrix}
\bar{I}_1 \\
\bar{I}_2 \\
\bar{I}_3
\end{bmatrix}
= \begin{bmatrix}
621 - 120^\circ \\
100 + 130^\circ \\
601 - 30^\circ
\end{bmatrix}
\tag{eq 13.7}
\]

We notice, as expected, that the \( \mathbf{Z} \) matrix is symmetrical about the diagonal of the matrix.

The solutions for \( \bar{I}_1, \bar{I}_2, \) and \( \bar{I}_3 \) by hand calculator are:

\[
\bar{I}_1 = (-0.586 - j0.522)A = 0.785/ -138.3^\circ \ A
\]
\[
\bar{I}_2 = (2.02 + j1.11)A = 2.30/ 28.8^\circ \ A
\]
\[
\bar{I}_3 = (0.739 - j0.018)A = 0.74/ -1.37^\circ \ A
\]

The corresponding time domain equations are: (steady state)

\[
I_1(t) = 0.785 \cos (377t - 138.3^\circ) \ A
\]
\[
I_2(t) = 2.30 \cos (377t + 28^\circ) \ A
\]
\[
I_3(t) = 0.74 \cos (377t - 1.37^\circ) \ A
\]
A MATLAB program for solving Eq. 13.7 is given below. This type of solution, as opposed to a hand calculator solution, would only be necessary when the number of meshes was greater than five. L13_ex13_3.m

% A simple program for solving for mesh currents
% in a three mesh AC circuit. Program: L13_ex13_3.m
% Written by W. Green: October 3, 2002, office PC

Z = [20+j*30 -20 0; -20 40+j*5 -j*20; 0 -j*20 40+j*15];

V = [-35.27-j*48.54; 86.6+j*50; 51.96-j*30];

% I cannot find a way to express polar form directly in MATLAB
% so I have converted the voltages to rectangular form. If you
% know how to express polar form, please let me know.

I = inv(Z)*V

% I = [I1; I2; I3]

L13_ex13_3

I =

-0.5269 - 0.5377i
2.0431 + 1.0990i
0.7464 - 0.0083i

>
Statements are added to the previous program in order to plot the currents. The modified program is given below with the output on the following page.

% A simple program for solving for mesh currents
% in a three mesh AC circuit. Program: L13_ex13_3.m
% Written by W. Green: October 3, 2002, office PC

Z = [20+j*30 -20 0; -20 40+j*5 -j*20; 0 -j*20 40+j*15];

V = [-35.27-j*48.54; 86.6+j*50; 51.96-j*30];

% I cannot find a way to express polar form directly in MATLAB
% so I have converted the voltages to rectangular form. If you
% know how to express polar form, please let me know.

I = inv(Z)*V

% I = [I1; I2; I3]

% To plot the three currents

t = 0:.0001:.03;

i1 = 0.785*cos(377*t -(138.3/180)*pi);
i2 = 2.3*cos(377*t +(28/180)*pi);
i3 = 0.74*cos(377*t -(1.37/180)*pi);

plot(t,i1,'-',t,i2,'*',t,i3, '--')
ylabel('Current Responses, Amps')
xlabel('time (sec)')
title('Mesh Currents for Example 13.3')
grid
gtext( ' __________  i1' )
gtext( '* * * * * * * * *  i2' )
gtext( ' - - - - - - - - -  i3' )
% A simple program for solving for mesh currents
% in a three mesh AC circuit. Program: L13_ex13_2.m
% Written by W. Green: October 3, 2002, office PC

Z = [20+j*30  -20  0;  -20  40+j*5  -j*20;  0  -j*20  40+j*15];
V = [-35.27-j*48.54; 86.6+j*50; 51.96-j*30];

% I cannot find a way to express polar form directly in MATLAB
% so I have converted the voltages to rectangular form. If you
% know how to express polar form, please let me know.

I = inv(Z)*V

% I = [I1; I2; I3]

% To plot the three currents

t = 0:.0001:.03;

i1 = 0.785*cos(377*t-(138.3/180)*pi);
i2 = 2.3*cos(377*t+(28/180)*pi);
i3 = 0.74*cos(377*t-(1.37/180)*pi);

plot(t,i1,'-',t,i2,'*-',t,i3,'--')
ylabel('Current Responses, Amps')
xlabel('time (sec)')
title('Mesh Currents for Example 13.2')
grid

gtext('-------------- i1')
gtext('** ** ** ** ** * * * * i2')
gtext('----- ----- ----- ----- i3')