# Lesson 16

# Power For AC Circuits Including Power Factor And Complex Power

## Notes for ECE 301

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# POWER IN AC CIRCUITS

## LESSON

We start by asking the question, what is power as related to electric circuits? One recalls from the study of dc circuits that

$$p(t) = \frac{\partial W(t)}{\partial t} \qquad \text{Eq. 16.1}$$

which is a mathematical statement that power is the rate at which energy is supplied to electrical devices. As we will see, one can speak of instantaneous power, average power, reactive power and complex power.

An important fundamental about power supplied to AC circuits is that we are mostly concerned about AVERAGE power, as opposed to instantaneous power. Furthermore, we will see that the inductor and capacitor absorb zero average power. Of the three passive elements, resistor, capacitor, inductor, only the resistor absorbs average power. However, inductors and capacitors can store energy.

We consider a hypothetical case of voltage from a source supplying current to a load:

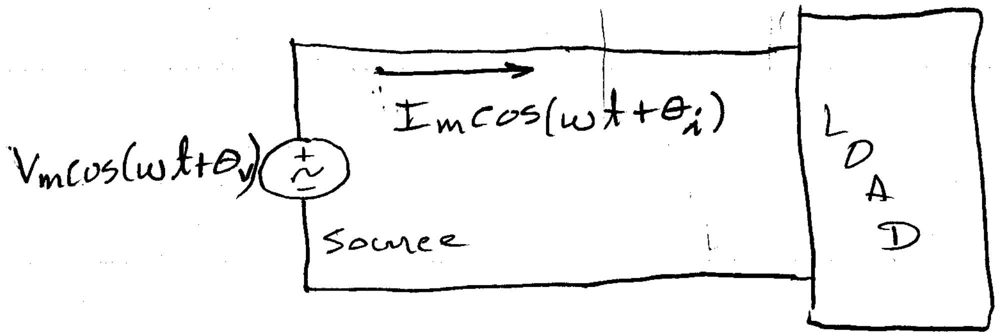



Figure 16.1: A Hypothetical AC circuit.

The instantaneous power supplied to the load is,

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t+\theta_v)\cos(\omega t+\theta_i) \qquad \text{Eq } 16.2$$

The power we are interested in is the average power, which can be obtained from

$$P_{AVG} = \frac{1}{T}\int_0^T p(t)\,dt \qquad \text{Eq } 16.3$$

Before attempting the integration, we use a well known trig identity to write the product of the two cosine functions in a form that can be readily integrated.

One recalls;

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right] \quad \text{Eq 16.4}$$

Applying this to equation 16.2, yields

$$V_m I_m \cos(\omega t+\theta_v)\cos(\omega t+\theta_i) = \frac{1}{2} V_m I_m \left[\cos(2\omega t+\theta_v+\theta_I) + \cos(\theta_v-\theta_i)\right] \quad \text{Eq 16.5}$$

We see two components in the above expression. One term, the first, is time varying, at double the frequency of either $v(t)$ or $i(t)$. The second term is constant. An example will be presented later that illustrates this with a computer solution (plot)

Now, the average power can be expressed as;

$$P_{AVG} = \frac{1}{T}\left[\int_0^T \frac{V_m I_m}{2}\cos(2\omega t+\theta_v+\theta_i)\,dt + \int_0^T \frac{V_m I_m}{2}\cos(\theta_v-\theta_i)\,dt\right] \quad \text{Eq 16.6}$$

One readily realizes that the integral of the first term is zero. We are essentially integrating over two cycles of the cosine $(2\omega t)$ and we know the net area under the curve is zero. This leaves,

$$P_{AVG} = P = \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\cos(\theta_V - \theta_i)\,dt \qquad \text{Eq } 16.7$$

so

$$\boxed{P = \frac{1}{2} V_m I_m \cos(\theta_V - \theta_i)} \qquad \text{Eq } 16.8$$

Note that $P = P_{AVG}$. We drop the subscript and adapt the convention of using upper case P for $P_{AVG}$ and reserve $p(t)$ for instantaneous power.

We recall from earlier study of AC circuits that

$$V_{Rms} = \frac{V_m}{\sqrt{2}}, \quad I_{Rms} = \frac{I_m}{\sqrt{2}} \qquad \text{Eq } 16.9$$

Then we can write

$$P = \frac{(\sqrt{2}V_{rms})(\sqrt{2}I_{Rms})}{2}\cos(\theta_V - \theta_i)$$

OR

$$\boxed{P = (V_{rms})(I_{Rms})\cos(\theta_V - \theta_i)} \qquad \text{Eq } 16.10$$

Equations 16.8 and 16.10 are equally valid for finding average power. Notice that we are dealing with $\cos(\theta_v - \theta_i)$ which is the cosine of the difference between $\theta_v$ and $\theta_i$. It makes no difference if either or both of $\theta_v$ and $\theta_i$ are positive or negative. We do need to keep up with whether we are using maximum or RMS values. More often than not, people in the power field use

$$\boxed{P = VI\cos(\theta)} \qquad \text{Eq 16.11}$$

for talking about (and determining) average power. One must keep in mind, in such case, that V and I are RMS values and $\theta$ is the difference in the phase between the voltage and the current.

We will now consider some examples.

## Example 16.1

The voltage supplied to a certain load is $v(t) = (115\sqrt{2})\cos(377t + 10^\circ)$. The current leaving the voltage source is $i(t) = (20\sqrt{20})\cos(377t - 40^\circ)$.

(a) Write a MATLAB program that will calculate and plot
$$p(t) = v(t)\,i(t)$$

(b) From the computer plot, estimate P (average).

(c) Hand calculate P (average) and compare to what you get in part (b).

MATLAB program shown below:

```
%  Program to illustrate instantaneous and average power
%  Program on home lap top.  pwrinst.m  Nov, 2002, w. green

t = 0:.0003:.06;

% the signal has a frequency of 60 so the period is 1/60 and we
% will cover 4 cycles or 4/60 or about 0.06.  .0003 is about (1/50)
% of one period and will provide a smooth plotted curve.

v = (115*sqrt(2))*cos(377*t+10*(pi/180));
i = (20*sqrt(2))*cos(377*t-40*(pi/180));

p = v.*i;

AVG = (115*20)*cos(50*(pi/180));
plot(t,p, t, v,'--',t,AVG,'x')
grid
title('Example 16.1 Showing instantaneous power, average power, and voltage')
ylabel('p(t)')
xlabel('t (sec)')
gtext('instantaneous power')
gtext('average power')
gtext('voltage')
```

The average power can be calculated from

$$P = \frac{1}{2} V_m I_m \cos(10-(-40))$$

OR

$$P = V_{RMS} I_{RMS} \cos(10-(-40))$$

OR

$$P = 115 \times 20 \cos(50)$$

$$P = 1478 \text{ WATTS}$$

Figure 16.2 shows the instantaneous power, average power and the voltage. The average is slightly less than 1500W, so one assumes it is 1478W. Note that the frequency of the instantaneous power is twice that of the voltage waveform, as expected.

## Example 16.2

Given the circuit shown below.

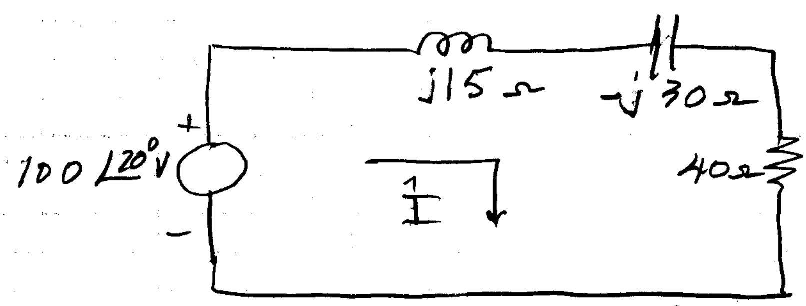



Figure 16.3: Circuit for Example 16.2.

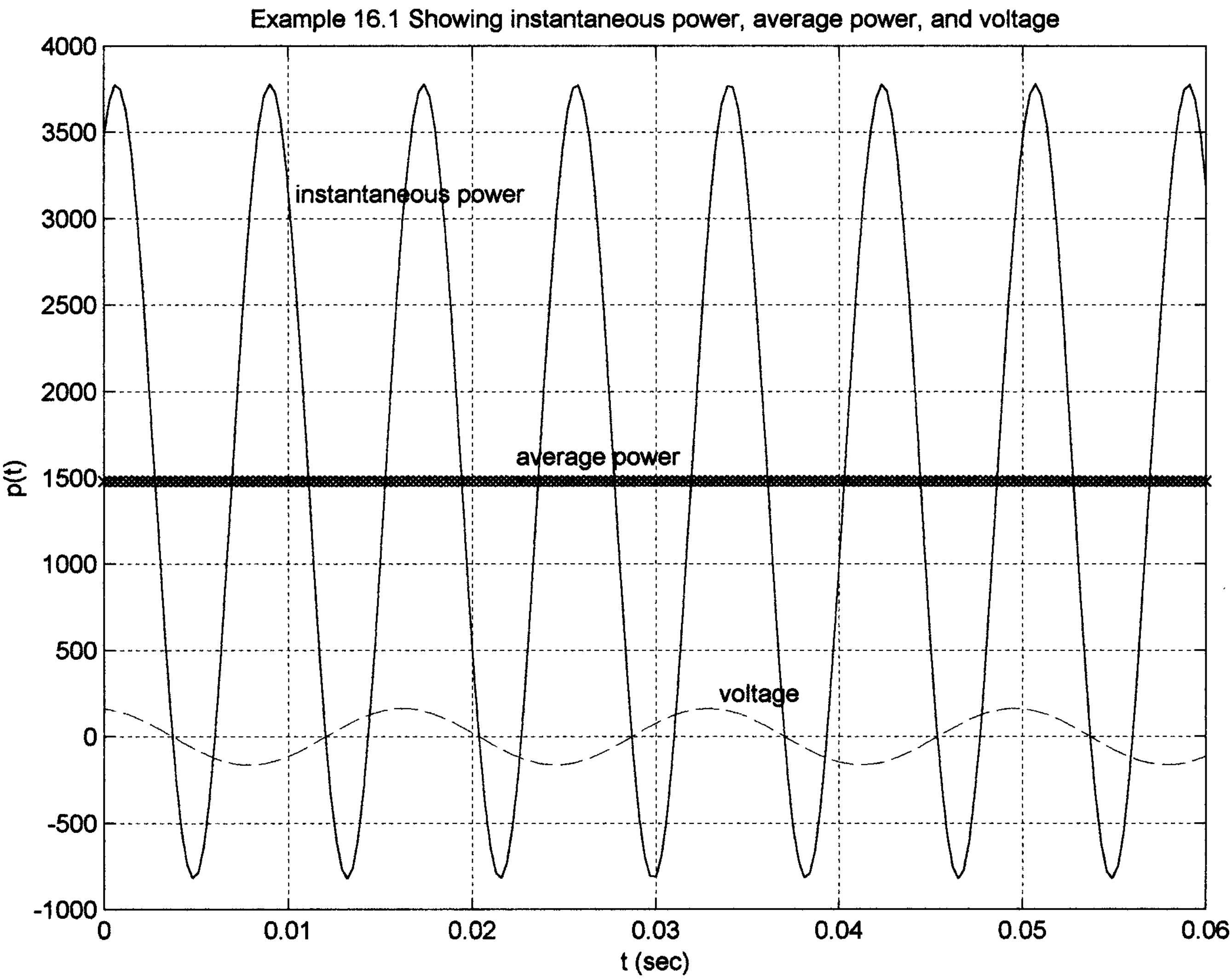



Figure 16.2: MATLAB output showing instantaneous power, average power, and voltage.

In the circuit of Figure 16.2, determine the following:

(a) The phasor current, $\hat{I}$.

(b) The average power delivered from the source.

(c) The average power absorbed by the inductor.

(d) The average power absorbed by the capacitor.

(e) The average power absorbed by the resistor.

(a) We find $\hat{I}$ as follows;

$$\hat{I} = \frac{\hat{V}_s}{Z} = \frac{100\angle 20}{j15 - j30 + 40}$$

$$\boxed{\hat{I} = \frac{100\angle 20}{40 - j15} = 2.34\angle 40.6^\circ \text{ A}}$$

(b) $$P_{source} = \frac{1}{2}|\hat{V}||\hat{I}|\cos(\theta_v - \theta_i)$$

$$P_{source} = \frac{1}{2} 100 \times 2.34 \cos(20 - 40.6)$$

$$\boxed{P_{source} = 109.5 \text{ W}}$$

(c) At this point we do not know how the average power is distributed among the inductor, the capacitor and the resistor but one thing we do know:
The average power absorbed by the inductor, the capacitor and the resistor will sum-up to be the average power supplied by the source, which is 190.5 W.

Now, the average power absorbed by the inductor will be

$$P_L = \frac{1}{2}|\hat{V}_L||\hat{I}_L|\cos(\theta_{V_L} - \theta_{i_L})$$

We know

$$\hat{I}_L = 2.34\angle 40.6 \text{ A}$$

(A series circuit)

and

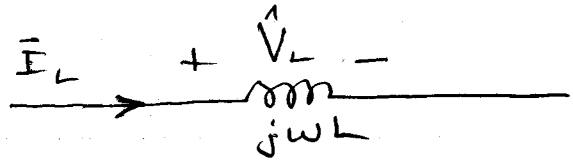
$$\hat{V}_L = \hat{I}_L \times Z_L = \hat{I}_L \times j15$$

$$\hat{V}_L = \hat{I}_L \times 15\angle 90 = |\hat{I}_L| \times 15 \angle \theta_I + 90$$

$$P_L = \frac{1}{2}|\hat{I}_L||I_L \times 15|\cos(\theta_I + 90 - \theta_I)$$

$$P_L = 0$$

This may surprise you at first but some thought on the matter gives that the angle of the phasor voltage for any pure inductor is always $+90^\circ$ more than the phasor current through the inductor. That is,




compare

$$\hat{V}_L = j\omega \bar{I}_L = \omega |\hat{I}_L| \angle \theta_I + 90^\circ = \omega |\hat{I}_L| \angle \theta_V$$

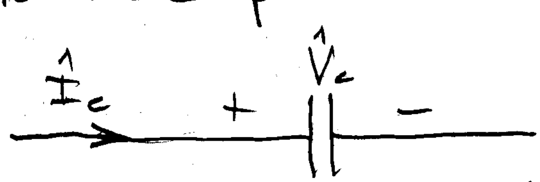
$$P_L = \frac{1}{2} |\hat{V}_L| |\hat{I}_L| \cos(\theta_V - \theta_I)$$

so

$$P_L = \frac{1}{2} |\hat{V}_L| |\hat{I}_L| \cos(\theta_I + 90 - \theta_I)$$

$$P_L = \frac{1}{2} |\hat{V}_L| |\hat{I}_L| \cos 90^\circ = 0$$

(d) The same reasoning applies to a capacitor:




$$-jX_c = -\frac{j}{\omega C}$$

$$\hat{V}_c = \hat{I}_c \times \frac{1}{wC}\angle -90 = |\hat{I}_c| \frac{1}{wC} \angle \theta_I - 90^\circ$$

$$\hat{V}_c \doteq |\hat{I}| \frac{1}{wC} \angle \theta_V$$

so

$$\theta_V = \theta_I - 90^\circ$$

and we see

$$\boxed{\theta_V - \theta_I = -90^\circ}$$

We see that the angle of the voltage is 90° behind (lags) the angle of the current (ICE)

Finally,

$$P_c = \frac{1}{2}|\hat{V}_c||\hat{V}_I| \cos(\theta_V - \theta_I)$$

$$\hat{P}_c = \frac{1}{2}|\hat{V}_c||\hat{V}_I| \cos(-90^\circ) = 0$$

Important Summary;

No average power (zero, nothing, none) is absorbed by either an inductor or capacitor in an AC circuit.

(e) The average power absorbed by the resistor is

$$P_R = \frac{1}{2} |\hat{V}_R||\hat{I}_R| \cos(\theta_{V_R} - \theta_I)$$

Now $\hat{I}_R = \hat{I}$ since we have a series circuit.

$$\hat{V}_R = \hat{I}_R R = 2.34\angle 40.6 \times 40$$

$$\hat{V}_R = 93.6\angle 40.6, \ V$$

$$\hat{I}_R = 2.34\angle 40.6, \ A$$

Remember that the voltage across a resistor has the same phase as the current through the resistor.

Now
$$P_R = \frac{1}{2} |\hat{V}_R||\hat{I}_R| \cos(40.6 - 40.6)$$

$$P_R = \frac{93.6 \times 2.34}{2}$$

$$P_R = 109.5 \ W$$

This is exactly the power delivered by the source, as calculated in part (b).

<u>end of example 16.2</u>