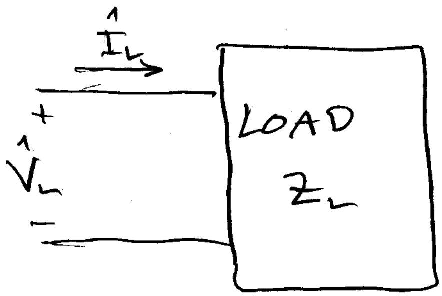
There are other expressions for calculating average power other than

$$\boxed{P = \frac{\hat{V}\hat{I}}{2}\cos(\theta_V - \theta_I)} \quad \text{Eq 16.12}$$

and they will be shown below. However, I think they are redundant since the above equation can always be used. Nevertheless, here we go:



For any load we can write

$$\hat{V}_L = \hat{I}_L Z_L = |\hat{V}_L| \angle \theta_V$$

Then $P_L = \frac{1}{2}|\hat{V}_L||\hat{I}_L|\cos(\theta_V - \theta_L)$

$$P_L = \frac{1}{2}|\hat{I}_L|^2|Z_L|\cos(\theta_V - \theta_I)$$

or just

$$\boxed{P_L = \frac{1}{2}|\hat{I}_L|^2|Z_L|\cos(\theta)} \quad \text{Eq 16.13}$$

Also we can write,

$$\hat{I}_L = \frac{\hat{V}_L}{Z_L} = |\hat{I}_L| \angle \theta_I = \frac{|\hat{V}_L|}{|Z_L|} \angle \theta_I$$

then

$$P_L = \frac{1}{2} |\hat{V}_L| |\hat{I}_L| \cos(\theta_V - \theta_I)$$

$$P_L = \frac{1}{2} \frac{|\hat{V}_L| |\hat{V}_L|}{|Z_L|} \cos(\theta_V - \theta_I)$$

OR

$$\boxed{P_L = \frac{|V_L|^2}{2|Z_L|} \cos(\theta_V - \theta_I)} \qquad \text{Eq 16.14}$$

Let's look at another example;

<u>Example 16.3</u>

Consider the circuit shown below.
(see Example 7.2, text)

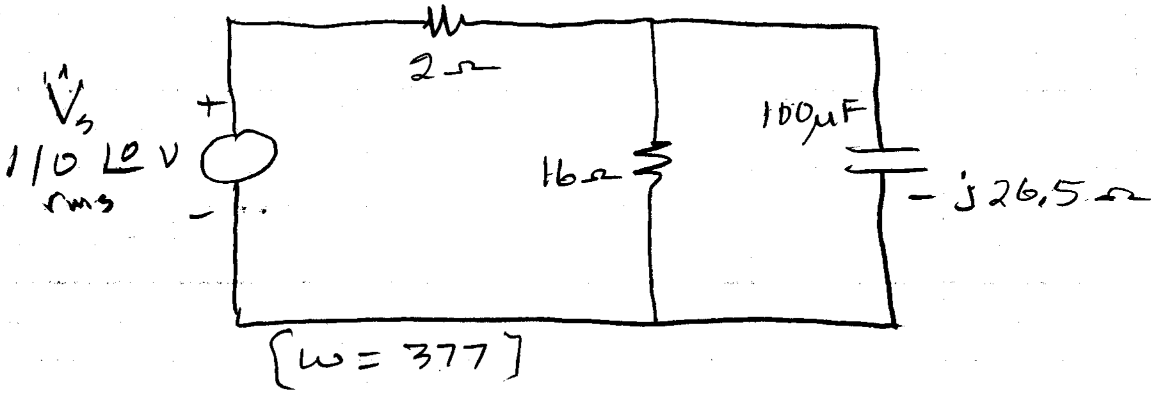


Figure 16.4 ; Circuit for example 16.3.

Required:

(a) Find the average power supplied by the source

(b) Find the average power absorbed by the 2Ω resistor.

(c) Find the average power absorbed by the 16Ω resistor.

Solution

First find $\hat{I}_{source} = \hat{I}_s$.

$$\hat{I}_s = \frac{\hat{V}_s}{Z_T}$$

$$Z_T = 2 + 16 \| (-j26.5)$$

$$= 2 + \frac{16 \times (26.5\angle{-90})}{16 - j26.5} = 2 + 13.7\angle{-31.12^\circ}$$

$$Z_T = 15.44\angle{-27.3^\circ}\ \Omega$$

$$\hat{I}_s = \frac{110\angle 0}{15.44\angle{-27.3}} = 7.12\angle{27.3^\circ}\ A_{rms}$$

$$P_s = \underset{(rms)}{\hat{V}_s I_s} \cos(\theta_{Vs} - \theta_{Is}) = 110 \times 7.12 \cos(27.3)\ W$$

$$\underline{P_s = 696\ W}$$

(b) $P_2 = |\hat{I}_s|^2 \times 2 = (7.12)^2 \times 2$

$P_2 = 101.4$ W

(c) P_{16} :

$$\hat{I}_{16} = \frac{\hat{I}_s \times (-j26.5)}{16 - j26.5}$$

$$\hat{I}_{16} = \frac{(7.12 \angle 27.3)(26.5 \angle -90)}{16 - j26.5}$$

$$\hat{I}_{16} = 6.1 \angle -3.82 \text{ A}_{rms}$$

$$P_{16} = (6.1)^2 \times 16 = 595.4 \text{ W}$$

$$P_2 + P_{16} = 101.4 + 595.4 = \underline{696.8 \text{ W}}$$

Compared to $P_s = \underline{696}$ W

Good enough.

Drill Problem (D.1)

Determine the average power absorbed by the 10 Ω resistor, Figure 16.5. $\hat{V}_s$ is a Peak voltage. Answer $P_{10} = 512$ W

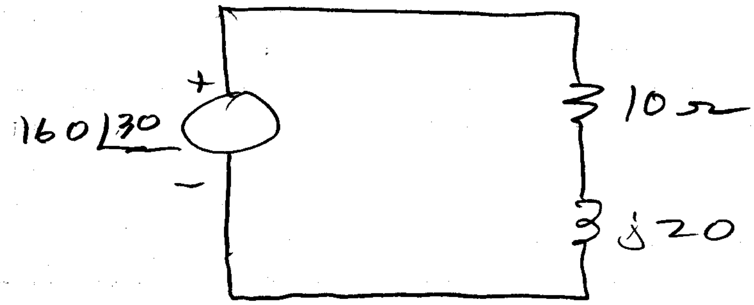
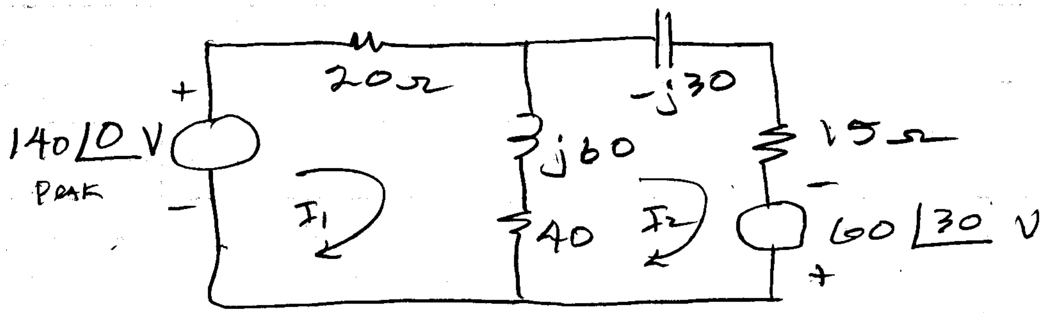


Figure 16.5: Circuit for D.1.

Drill Problem: D.2

Given the following circuit.



Find the power supplied to the 10Ω resistor. Ans. $P_{15} = 273.5\text{ W}$, $\hat{I}_1 = 3.32\angle 41.9$, A
$I_2 = 4.27\angle 57.7°$ A

Power Factor

The power factor associated with a load, or a source delivering power to a system, is an important concept. Power companies want the customer to have a pure resistive load. We will see why this is true later. On the other hand, loads cannot, in general, be all resistive. Most load will have an inductive component — loads such as motor, electric ranges, for example.

We recall

$$P = \frac{1}{2} \hat{V} \hat{I} \cos(\theta_V - \theta_I)$$

The power factor is defined as

$$P.f. = \cos(\theta_V - \theta_I)$$

If the load is entirely resistive then P.f. = 1. If the load is entirely reactive (pure inductor or capacitor) The power factor is 0. Thus, for a mixed load of $R + jX$ the power factor falls between 0 and 1. Generally;

$$\boxed{0 \leq P.f. \leq 1}$$

Complex Power

To provide a better understanding of total power (not just average power) in an AC circuit, we introduce complex power. Complex power is made up of Average power and the power associated with reactance. We write this as

$$\hat{S} = V_{rms} I_{rms}^{*}, \text{ (VA)} \qquad \text{Eq } 16.15$$

Units of complex power is volt-amps (VA)

Consider the following:

$$\hat{S} = \left(|\hat{V}|_{rms} \angle \theta_V\right)\left[\left(|\hat{I}|_{rms} \angle \theta_I\right)\right]^{*} = V_{rms} \angle \theta_V \times I_{rms} \angle -\theta_I$$

$$\hat{S} = |\hat{V}|_{rms} |\hat{I}|_{rms} \angle \theta_V - \theta_I \qquad \text{Eq } 16.16$$

$$\hat{S} = |\hat{V}_{rms}||\hat{I}_{rms}| e^{j(\theta_V - \theta_I)} \qquad \text{Eq } 16.17$$

$$\hat{S} = |\hat{V}_{rms}||\hat{I}_{rms}| \left[\cos(\theta_V - \theta_i) + j\sin(\theta_V - \theta_I)\right]$$

$$\hat{S} = P + jQ \qquad \text{Eq } 16.18$$

$$P = |\hat{V}_{rms}||\hat{I}_{rms}| \cos(\theta_V - \theta_I) \qquad \text{Eq } 16.19$$

$$Q = |\hat{V}_{rms}||\hat{I}_{rms}| \sin(\theta_V - \theta_I) \qquad \text{Eq } 16.20$$

As we already know,

$$P = |\hat{V}_{rms}||\hat{I}_{rms}|\cos(\theta_V - \theta_I)$$

is the average power. We call this the <u>real power.</u>

$$Q = |\hat{V}_{rms}||\hat{I}_{rms}|\sin(\theta_V - \theta_I) \qquad \text{Eq } 16.21$$

we call this the <u>reactive power</u> or <u>quature power</u>. Q has units of VARs (<u>V</u>olts <u>A</u>mps <u>R</u>eactive).

We note that

$$P = \text{Re}[\hat{S}] = \text{Re}[\hat{V}\hat{I}^*] \qquad \text{Eq } 16.21$$

$$S = \text{Im}[\hat{S}] = \text{Im}[\hat{V}\hat{I}^*] \qquad \text{Eq } 16.22$$

Now it is true that

$$\hat{Z} = \text{Re}\,\hat{Z} + j\,\text{Im}\,\hat{Z} = \frac{\hat{V}_{rms}}{\hat{I}_{rms}} = \frac{|V_{rms}|}{|I_{rms}|} e^{j(\theta_V - \theta_I)}$$

or

$$\text{Re}\,\hat{Z} + j\,\text{Im}\,\hat{Z} = |Z|\left[\cos(\theta_V - \theta_I) + j\sin(\theta_V - \theta_I)\right) \qquad \text{Eq } 16.23$$

Set the real on the left = real on the right and the j part on the left equal to the j part on the right gives us the following:

$$\cos(\theta_V - \theta_I) = \text{p.f.} = \frac{\operatorname{Re} \hat{Z}}{|\hat{Z}|} \qquad \text{Eq } 16.23$$

$$\sin(\theta_V - \theta_I) = \frac{\operatorname{Im} \hat{Z}}{|\hat{Z}|} \qquad \text{Eq } 16.24$$

Using Eq 16.23 and 16.24 in Equation 16.21 and 16.22 gives

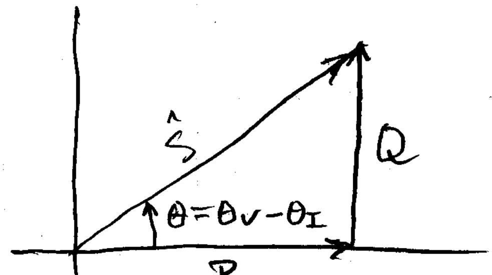
$$P = |I_{rms}|^2 \operatorname{Re} \hat{Z} \qquad \text{Eq } 16.25$$

$$Q = |I_{rms}|^2 \operatorname{Im} \hat{Z} \qquad \text{Eq } 16.26$$

Also,

$$\hat{S} = |I_{rms}|^2 \hat{Z} \qquad \text{Eq } 16.27$$

We can draw a triangle that perhaps helps to see the nature of $\hat{S}$. Such a triangle is shown in Figure 16.5 below.



Assuming $\theta_V - \theta_I > 0$

Figure 16.5: Triangle showing complex power.

One often speaks of apparent power and complex power. Apparent power is simply the magnitude of complex power.

$$\underset{\text{complex power}}{\hat{S}} = \hat{V}_{rms} \hat{I}_{rms} = \frac{\hat{V}_p \hat{I}_p}{2}, \text{ VA} \quad \text{Eq } 16,28$$

$$\underset{\text{apparent power}}{|\hat{S}|} = |V_{rms} I_{rms}| = \frac{|\hat{V}_p I_p|}{2}, \text{ VA} \quad \text{Eq } 16,29$$

Another word about power factor;

When stating the power factor, one must always state whether it is a leading or lagging power factor. This is relatively easy to keep up with.

The power factor is <u>positive</u> if the current lead the voltage. It follows that it is negative if the voltage leads the current. We say the voltage leads the current if the phase angle of the voltage is greater than the phase angle of the current.

Illustrations of power factor.

Illustration 16.1

The voltage across a load is $140\angle 30^\circ$ V, rms; The current for the load is $22.5\angle -10^\circ$ A, rms. Find the p.f.

Now, the phase of the voltage is 30° and the phase of the current is -10°. Therefore the p.f. is lagging.
(voltage leads the current)

$$p.f. = \cos(\theta_V - \theta_I) = \cos(30-(-10))$$

$$p.f. = 0.766 \text{ lagging}$$

Illustration 2

The impedance of a certain load is

$$\hat{Z} = 200\angle -40^\circ \ \Omega$$

Find the load power factor.

Now, $\hat{Z} = \dfrac{\hat{V}}{\hat{I}} = \dfrac{|\hat{V}|\angle\theta_V}{|\hat{I}|\angle\theta_I} = \dfrac{|\hat{V}|}{|\hat{I}|}\angle\theta_V - \theta_I$

If the phase angle of the impedance is positive, the p.f. is lagging (current is lagging the voltage.

If the phase angle of the impedance is negative, the p.f. is leading (current leads voltage)

In this illustration, p.f. is leading.

Illustration 3

A certain load has a complex power of $\hat{S} = 150 + j90$ VARS.
Determine the power factor.

We know,

$$P = |V_{rms}||I_{rms}| \cos(\theta_V - \theta_I)$$

and we know

$$|S| = |V_{rms}||I_{rms}|$$

Thus

$$P = |S| \cos(\theta_V - \theta_I)$$

$$\cos(\theta_V - \theta_I) = \text{p.f.} = \frac{P}{|S|} \qquad \text{Eq } 16.30$$

This will tell us the value of the p.f, but would not determine if it were leading or lagging.

We see from Eq 16.22, page 16.22 that

$$\hat{S} = |I_{rms}|^2 \hat{Z}$$

So whatever angle $\hat{Z}$, $\hat{S}$ will have the same angle. We know that if $\hat{Z}$ has a positive phase angle, the p.f. is lagging. So in this case the p.f. is $\cos(\theta_V - \theta_I) = \dfrac{150}{\sqrt{150^2 + 90^2}} = 0.6575$ lagging

A summary of power equations

Real Power $= P = |\hat{V}_{rms}\hat{I}_{rms}|\cos(\theta_V - \theta_I)$, WATTS

$$P = |\hat{S}|\cos(\theta_V - \theta_I) = \mathrm{Re}[\hat{S}], W$$

units of watts, W.

Reactive Power $= Q = |\hat{V}_{rms}||\hat{I}_{rms}|\sin(\theta_V - \theta_I)$

$$Q = |\hat{S}|\sin(\theta_V - \theta_I) = \mathrm{Im}[\hat{S}]$$

units of Volt amps reactive (VARs)

Apparent Power $= |\hat{S}| = |V_{rms}||I_{rms}|$

$$= \sqrt{P^2 + Q^2}$$

Complex Power: $= \hat{S} = P + jQ$ (VA)

$$= \hat{V}_{rms}(\hat{I})^*_{rms}$$

$$= \frac{1}{2}\hat{V}(\hat{I})^*$$

Power Factor: $= \dfrac{P}{|\hat{S}|} = \cos(\theta_V - \theta_I)$

Leading P.f. if $\angle\hat{Z}$ is negative

Lagging P.f. if $\angle\hat{Z}$ is positive