

Example 16.4

16.27

Consider the following configuration.

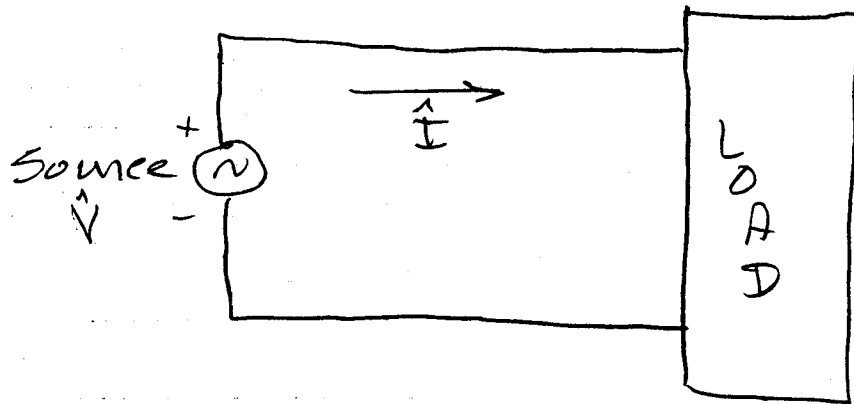


Figure 16.6: General configuration for Example 16.4.

The source voltage is

$$v(t) = 160 \cos(100\pi - 30^\circ) \text{ V}$$

and the resulting current to a series load ($R + jX$) is

$$i(t) = 6 \cos(100\pi + 20^\circ) \text{ A}$$

Find the ^(a) apparent power and the ^(b) power factor of the load. Determine the values of the series elements.

$$\text{Apparent Power} = |\vec{S}| = |\hat{V}_{\text{rms}}| |\hat{I}_{\text{rms}}|$$

$$|\vec{S}| = \frac{|\hat{V}| |\hat{I}|}{2}$$

$$(a) \quad \underline{|\vec{S}|} = 160 \times 6 = \underline{484 \text{ VA}}$$

Example 16.4 cont.

(b)

$$\text{power factor} = \text{P.F.} = \cos(\theta_v - \theta_i)$$

$$= \cos(-30 - 20)$$

$$\text{P.f.} = 0.6428 \text{ lagging}$$

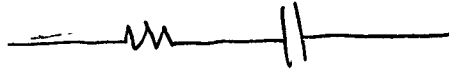
lagging since $\angle I$ is greater $\angle V$.

(c)

$$\underline{Z} = \frac{160 \angle -30}{6 \angle 20} = 26.7 \angle -50$$

$$\underline{Z} = 26.7 \angle -50 = R + jX$$

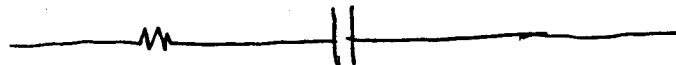
$$\underline{Z} = 17.16 - j20.45$$



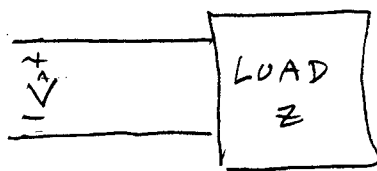
$$17.16 \quad -j20.45 = \frac{-j}{\omega C}$$

$$\omega = 100\pi$$

$$C = \frac{1}{(20.45)(100)(\pi)} = 0.0156 \mu\text{F}$$



$$R = 17.16 \Omega \quad C = 0.0156 \mu\text{F}$$

Example 16.5

The load Z is $200 \angle -30^\circ \Omega$.

The load voltage is $\hat{V} = 115 \angle 0^\circ \text{ V}_{\text{rms}}$.

Determine the power factor.

$$\text{P.f.} = \cos(\theta_V - \theta_I) = \cos(\angle Z)$$

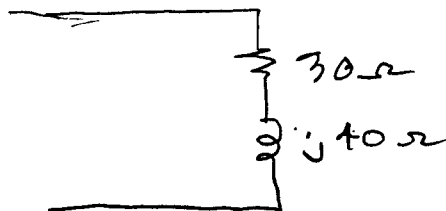
Recall:

$$\hat{Z} = \frac{\hat{V}}{\hat{I}} = \frac{|\hat{V}|}{|\hat{I}|} \angle \theta_V - \theta_I$$

$$\text{P.f.} = \cos(-30) = 0.866 \text{ leading}$$

Example 16.6

Given the following load



Determine the power factor.

$$\text{Since } \hat{Z} = 30 + j40 = 50 \angle 53.1^\circ \Omega$$

$$\text{P.f.} = \cos(53.13) = 0.6 \text{ lagging}$$

Example 16.7

A certain load has $\hat{V}_{rms} = 110 \angle 85^\circ$ V and $\hat{I}_{rms} = 0.4 \angle 15^\circ$ A. Determine

- the complex and apparent powers,
- the real and reactive powers,
- the power factor and load impedance.

$$(a) \quad \hat{S} = \hat{V}_{rms} (\hat{I}_{rms})^* = (110 \angle 85^\circ) (0.4 \angle 15^\circ)^*$$

$$\hat{S} = 44 \angle 70^\circ \text{ VA}$$

$$|\hat{S}| = \text{apparent power} = 44 \text{ VA}$$

$$(b) \quad P = \text{Re}[\hat{S}] = 44 \cos 70$$

$$P = 15.05 \text{ WATTS}$$

$$Q = \text{Im}[\hat{S}] = 44 \sin 70$$

$$Q = 41.35 \text{ VARs}$$

$$(c) \quad \text{P.F.} = \cos(\angle \hat{S}) = \cos(70) = 0.342 \text{ lag.}$$

$$\angle \hat{S} = \angle Z = (\theta_V - \theta_I); \quad \hat{S} = \hat{V}_r \hat{I}_r^* = Z \hat{I}_r \hat{I}_r^*$$

$$\hat{I}_r \hat{I}_r^* = |\hat{I}|^2$$

$$-\hat{S} = |\hat{I}_{rms}|^2 Z$$

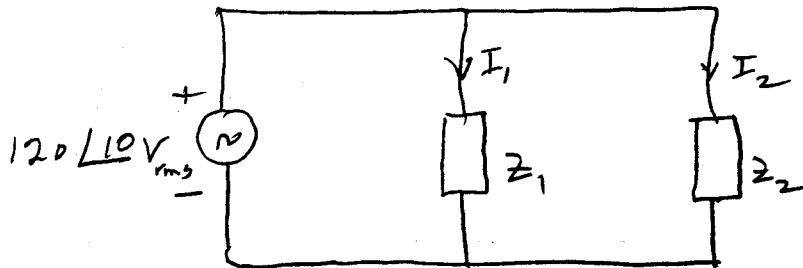
$$Z = -\hat{S} / |\hat{I}_{rms}|^2$$

$$\hat{Z} = \frac{V_L}{I_L} = \frac{110 \angle 85^\circ}{0.4 \angle 15^\circ} = 275 \angle 70^\circ \Omega$$

Example 16.8

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You are given the following circuit.



$$Z_1 = 60 \angle -30^\circ \Omega, \quad Z_2 = 40 \angle 45^\circ \Omega$$

Determine the following.

- (a) the apparent power
- (b) the real power
- (c) the reactive power
- (d) the power factor.

$$(a) \quad \hat{I}_1 = \frac{120 \angle 10^\circ}{Z_1} = \frac{120 \angle 10^\circ}{60 \angle -30^\circ} = 2 \angle 40^\circ \text{ A}$$

$$\hat{I}_2 = \frac{120 \angle 10^\circ}{Z_2} = \frac{120 \angle 10^\circ}{40 \angle 45^\circ} = 3 \angle -35^\circ \text{ A}$$

$$\hat{I} = \hat{I}_1 + \hat{I}_2 = 2 \angle 40^\circ + 3 \angle -35^\circ = 4.01 \angle -6.2^\circ$$

$$\hat{S} = V \hat{I}^* = (120 \angle 10^\circ) (4.01 \angle 6.2^\circ) = 481.2 \angle 16.2^\circ$$

$$S = (462.1 + j134.3) \text{ VA}$$

$$|S| = \text{apparent power} = \underline{481.2}$$

(cont)

Ex. 16.8
(a)

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You can also say,

$$\hat{S}_1 = \hat{V} \times I_1^* = \frac{\hat{V} \hat{V}^*}{Z_1^*} = \frac{V^2}{Z_1^*} = \frac{120^2}{60 \angle 30}$$

$$\hat{S}_1 = 240 \angle -30 \text{ VA}$$

$$S_2 = \frac{V^2}{Z_2^*} = \frac{120^2}{40 \angle -45} = 360 \angle 45$$

$$\hat{S} = \hat{S}_1 + \hat{S}_2 = 240 \angle -30 + 360 \angle 45$$

$$\hat{S} = 481.6 \angle 16.2 \text{ VA}$$

$|\hat{S}| = \text{apparent power} = 481.6 \text{ As before}$

(b) Real power = $P = \text{Re}[\hat{S}] = 462 \text{ W}$

(c) reactive power = $Q = \text{Im}[\hat{S}] = 134.3 \text{ VARs}$

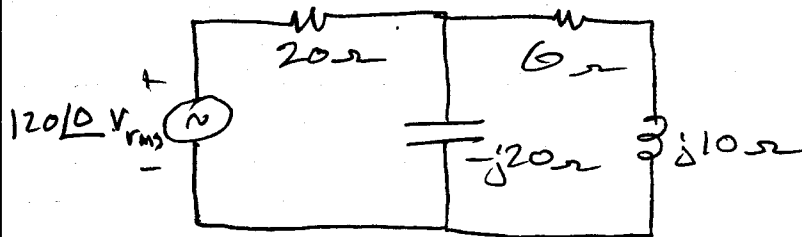
(d) power factor

$$\text{P.f.} = \frac{P}{|\hat{S}|} = \frac{462}{481.6} = 0.96 \text{ lagging}$$

(lagging because \underline{S} is positive, which means \underline{Z} is positive, which means voltage lags current, thus lagging P.f.)

Example 16.9

You are given the following circuit.



- What is the average power (real) supplied by the source.
- What is the complex power supplied by the source.
- What is the p.f. with respect to the source.
- What is the real power ~~delivered~~ to the 6Ω resistor?
- What is the real power ~~delivered~~ to the 20Ω resistor.
- Verify that the real power supplied by the source = real power absorbed by the 20Ω resistor + real power absorbed by the 6Ω resistor.

(a)

$$Z_{\text{LOAD}} = 20 + \frac{(-j20)(6+j10)}{6+j10-j20}$$

$$Z_{\text{LOAD}} = 38.8 \angle 14^\circ \Omega$$

$$\hat{I}_s = \frac{\hat{V}_s}{Z_{\text{LOAD}}} = \frac{120 \angle 0}{38.8 \angle 14} = 3.09 \angle -14 \text{ A}$$

$$P_s = |\hat{V}_s \hat{I}_s| \cos(\theta_{V_s} - \theta_{I_s}) = 120 \times 3.09 \cos(0 - (-14))$$

$$P_s = 359.8 \text{ W}$$

$$(b) \hat{S} = \hat{V}_s \hat{I}_s^* = 120 \times 3.09 \angle 14 = 370.8 \angle 14 \text{ VA}$$

$$(c) \text{p.f.} = \frac{P}{|S|} = \frac{359.8}{370.8} = 0.9703 \text{ lagging} = \cos 14^\circ$$

$$(d) \hat{I}_6 = \frac{3.09 \angle -14 (-j20)}{-j20 + 6 + j10} = 5.3 \angle -45^\circ \text{ A}$$

$$P_6 = (|\hat{I}_6|)^2 \times 6 = (5.3)^2 \times 6 = 168.54 \text{ W}$$

$$(e) P_{20} = (|\hat{I}_s|)^2 \times 20 = (3.09)^2 \times 20 = 190.96 \text{ W}$$

(f) Checking for $P_s = P_{20} + P_6$

$$359.8 \stackrel{?}{=} 190.96 + 168.54 = 359.5 \text{ W}$$

Very close, error in numerical round-off.

QED

Example 16.10

Given the network shown in the following diagram, Determine the input voltage \hat{V}_s .

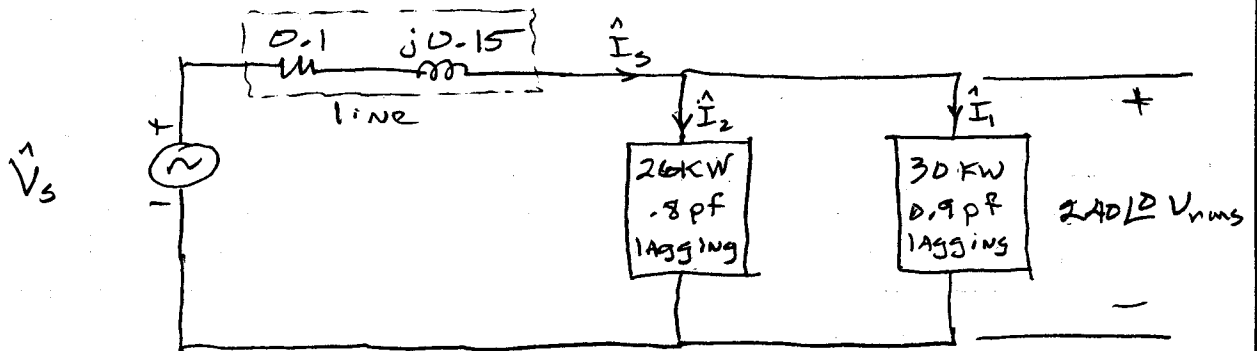
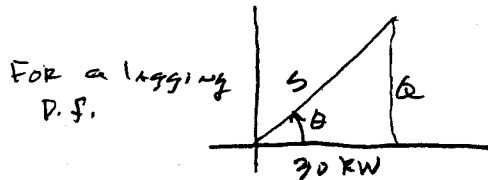


Figure 16.7: System for Example 16.10.

Approach

Find \hat{I}_1 & \hat{I}_2 ; $\hat{I}_s = \hat{I}_1 + \hat{I}_2$
 then $\hat{V}_s = (0.1 + j0.15)\hat{I}_s + 240 \angle 0^\circ \text{ V}_{rms}$

For \hat{I}_1 : Lagging P.f.



$$\cos \theta_1 = 0.9 \quad \theta_1 = \cos^{-1} 0.9 = 24.8$$

$$\frac{P_1}{|S_1|} = \cos \theta$$

$$|S_1| = \frac{P_1}{\cos \theta} = \frac{30 \text{ kW}}{0.9} = 33.33$$

$$\hat{S}_1 = 33.33 \angle 24.8$$

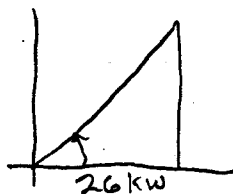
$$\hat{S} = \hat{V} \hat{I}^*$$

$$\hat{I}_1^* = \frac{33.33 \angle 24.8}{240} \text{ k} = 138.9 \angle 24.8 \text{ A}_{rms}$$

$$\hat{I}_1 = 138.9 \angle -24.8 \text{ A}_{rms}$$

For \hat{I}_2 :

Lagging P.f.



$$\cos \theta_2 = 0.8 \quad \theta_2 = 36.87$$

$$|S_2| = \frac{P_2}{\cos \theta} = \frac{26 \text{ kW}}{0.8} = 32.5 \text{ KVA}$$

$$\hat{S}_2 = 32.5 \angle 36.87^\circ \text{ KVA} = \hat{V} \hat{I}_2^*$$

$$\hat{I}_2^* = \frac{32.5 \angle 36.87}{240 \angle 0} = 135.4 \angle 36.87$$

$$\hat{I}_2 = 135.4 \angle -36.87 \text{ A}_{rms}$$

Ex 16.10 cont,

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$$\hat{I}_s = \hat{I}_1 + \hat{I}_2 = 138.9 \angle -24.8^\circ + 135.4 \angle -36.87^\circ$$

$$\hat{I}_s = 271.9 \angle -30.8^\circ \text{ A}_{\text{rms}}$$

$$\hat{V}_s = (0.1 + j0.15)(271.9 \angle -30.8^\circ) + 240 \angle 0^\circ$$

$$\hat{V}_s = 284.9 \angle 4.2^\circ \text{ V}_{\text{rms}} \quad \text{Ans.}$$

Note: If one or both of the loads are given as KVA you must take this into account on the complex power triangle.