Example 16.4

Consider the following configuration.

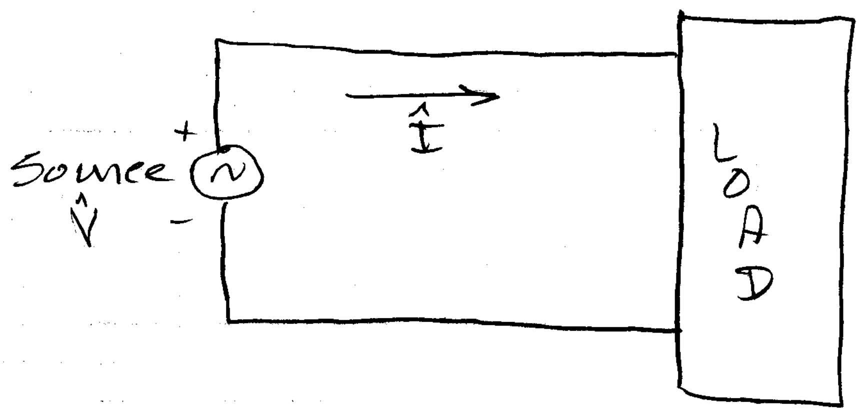


Figure 16.6: General configuration for Example 16.4.

The source voltage is

$$v(t) = 160\cos(100\pi - 30^\circ)\ V$$

and the resulting current to a series load $(R + jX)$ is

$$i(t) = 6\cos(100\pi + 20^\circ)\ A$$

Find the (a) apparent power and the (b) power factor of the load. Determine the (c) values of the series elements.

$$\text{Apparent Power} = |\hat{S}| = |\hat{V}_{rms}|\,|\hat{I}_{rms}|$$

$$|\hat{S}| = \frac{|\hat{V}|\,|\hat{I}|}{2}$$

(a) $|\hat{S}| = \dfrac{160 \times 6}{2} = 480\ VA$

Example 16.4 cont.

(b)

power factor $= p.f. = \cos(\theta_V - \theta_I)$

$= \cos(-30-20)$

$p.f. = 0.6428$ leading

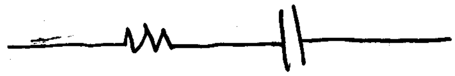
lagging since $\angle I$ is greater $\angle V$.

(c)

$$Z = \frac{160\angle -30}{6\angle 20} = 26.7\angle -50$$

$$Z = 26.7\angle -50 = R + jX$$

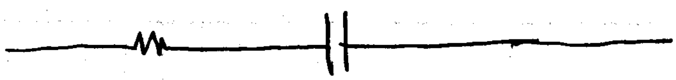
$$Z = 17.16 - j20.45$$



17.16 $\quad -j20.45 = \frac{-j}{\omega C}$

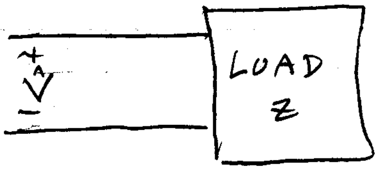
$\omega = 100\pi$

$$C = \frac{1}{(20.45)(100)(\pi)} = .0156\mu F$$



$R = 17.16\Omega \quad C = 0.0156\mu F$

Example 16.5



The load Z is $200\angle{-30°}\ \Omega$.

The load voltage is $\hat{V} = 115\angle{0}\ V_{rms}$.

Determine the power factor.

$$P.f. = \cos(\theta_V - \theta_I) = \cos(\angle Z)$$

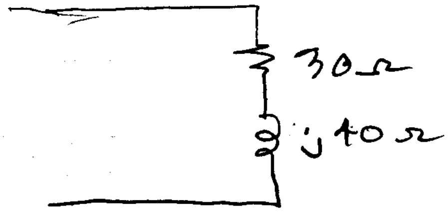
recall;

$$\hat{Z} = \frac{\hat{V}}{\hat{I}} = \frac{|\hat{V}|}{|\hat{I}|}\angle{\theta_V - \theta_I}$$

$$P.f. = \cos(-30) = 0.666 \text{ leading}$$

Example 16.6

Given the following load



Determine the power factor.

Since $\hat{Z} = 30 + j40 = 50\angle{53.1°}\ \Omega$

$$P.f. = \cos(53.13) = 0.6 \text{ lagging}$$

Example 16.7

A certain load has $\hat{V}_{rms} = 110\angle 85^\circ$ V and $\hat{I}_{rms} = 0.4\angle 15^\circ$ A. Determine

(a) the complex and apparent powers,
(b) the real and reactive powers,
(c) the power factor and load impedance.

(a) $\hat{S} = \hat{V}_{rms}(\hat{I})^*_{rms} = (110\angle 85)(0.4\angle 15)^*$

$\underline{\hat{S} = 44\angle 70^\circ \text{ VA}}$

$|\hat{S}|$ = apparent power = 44 VA

(b) $P = \text{Re}[\hat{S}] = 44\cos 70$

$P = 15.05$ WATTS

$Q = \text{Im}[\hat{S}] = 44\sin 70$

$Q = 41.35$ VARs

(c) $\text{P.f.} = \cos(\angle\hat{S}) = \cos(70) = 0.342$ lagging

$\angle\hat{S} = \angle Z = \angle(\theta_V - \theta_I)$; $\hat{S} = \hat{V}_r\hat{I}_r^* = Z\hat{I}_r\hat{I}^*$

$\hat{I}\hat{I}^* = |I|^2$

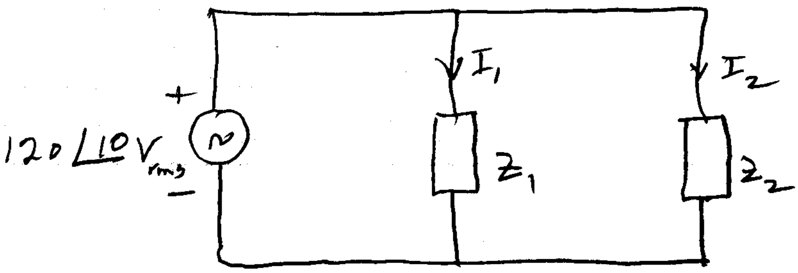
$\hat{S} = |I_{rms}|^2\hat{Z}$

$\angle S = \angle Z$

$\hat{Z} = \dfrac{V_L}{I_L} = \dfrac{110\angle 85}{0.4\angle 15} = 275\angle 70^\circ\ \Omega$

Example 16.8

You are given the following circuit.



$Z_1 = 60\angle -30^\circ\ \Omega, \quad Z_2 = 40\angle 45^\circ\ \Omega$

Determine the following.

(a) the apparent power
(b) the real power
(c) the reactive power
(d) the power factor.

(a)

$$\hat{I}_1 = \frac{120\angle 10}{Z_1} = \frac{120\angle 10}{60\angle -30} = 2\angle 40^\circ\ A$$

$$\hat{I}_2 = \frac{120\angle 10}{Z_2} = \frac{120\angle 10}{40\angle 45} = 3\angle -35^\circ A$$

$$\hat{I} = \hat{I}_1 + \hat{I}_2 = 2\angle 40 + 3\angle -35 = 4.01\angle -6.2\ A$$

$$\hat{S} = V I^* = (120\angle 10)(4.01\angle 6.2) = 481.2\angle 16.2\ VA$$

$$S = (462.1 + j\,134.3)\ VA$$

$|S|$ = apparent power = $\underline{\underline{481.2}}$ VA

(cont)

(a)

You can also say,

$$\hat{S}_1 = \hat{V} \times I_1^* = \frac{\hat{V}\hat{V}^*}{Z_1^*} = \frac{V^2}{Z_1^*} = \frac{120^2}{60\angle 30}$$

$$\hat{S}_1 = 240\angle{-30} \text{ VA}$$

$$S_2 = \frac{V^2}{Z_2^*} = \frac{120^2}{40\angle{-45}} = 360\angle 45$$

$$\hat{S} = \hat{S}_1 + \hat{S}_2 = 240\angle{-30} + 360\angle 45$$

$$\hat{S} = 481.6\angle 16.2 \text{ VA}$$

$|\hat{S}|$ = apparent power = 481.6VA, As before.

(b) Real power $= P = \mathrm{Re}[\hat{S}] = 462$ W

(c) reactive power $= Q = \mathrm{Im}[\hat{S}] = 134.3$ VARs

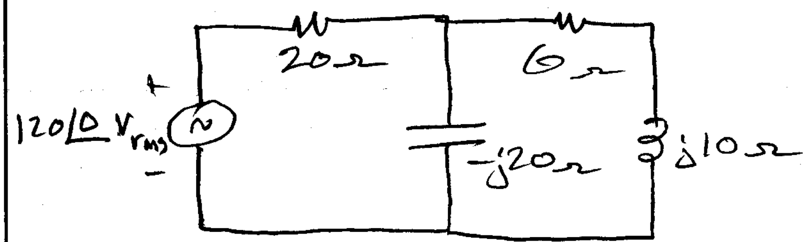
(d) power factor

$$\text{P.f.} = \frac{P}{|S_T|} = \frac{462}{481.6} = 0.96 \text{ lagging}$$

(lagging because $\angle S$ is positive, which means $\angle Z$ is positive, which means voltage leads current, thus lagging p.f.

Example 16.9

You are given the following circuit.



(a) What is the average power (real) supplied by the source.

(b) What is the complex power supplied by the source.

(c) What is the p.f. with respect to the source.

(d) What is the real power delivered to the 6Ω resistor?

(e) What is the real power delivered to the 20Ω resistor.

(f) Verify that the real power supplied by the source = real power absorbed by the 20Ω resistor + real power absorbed by the 6Ω resistor.

(a) $$Z_{LOAD} = 20 + \frac{(-j20)(6+j10)}{6+j10-j20}$$

$$Z_{LOAD} = 38.8 \angle 14^\circ \ \Omega$$

$$\hat{I}_s = \frac{\hat{V}_s}{Z_{LOAD}} = \frac{120\angle 0}{38.8 \angle 14} = 3.09\angle -14 \ A$$

$$P_s = |\hat{V}_s I_s| \cos(\theta_{Vs} - \theta_{Is}) = 120 \times 3.09 \cos(0-(-14))$$

$$P_s = 359.8 W$$

(b) $$\hat{S} = V_s I_s^* = 120 \times 3.09 \angle 14 = 370.8 \angle 14 \ VA$$

(c) $$p.f. = \frac{P}{|S|} = \frac{359.8}{370.8} = 0.9703 \text{ lagging} = \cos 14^\circ$$

(d) $$\hat{I}_6 = \frac{3.09\angle -14(-j20)}{-j20+6+j10} = 5.3 \angle -45^\circ \ A$$

$$P_6 = (|\hat{I}_6|)^2 \times 6 = (5.3)^2 \times 6 = 168.54 W$$

(e) $$P_{20} = |\hat{I}_s|^2 \times 20 = (3.09)^2 \times 20 = 190.96 W$$

(f) Checking for $P_s = P_{20} + P_6$

$$359.8 \stackrel{?}{=} 190.96 + 168.54 = 359.5 W$$

Very close, error in numerical round-off.

QED

Example 16.10

Given the network shown in the following diagram, Determine the input voltage $\hat{V}_s$.

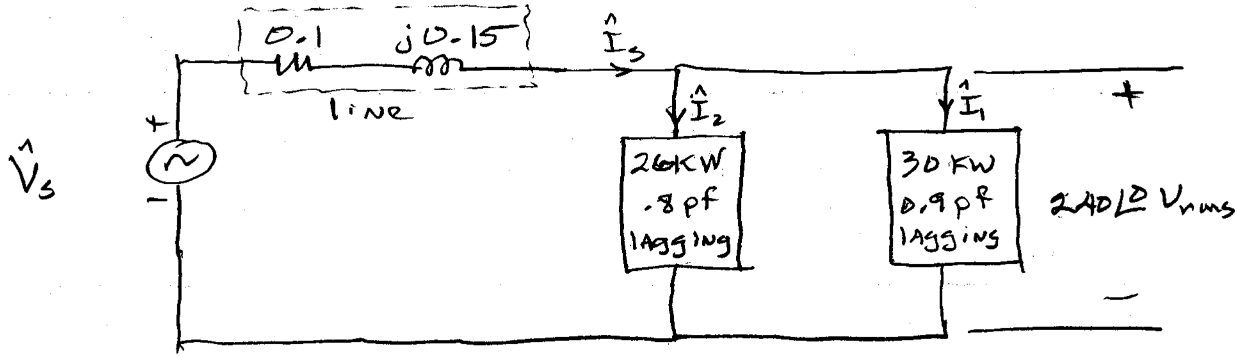


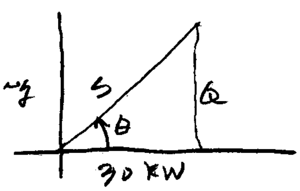
Figure 16.7: System for Example 16.10.

Approach: Find $\hat{I}_1$ & $\hat{I}_2$; $\hat{I}_s = \hat{I}_1 + \hat{I}_2$

then $\hat{V}_s = (0.1 + j0.15)\hat{I}_s + 240\angle 0$ V rms

For I_1: Lagging p.f.

For a lagging P.f.



$\cos\theta_1 = .9 \qquad \theta_1 = \cos^{-1}.9 = 24.8$

$\frac{P_1}{|S|} = \cos\theta$

$|S_1| = \frac{P_1}{.9} = \frac{30KW}{.9} = 33.33$

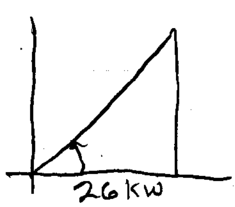
$\hat{S}_1 = 33.33\angle 24.8$

$\hat{S} = \hat{V} I_1^* \qquad I_1^* = \frac{33.3\angle 24.8}{240}k = 138.9\angle 24.8$ A rms

$\hat{I}_1 = 138.9\angle -24.8$ A rms

For I_2:

Lagging P.f



$\cos\theta_2 = .8 \qquad \theta_2 = 36.87$

$|S_2| = \frac{P_2}{.8} = \frac{26k}{.8} = 32.5$ KVA

$\hat{S}_2 = 32.5\angle 36.87^\circ$ KVA $= \hat{V} I_2^* \qquad I_2^* = \frac{32.5\angle 36.87}{240\angle 0} = 135.4\angle 36.87$

$\hat{I}_2 = 135.4\angle -36.87$ Arms

Ex 16.10 cont.

$$\hat{I}_s = \hat{I}_1 + \hat{I}_2 = 138.9\angle{-24.8} + 135.4\angle{-36.87}$$

$$\hat{I}_s = 271.9\angle{-30.8}\ A_{rms}$$

$$\hat{V}_s = (0.1+j0.15)(271.9\angle{-30.8}) + 240\angle{0}$$

$$\boxed{\hat{V}_s = 284.9\angle{4.2^\circ}\ V_{rms}} \quad \text{Ans.}$$

<u>Note</u>: If one or both of the loads are given as KVA you must take this into account on the complex power triangle.