# Lesson 13

## Solving AC Circuit Problems

This lesson is dedicated to solving AC circuit problems.

### Example 13.1

For the circuit shown in Figure 13.1, solve for $\bar{V}_R$, $\bar{V}_C$ and $\bar{V}_L$. Prepare the phasor diagram for the above phasor voltages. Finally, write the time domain equation for the steady state current in the circuit.

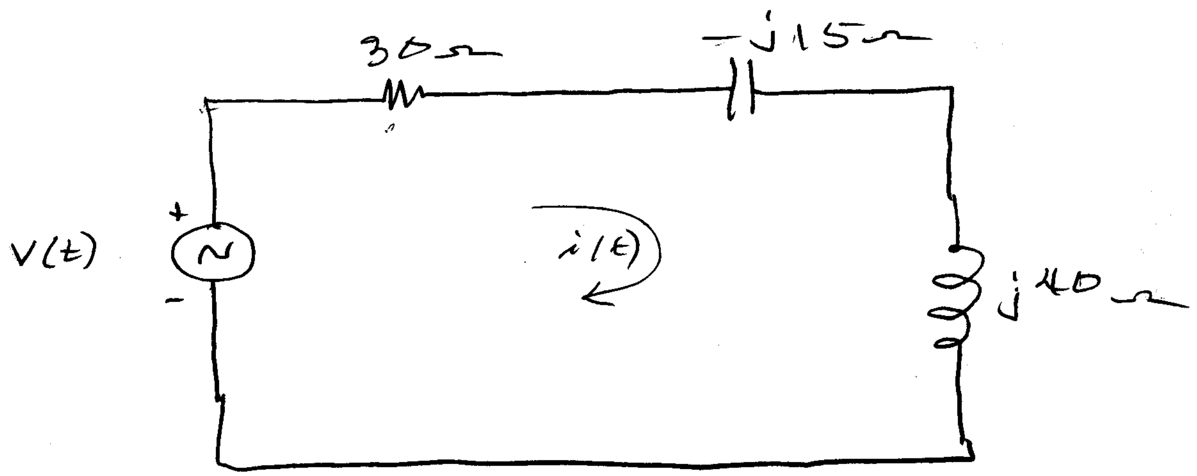



Figure 13.1: Circuit for Example 13.1

becomes

$v(t) = 150 \sin(377t + 120^\circ)$ V

$v(t) = 150 \cos(377t + 30^\circ)$ V.

$$\bar{V} = 150 \angle 30^\circ \text{ V}$$

We can write;

$$\bar{I} = \frac{150 \angle 30^\circ}{30 - j15 + j40} = 3.84 \angle -9.8^\circ \text{ A} \qquad \text{Eq 13.1}$$

Now calculate the voltages:

$$\bar{V}_R = 30 \times \bar{I} = 115.2 \angle -9.8^\circ \text{ V}$$

$$\bar{V}_c = (15 \angle -90)(3.84 \angle -9.8^\circ) = 57.6 \angle -99.8^\circ \text{ V}$$

$$\bar{V}_L = (40 \angle 90)(3.84)\angle -9.8) = 153.6 \angle 80.2 \text{ V}$$

By KVL;

$$\bar{V} = \bar{V}_R + \bar{V}_c + \bar{V}_L \qquad \text{Eq 13.2}$$

Putting in the above values gives,

$$150 \angle 30^\circ = 115.2 \angle -9.8 + 57.6 \angle -99.8 + 153.6 \angle 80.2$$

According to my calculator, the right hand side of the above equation is; $149.957 \angle 30^\circ$ V.

This is a good check since we have rounded numbers along the way.

What the phasor diagram shows is that $\bar{V}_R + \bar{V}_c + \bar{V}_L$ will be equal to $\bar{V}$. We now show this, roughly.

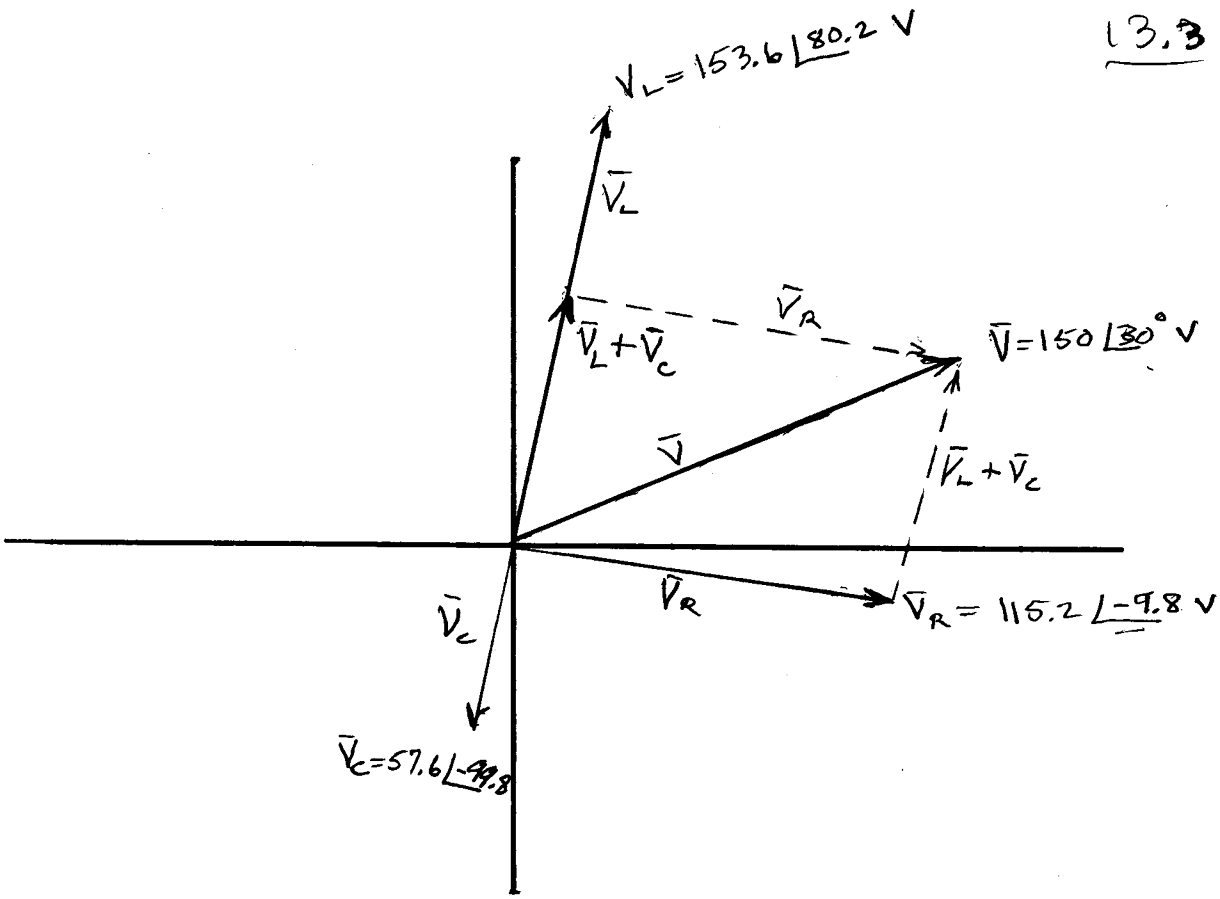



Figure 13.2: Phasor diagram of voltages for Example 13.1

We note the following:

- $V_L$ and $V_C$ are in opposite directions. This will always happen ($+j$ for inductor, $-j$ for capacitor)
- The graphical sum of $\bar{V}_R + \bar{V}_L + \bar{V}_C = \bar{V}$. Verification of KVL.

Finally, in the time domain, $i(t)$ is

$$i(t) = 3.84\cos(377t - 9.8)\ \text{A} \qquad \text{Eq } 13.3$$

## Example 13.2

Find the mesh currents $\bar{I}_1$ and $\bar{I}_2$ in the following circuit.

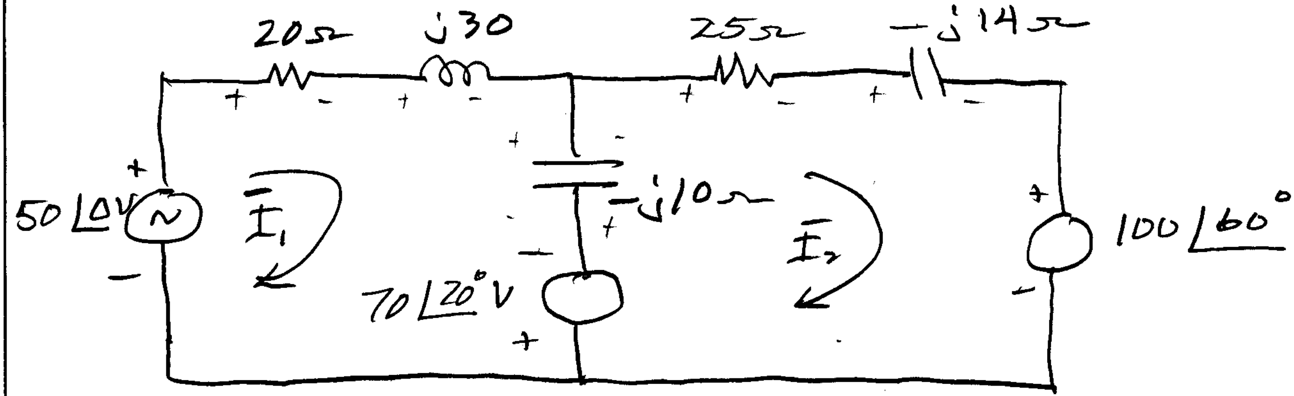



Figure 13.3: Circuit for Example 13.2.

This is about a tough as an AC circuit problems get to be.

We write the following two mesh equations, using $\sum$ vises = 0.

$$50\angle 0 - (20+j30)\bar{I}_1 - (-j10)(\bar{I}_1 - \bar{I}_2) + 70\angle 20^\circ = 0$$

Simplifying gives.

$$\boxed{(20+j20)\bar{I}_1 + (j10)\bar{I}_2 = 50\angle 0 + 70\angle 20} \quad \text{Eq 13.4}$$

Around mesh 2 we have

$$-70\angle 20 - (\bar{I}_2 - \bar{I}_1)(-j10) - (25-j14)\bar{I}_2 - 100\angle 60 = 0$$

or

$$\boxed{+j10\,\bar{I}_1 + (25-j10-j14)\bar{I}_2 = -70\angle 20 - 100\angle 60} \quad \text{Eq 13.5}$$

Eq's 13.4 & 13.5 can be put in the form

$$(20+j20)\bar{I}_1 + j10\,\bar{I}_2 = 118.23\angle 11.7^\circ$$
$$j10\,\bar{I}_1 + (25-j24)\bar{I}_2 = 160\angle -136.3^\circ$$

OR

$$\begin{bmatrix} 20+j20 & j10 \\ j10 & 25-j24 \end{bmatrix}\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = \begin{bmatrix} 118.23\angle 11.7 \\ 160\angle -136.3 \end{bmatrix}$$

From this we find;

$$\bar{I}_1 = 2.385\angle -24.45^\circ \text{ A}$$
$$\bar{I}_2 = 5.26\angle -89.7^\circ \text{ A}$$

If we want to express these currents in the time domain we have

$$i_1(t) = 2.385\cos(\omega t - 24.45) \text{ A}$$
$$i_2(t) = 5.26\cos(\omega t - 89.7) \text{ A}$$

We would need to know $\omega$ if we wanted to plot the above equations as time functions.

## Example 13.3-

We make this a 3 mesh problem and write the equations by inspection. Given the circuit in Figure 13.4. Assume $\omega = 377$ rad/sec.

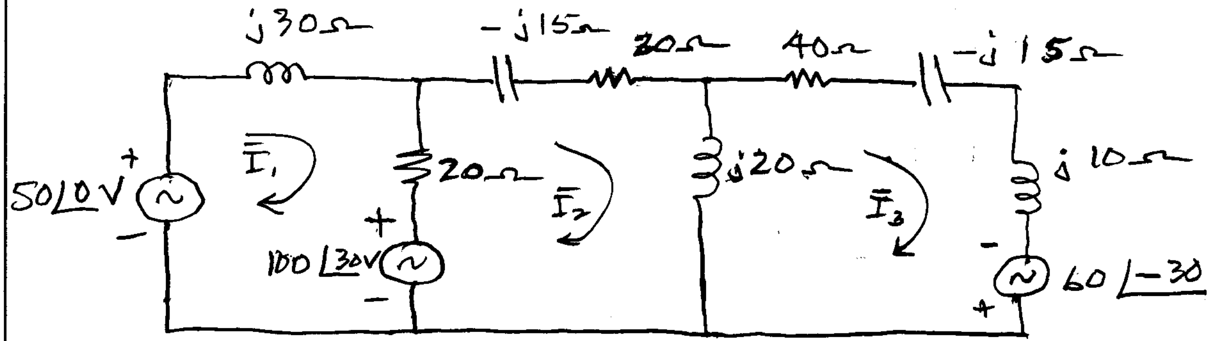



Figure 13.4: Circuit for Example 13.2.

The equations of this circuit are of the form;

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \end{bmatrix} = \begin{bmatrix} \Sigma V_1 \\ \Sigma V_2 \\ \Sigma V_3 \end{bmatrix} \qquad \text{Eq 13.6}$$

7500

$1 \times 10^{-6} \times 2.5 \times 10^{3}$

$\Sigma V_1$ = sum of the rises around mesh 1

$\Sigma V_2$ = sum of the rises around mesh 2

$\Sigma V_3$ = sum of the rises around mesh 3

For this case:

$$\Sigma V_1 = 50\angle 0 - 100\angle 30 = 61.97\angle -126.2 \text{ V}$$

$$\Sigma V_2 = 100\angle 30 \text{ V}$$

$$\Sigma V_3 = 60\angle -30^\circ \text{ V}$$

$Z_{11}$ = impedance of mesh one, common to mesh current $\bar{I}_1$

$\boxed{Z_{11} = 20 + j30}\ \Omega$

$Z_{12}$ = impedance shared between mesh one and mesh two. There is a negative sign placed before the impedance if mesh currents $\bar{I}_1$ and $\bar{I}_2$ are in opposition. They usually are in opposition if one follows the establish method for mesh analysis.

$\boxed{Z_{12} = -[20]\ \Omega}$

$Z_{13}$ = impedance shared between mesh one and mesh three. Again, there is a negative sign place before the impedance if $\bar{I}_1$ and $\bar{I}_3$ are in opposition.

$\boxed{Z_{13} = 0}$

$Z_{21} = Z_{12}$. True for RLC linear circuits. May not be true when we have transformers.

$\boxed{Z_{21} = -20\ \Omega}$

$Z_{22}$ = impedance of mesh two, common to mesh current $\bar{I}_2$.

$$Z_{22} = 20 - j15 + 20 + j20$$

$$\boxed{Z_{22} = (40 + j5)\,\Omega}$$

$Z_{23}$ = impedance shared between mesh two and mesh three. Again, there is a negative sign placed before the impedance when $\bar{I}_2$ and $\bar{I}_3$ are opposition through this impedance.

$$Z_{23} = -[j20]\,\Omega$$

$Z_{31} = Z_{13}$ for the same reason as stated for $Z_{12}$, $Z_{21}$.

$$\boxed{Z_{31} = 0}$$

$Z_{32} = Z_{23}$ for the reason stated above.

$Z_{33}$ = impedance of mesh three, common to mesh current $\bar{I}_3$.

$$Z_{33} = j20 + 40 - j15 + j10$$

$$Z_{33} = (40 + j15)\,\Omega$$

Substituting into eq 13.6 gives

$$\underbrace{\begin{bmatrix} 20+j30 & -20 & 0 \\ -20 & 40+j5 & -j20 \\ 0 & -j20 & 40+j15 \end{bmatrix}}_{\mathbf{Z}} \underbrace{\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \end{bmatrix}}_{\mathbf{I}} = \underbrace{\begin{bmatrix} 62\angle{-126^\circ} \\ 100\angle{30^\circ} \\ 60\angle{-30^\circ} \end{bmatrix}}_{\mathbf{V}} \qquad \text{Eq 13.7}$$

We notice, as expected, that the $\mathbf{Z}$ matrix is symmetrical about the diagonal of the matrix.

The solutions for $\bar{I}_1$, $\bar{I}_2$, and $\bar{I}_3$ by hand calculator are:

$$\bar{I}_1 = (-0.586 - j0.522)\,A = 0.785\angle{-138.3^\circ}\,A$$

$$\bar{I}_2 = (2.02 + j1.11)\,A = 2.30\angle{28.8^\circ}\,A$$

$$\bar{I}_3 = (0.739 - j0.018)\,A = 0.74\angle{-1.37^\circ}\,A$$

The corresponding time domain equations are: (steady state)

$$i_1(t) = 0.785\cos(377t - 138.3^\circ)\,A$$

$$i_2(t) = 2.3\cos(377t + 28^\circ)\,A$$

$$i_3(t) = 0.74\cos(377t - 1.37^\circ)\,A$$

A MATLAB program for solving Eq 13.7 is given below. This type of solution, as opposed to a hand calculator solution, would only be necessary when the number of meshes was greater than five. L13_ex13_3.m

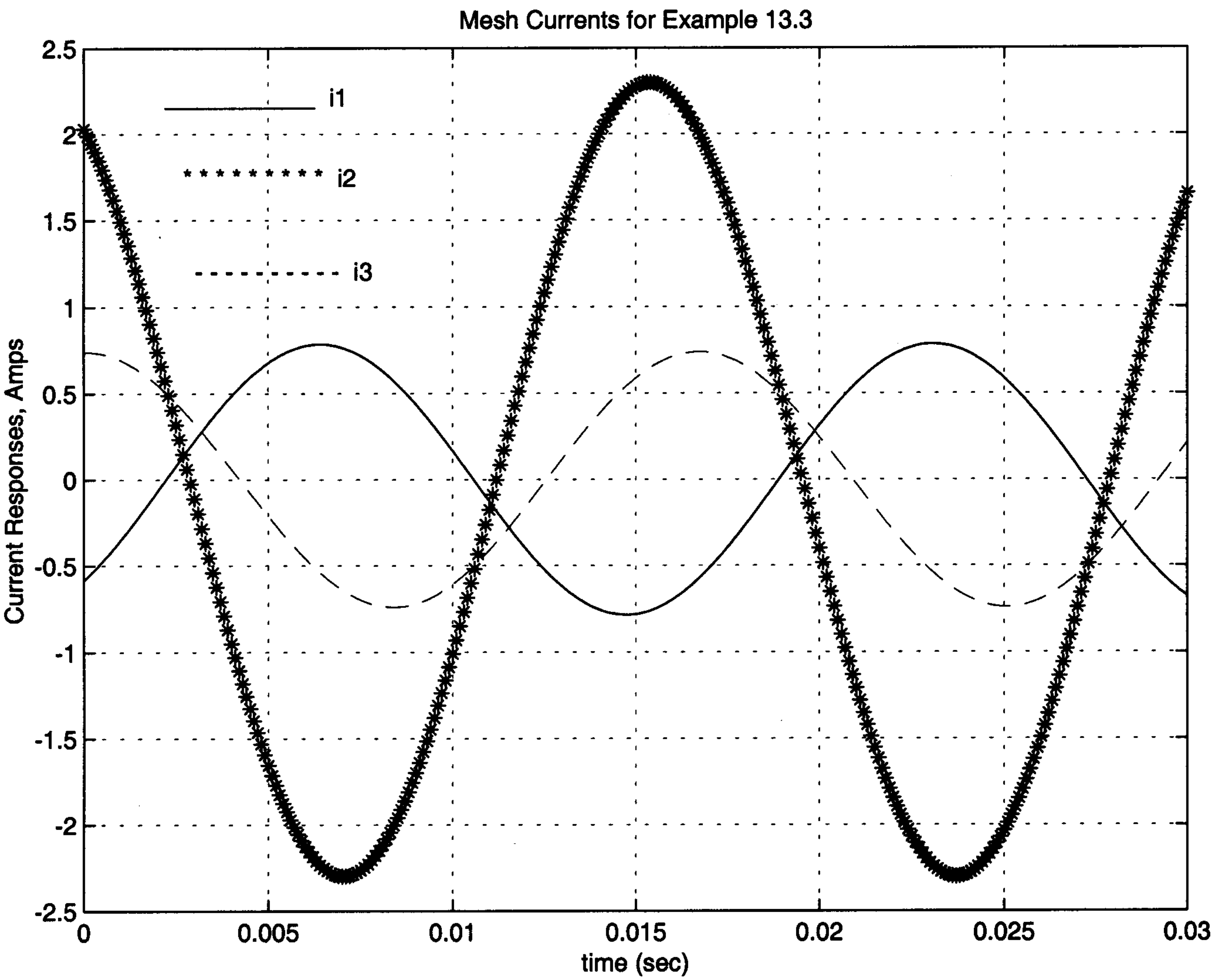
```
%  A simple program for solving for mesh currents
%  in a three mesh AC circuit.  Program: L13_ex13_3.m
%  Written by W. Green:  October 3,2002, office PC

Z = [20+j*30  -20  0;  -20  40+j*5  -j*20;  0  -j*20  40+j*15];

V = [-35.27-j*48.54; 86.6+j*50; 51.96-j*30];

%  I cannot find a way to express polar form directly in MATLAB
%  so I have converted the voltages to rectangular form.  If you
%  know how to express polar form, please let me know.

I = inv(Z)*V

% I = [I1; I2; I3]
```

```
» L13_ex13_3

I =

  -0.5269 - 0.5377i
   2.0431 + 1.0990i
   0.7464 - 0.0083i

»
```

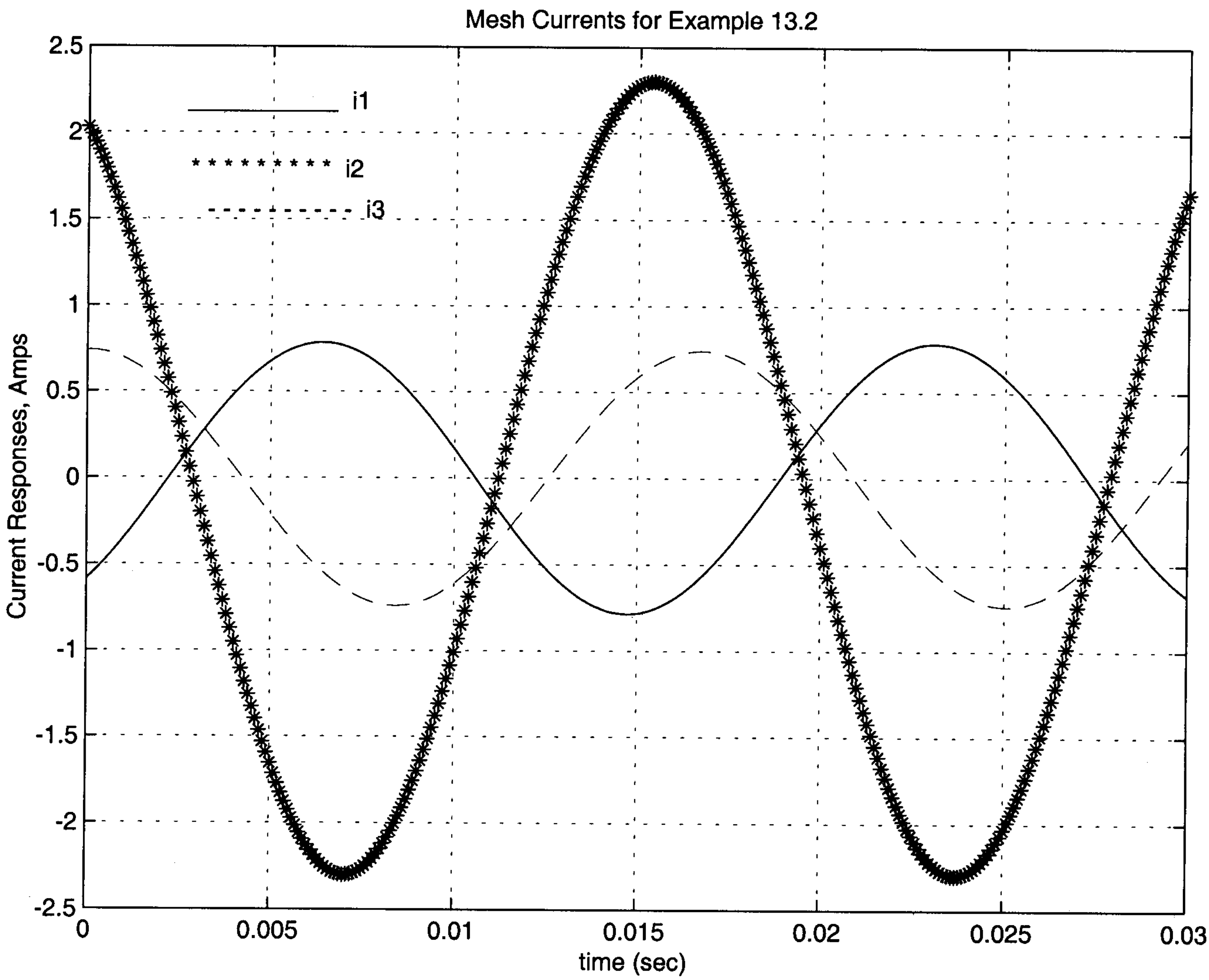
Statements are added to the previous program in order to plot the currents. The modified program is given below with the output on the following page.

```
%  A simple program for solving for mesh currents
%  in a three mesh AC circuit.  Program: L13_ex13_3.m
%  Written by W. Green:  October 3,2002, office PC

Z = [20+j*30  -20  0;  -20  40+j*5  -j*20;  0  -j*20  40+j*15];

V = [-35.27-j*48.54; 86.6+j*50; 51.96-j*30];

%  I cannot find a way to express polar form directly in MATLAB
%  so I have converted the voltages to rectangular form.  If you
%  know how to express polar form, please let me know.

I = inv(Z)*V

% I = [I1; I2; I3]

% To plot the three currents

t = 0:.0001:.03;

i1 = 0.785*cos(377*t-(138.3/180)*pi);
i2 = 2.3*cos(377*t+(28/180)*pi);
i3 = 0.74*cos(377*t-(1.37/180)*pi);

plot(t,i1,'-',t,i2,'*',t,i3,'--')
ylabel('Current Responses, Amps')
xlabel('time (sec)')
title('Mesh Currents for Example 13.3')
grid
gtext(' ___________  i1')
gtext('* * * * * * * * *  i2')
gtext(' - - - - - - - - - -  i3')
```

13.12


Mesh Currents for Example 13.3
i1
i2
i3
Current Responses, Amps
time (sec)
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
0.005
0.01
0.015
0.02
0.025
0.03

```
%  A simple program for solving for mesh currents
%  in a three mesh AC circuit.  Program: L13_ex13_2.m
%  Written by W. Green:  October 3,2002, office PC

Z = [20+j*30  -20  0;  -20  40+j*5  -j*20;  0  -j*20  40+j*15];

V = [-35.27-j*48.54; 86.6+j*50; 51.96-j*30];

%  I cannot find a way to express polar form directly in MATLAB
%  so I have converted the voltages to rectangular form.  If you
%  know how to express polar form, please let me know.

I = inv(Z)*V

% I = [I1; I2; I3]

% To plot the three currents

t = 0:.0001:.03;

i1 = 0.785*cos(377*t-(138.3/180)*pi);
i2 = 2.3*cos(377*t+(28/180)*pi);
i3 = 0.74*cos(377*t-(1.37/180)*pi);

plot(t,i1,'-',t,i2,'*',t,i3,'--')
ylabel('Current Responses, Amps')
xlabel('time (sec)')
title('Mesh Currents for Example 13.2')
grid
gtext(' ___________  i1')
gtext('* * * * * * * * *  i2')
gtext(' - - - - - - - - - - -  i3')
```


Mesh Currents for Example 13.2
i1
i2
i3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
Current Responses, Amps
0
0.005
0.01
0.015
0.02
0.025
0.03
time (sec)