Lesson 17

Magnetically Coupled Circuits
Linear and Ideal Transformers

Notes for ECE 301

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Equations 17.27 and 17.28 correspond to the dot polarities, assumed voltage polarities and assumed current directions as shown in Figure 17.9.

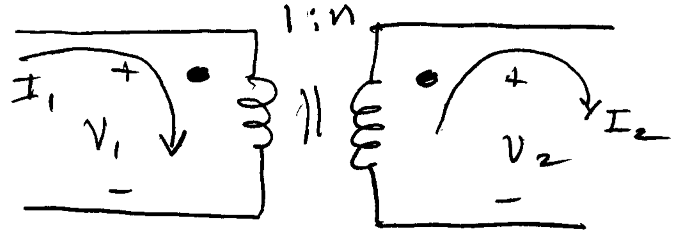
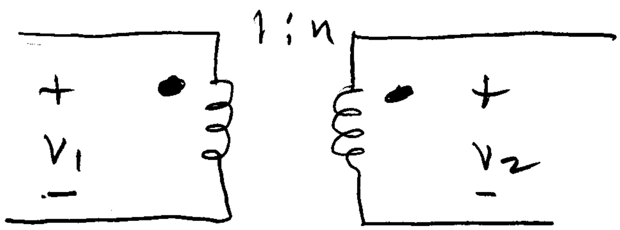


Figure 17.9: Configuration of circuit and assumed voltages and currents for $\frac{V_2}{V_1} = n$, $\frac{I_2}{I_1} = \frac{1}{n}$.

We establish the sign for the voltage and current ratios as follow:

<u>Sign For the voltage ratio,</u> $\frac{V_2}{V_1}$.

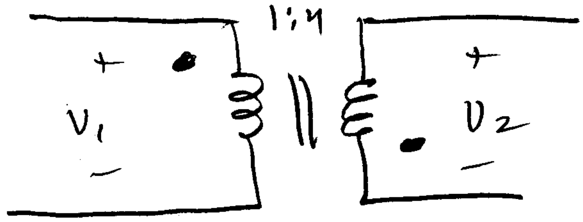
Consider the ideal transformer shown in Figure 17.10.



$$\frac{V_2}{V_1} = n$$

Figure 17.10: An assumed configuration for dots and polarity of V_1 and V_2.

Now consider Figure 17.11.



$$\frac{V_2}{V_1} = -4$$

Figure 17.11: configuration where V_2/V_1 is negative.

The general case for $\frac{V_2}{V_1}$ is;

- If V_1 and V_2 are both positive or both negative at the dotted terminals, $+n$ is used; otherwise use a $-$ sign. This applies regardless of the assumed directions for I_1 and I_2.

The general case for $\frac{I_2}{I_1}$ is

- If I_1 and I_2 both enter or both leave dotted terminals a $-$ sign is used in the current ratio. Otherwise a $+$ sign is used.

There are very sound reasons for the above statements. Just as current cannot change instantaneously, flux also cannot change instantaneously in coupled circuits. We consider the transformer with given windings.

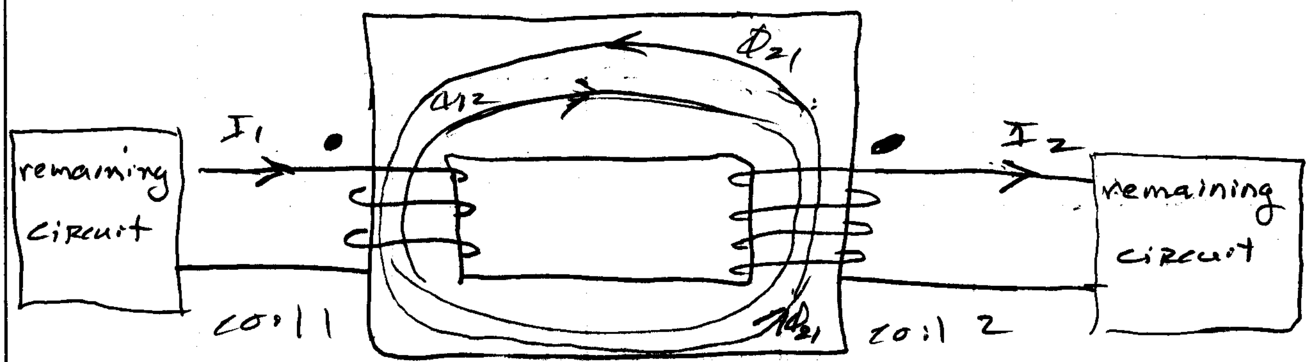


Figure 17.12: Circuit used to explain flux linkage.

Basically, with current I_1 setting up (tending to set-up) the flux Φ_{12}, there will be a voltage induced at the terminals of coil two that produces a current I_2 that sets up a flux, Φ_{21}, that opposes the flux Φ_{12}. The current I_2 leaves the dot of coil 2. We then say $\frac{I_2}{I_1} = \frac{1}{n}$. If we apply a positive voltage at the dotted terminal of coil 1, then the dotted terminal of coil 2 will be positive. This might be easier to understand if we consider ~~the~~ task of making dots on the transformer.

An Illustration

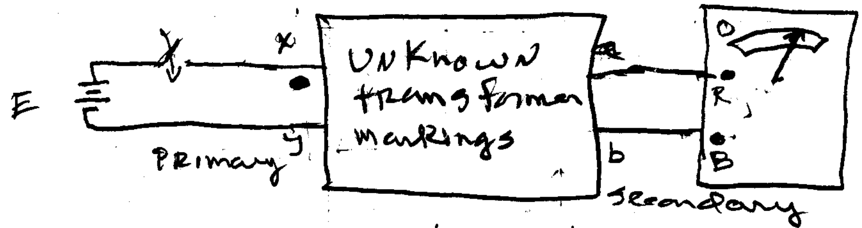


Figure 17.13: Determining dots on an unmarked transformer.

Suppose we have a transformer and all we have available are the terminals x-y of the primary and terminals a-b of the secondary. Our job is to make the appropriate dots for the primary and secondary. We start by using a switch and connecting a known positive voltage to terminal X. We automatically assume X is the dot terminal of the primary. We connect the red lead of a voltmeter to terminal "a". We suddenly close the switch on the primary side.

If the meter flips up-scale and back, we know a positive voltage was induced at terminal "a" and we make a dot there. If the meter needle flips to the left we mark "b" with the dot.

Example 17.3

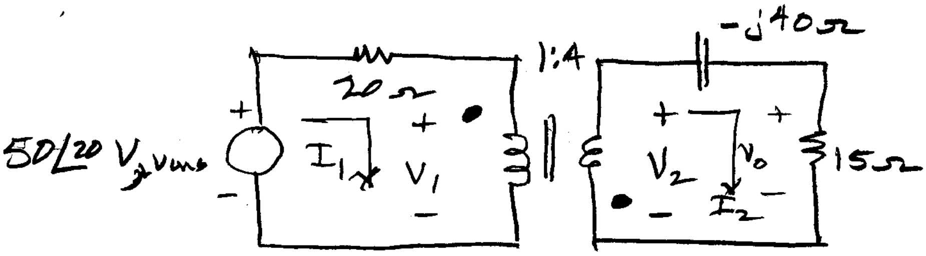


Figure 17.14: Circuit for Example 17.3

For the above circuit, solve for I_1, I_2, V_1 and V_2:

Solution:

We can write 4 equations in terms of 4 unknowns as follows

$$\frac{V_2}{V_1} = -4 \Rightarrow \boxed{4V_1 + V_2 = 0}$$

$$\frac{I_2}{I_1} = -\frac{1}{4} \Rightarrow \boxed{I_1 + 4I_2 = 0}$$

$$\boxed{V_1 + 20I_1 = 50\angle 20}$$

$$\boxed{-V_2 + (15 - j40)I_2 = 0}$$

$$\begin{array}{cccc} V_1 & V_2 & I_1 & I_2 \end{array}$$
$$\begin{bmatrix} 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 1 & 0 & 20 & 0 \\ 0 & -1 & 0 & (15-j40) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 50\angle 20 \\ 0 \end{bmatrix} \qquad \text{Eq 17.29}$$

$V_1 = 6.33\angle{-42.6}$, $V_2 = 25.3\angle{+37.4}$ V, rms

$I_1 = 2.37\angle 26.8$ A, rms $I_2 = 0.593\angle{-153.2}$ A, rms

Example 17.4

Given the following ideal transformer

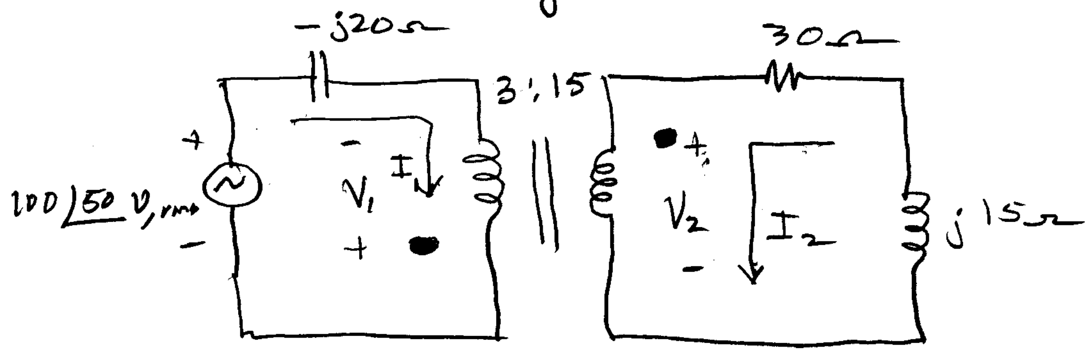


Figure 17.15: Circuit for Example 17.4.

Again, we have 4 equations and 4 unknowns.

$$n = \frac{15}{3} = 5$$

$$\frac{V_2}{V_1} = 5 \implies \boxed{5V_1 - V_2 = 0}$$

$$\frac{I_2}{I_1} = \frac{1}{5} \implies \boxed{I_1 - 5I_2 = 0}$$

$$\boxed{-j20I_1 - V_1 = 100\angle 50}$$

$$\boxed{V_2 + (30+j15)I_2 = 0}$$

$$\begin{array}{c} \begin{array}{cccc} V_1 & V_2 & I_1 & I_2 \end{array} \\ \begin{bmatrix} 5 & -1 & 0 & 0 \\ 0 & 0 & 1 & -5 \\ -1 & 0 & -j20 & 0 \\ 0, & 1, & 0, & (30+j15) \end{bmatrix} \end{array} \begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 100\angle 50 \\ 0 \end{bmatrix} \quad \text{Eq 17.30}$$

$V_1 = 6.9\angle -17^\circ$ V, rms $\quad V_2 = 34.5\angle -17$ V, rms

$I_1 = 5.145\angle 136.5^\circ$ A, rms $\quad I_2 = 1.03\angle 136.5^\circ$ A, rms

Thevenin's Theorem & The Ideal Transformer:

Consider the following transformer configuration.

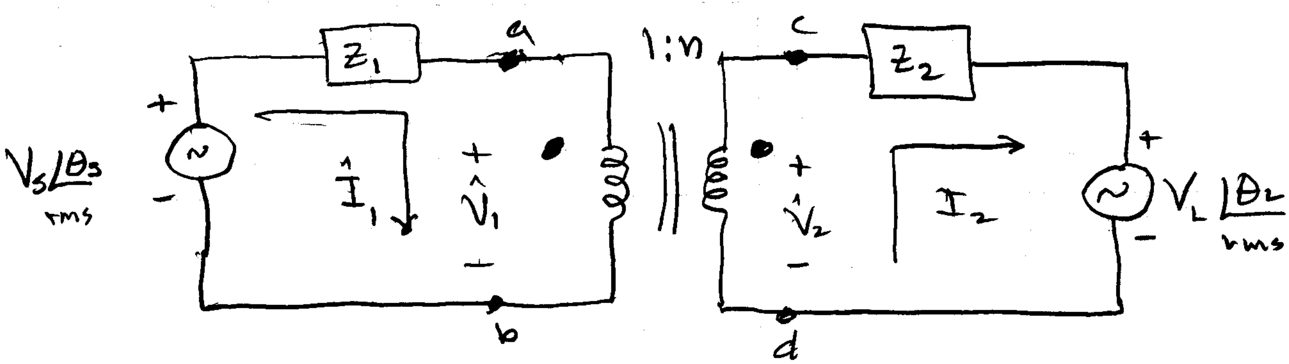


Figure 17.16: Illustrating Thevenin reflected to the primary of the ideal transformer.

We open the primary to find the Theven voltage and impedance to the right of terminals a-b.

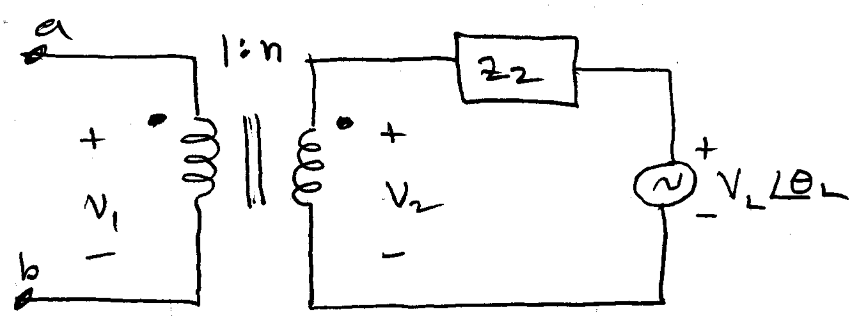


Figure 17.17: Find Thevenin's equivalent ckt.

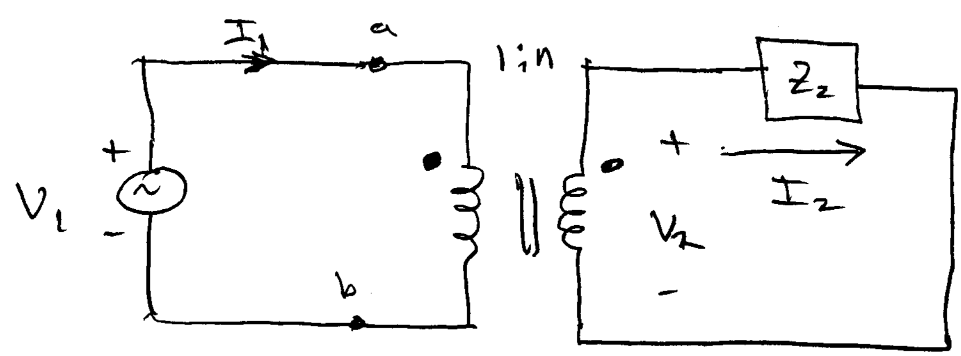
For the circuit in Figure 17.17, $\hat{I}_1 = 0$. When $I_1 = 0$, $I_2 = 0$; $\frac{I_2}{I_1} = \frac{1}{n}$ and with $I_1 = 0$, $I_2 = 0$ (L_2, M are extremely large inductances and $I_2 = 0$.

From Figure 17.17, $\hat{V}_2 = \hat{V}_L$.

$$\frac{V_2}{V_1} = n$$

so $\boxed{V_1 = \frac{V_2}{n};} = V_{TH}$

To find the reflected impedance, we short (deactivate) the load voltage. We Apply a voltage of V_1 at a-b and find the ratio of $V_1/I_1 = Z_1$



$$\frac{V_1}{I_1} = Z_{ref}$$

$$\frac{V_2}{V_1} = n$$

$$\frac{I_2}{I_1} = \frac{1}{n}$$

use $V_1 = \frac{V_2}{n}$; $I_1 = n I_2$

$$\frac{V_1}{I_1} = \frac{V_2}{n} \times \frac{1}{n I_2} = \frac{1}{n^2} \times \frac{V_2}{I_2}$$

but $V_2 = I_2 Z_2$

$$\frac{V_2}{I_2} = Z_L$$

$$\frac{V_1}{I_1} = \frac{Z_L}{n^2}$$

The equivalent circuit reflected to the primary is shown below.

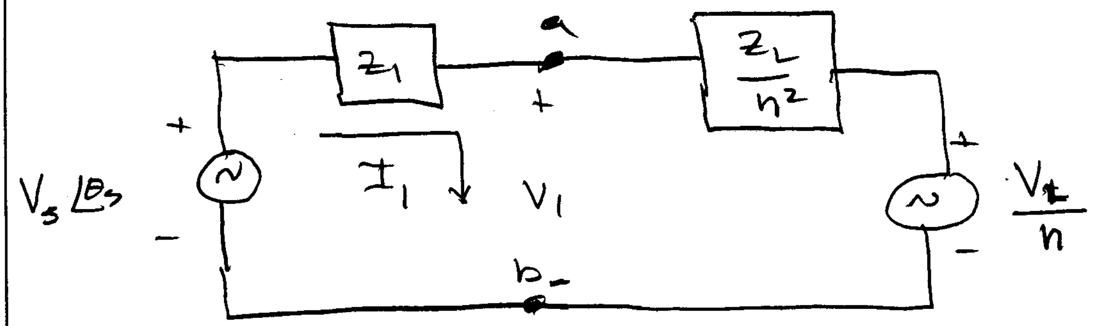
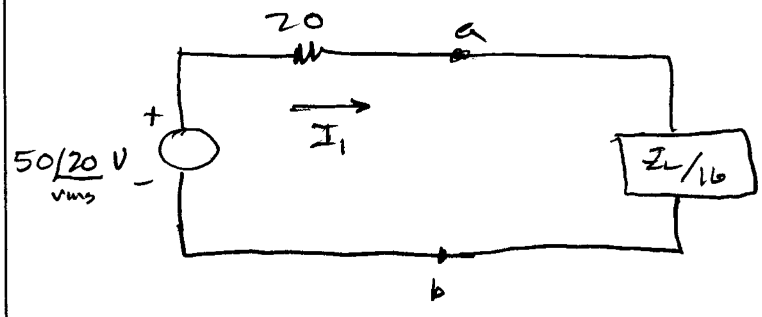


Figure 17.18: Secondary reflected to the primary for transformer markings, polarities and current directions of Figure 17.16.

Let us apply the above circuit to find I_1 & V_1 of Example 17.3. We have



$= \frac{15-j40}{16}$

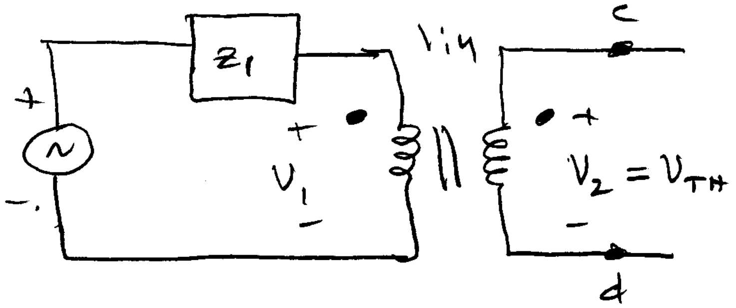
$$\hat{I}_1 = \frac{50\angle 20}{20 + \frac{15-j40}{16}} = 2.37\angle 26.8 \text{ A, rms} \quad \text{(check)}$$

$$\hat{V}_1 = 50\angle 20 - 20 I_1 = 6.33\angle -42.4 \text{ V, rms}$$

The above answers agree with the solutions of Example 17.3

Reflecting The Primary To The Secondary For The Ideal Transformer

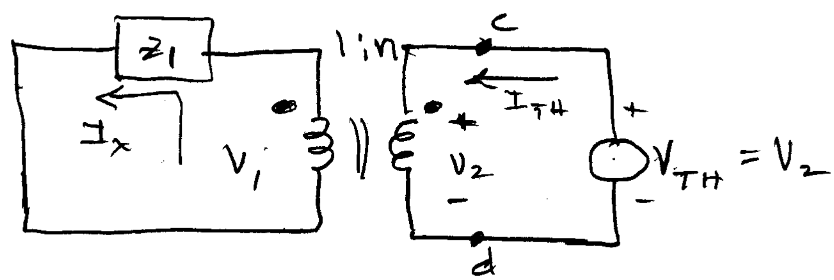
We start by considering the secondary open



Since $I_2 = 0$, $I_1 = 0$.

$V_1 = V_s \angle \theta$, $V_2 = n V_1 = n V_s \angle \theta_s = V_{TH}$

To find the reflected impedance



$\frac{I_x}{I_{TH}} = n$

$$Z_{ref} = Z_{TH} = \frac{V_{TH}}{I_{TH}} = \frac{V_2}{I_{TH}}$$

$V_2 = V_1 n$, $\quad I_x = n I_{TH}$ $\qquad \frac{I_x}{I_{TH}} = \frac{1}{n}$

$$Z_{ref} = \frac{V_1 n^2}{I_x} \quad \text{but } V_1 = Z_1 I_x \Rightarrow Z_1 = \frac{V_1}{I_x}$$

$$Z_{ref} = n^2 Z_1$$

We now have the equivalent circuit as shown in Figure 17.19

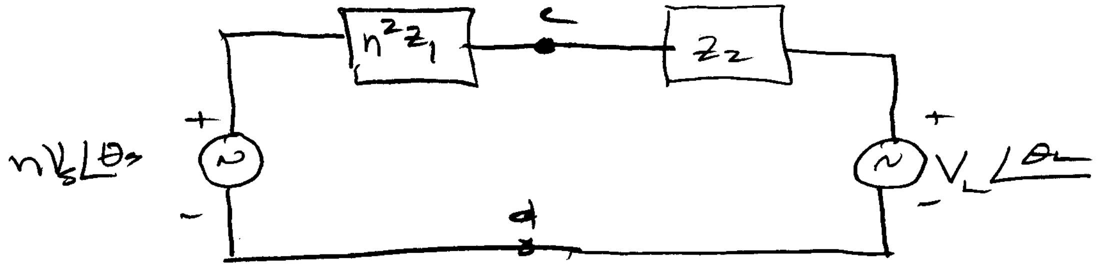


Figure 17.19: Primary reflected to secondary. Relevant to Figure 17.16.

The following is true in general:

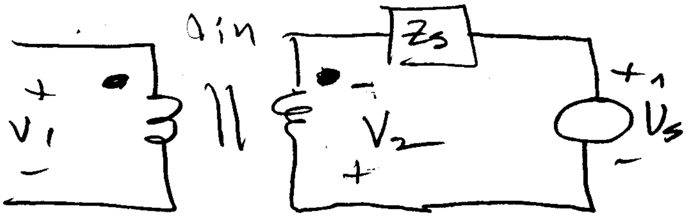
- The impedance reflected from the secondary to the primary is $\frac{Z_s}{n}$ (Z_s is the secondary impedance).
- The impedance reflected from the primary to the secondary is $n^2 Z_p$ where Z_p is the primary impedance

The above is true regardless of dot marking, voltage polarity, current direction.

- The voltage source of the secondary reflected to the primary is $\frac{|V_s|}{n}$
- The voltage source of the primary reflected to the secondary is $n|V_p|$.

Illustration A

What is the voltage source value and polarity reflected from the secondary to the primary?



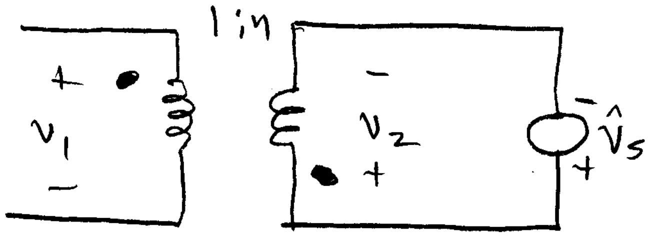
$I_1 = 0 \Rightarrow I_2 = 0 \Rightarrow V_2 = -V_s$

but

$$\frac{V_2}{V_1} = -n \qquad \text{OR} \quad V_1 = -\frac{V_2}{n}$$

Then $V_1 = -\dfrac{(-V_s)}{n} = \dfrac{V_s}{n}$

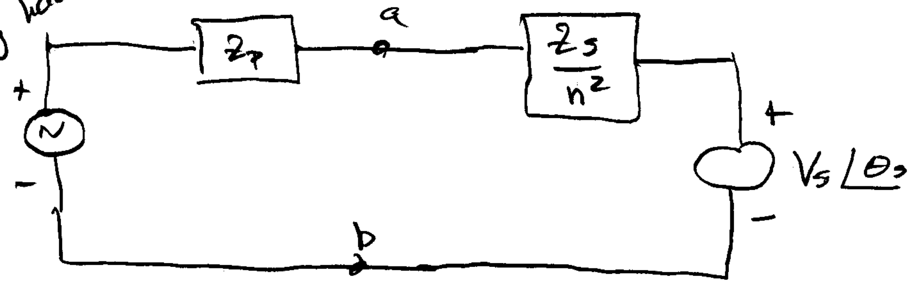
Illustration B



$$V_1 = \frac{V_2}{n} = \frac{V_s}{n}$$

So the equivalent ckt is

whatever V_p polarity had



Verification of Illustration B

Example 17.5

Find $\hat{I}_1$ in the following ideal transformer circuit by (a) direct circuit analysis (b) reflecting secondary to primary using information from Illustration B.

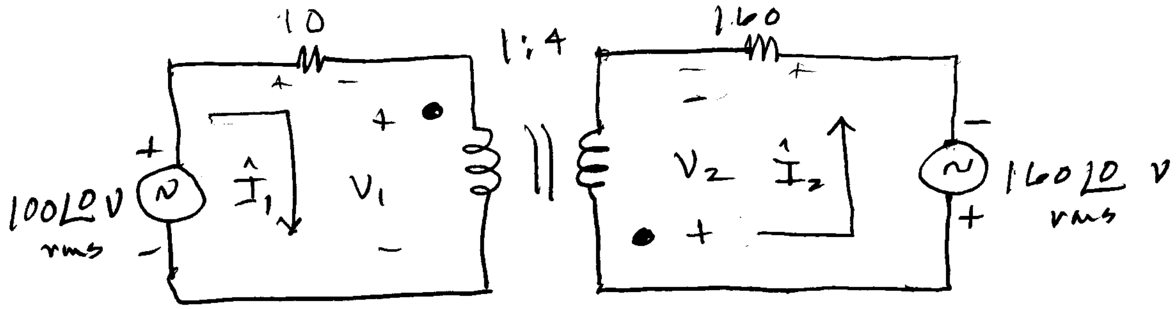


Figure 17.20 : Circuit for example 17.5.

$$\frac{\hat{V}_2}{\hat{V}_1} = 4 \Rightarrow \boxed{4\hat{V}_1 - \hat{V}_2 = 0}$$

$$\frac{I_2}{I_1} = \frac{1}{4} \Rightarrow \boxed{I_1 - 4I_2 = 0}$$

$$\boxed{V_1 + 10I_1 = 100}$$

$$\boxed{-V_2 + 160I_2 = -160}$$

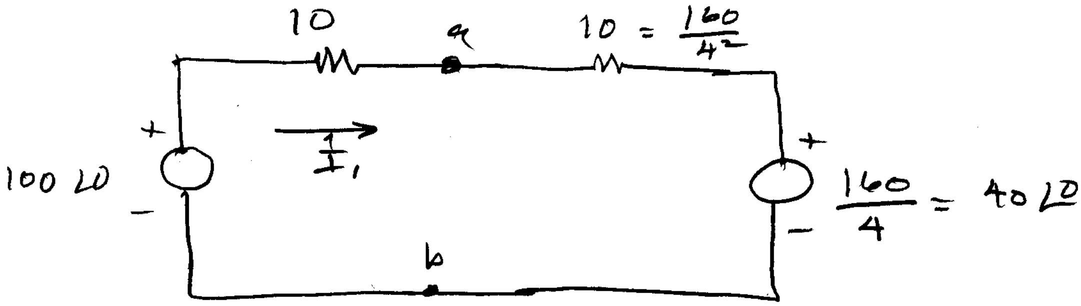
$$\begin{array}{cccc} V_1 & V_2 & I_1 & I_2 \end{array}$$

$$\begin{bmatrix} 4 & -1 & 0 & 0 \\ 0 & 0 & 1 & -4 \\ 1 & 0 & 10 & 0 \\ 0 & -1 & 0 & 160 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 100 \\ -160 \end{bmatrix}$$

$\hat{I}_1 = 3\angle 0$ A

Example 17.5

Reflecting the secondary as per Illustration B.



$$\hat{I}_1 = \frac{100 - 40}{20} = 3 \angle 0 \text{ A}$$

This checks with circuit analysis method.

Example 17.6

Find I_2 for the following circuit using (a) regular circuit analysis. (b) Reflect primary to secondary.

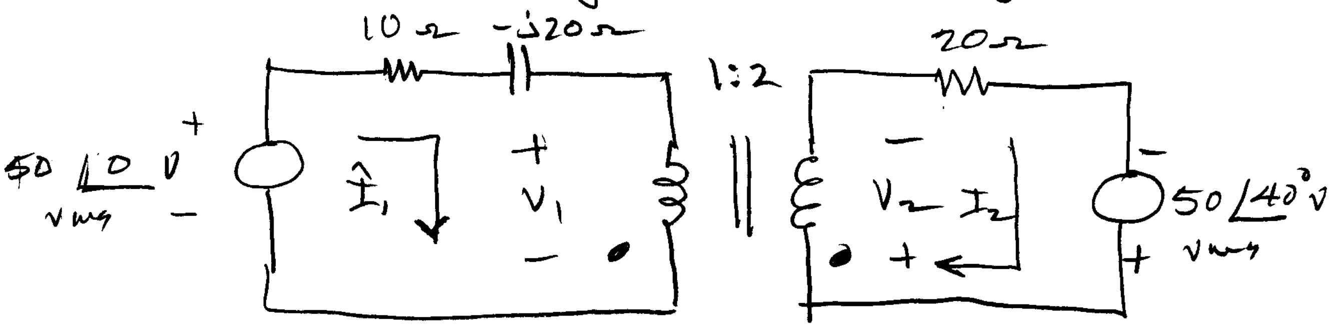


Figure 17.21: Circuit for example 17.6.

(a) First by regular circuit analysis.

$$\frac{V_2}{V_1} = -2 \quad \Rightarrow \quad \boxed{2V_1 + V_2 = 0}$$

$$\frac{I_2}{I_1} = \frac{1}{2} \Rightarrow \boxed{I_1 - 2I_2 = 0}$$

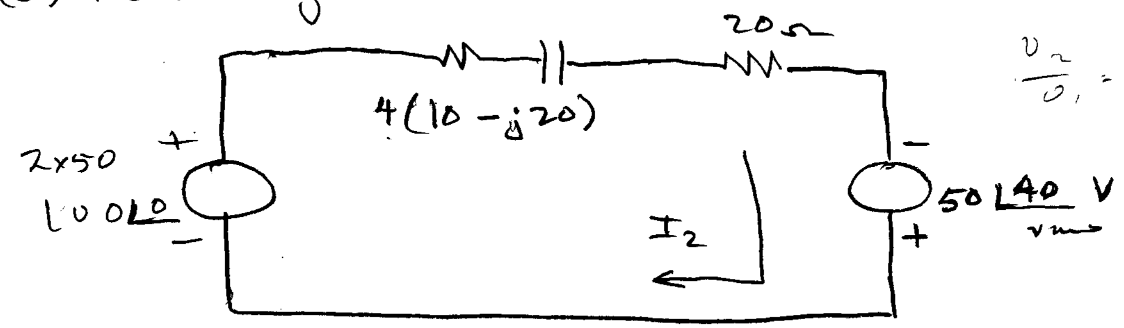
$$\boxed{V_1 + (10 - j20) I_1 = 50\angle 0}$$

$$\boxed{V_2 + 20 I_2 = 50\angle 40}$$

$$\begin{array}{cccc} V_1 & V_2 & I_1 & I_2 \\ \end{array}$$
$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 1 & 0 & (10-j20) & 0 \\ 0 & 1 & 0 & 20 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 50\angle 0 \\ 50\angle 40 \end{bmatrix}$$

$\hat{I}_2 = 1.42\angle 66.2$ A, rms

(b) Reflecting



$\frac{v_2}{v_1} = -2$

$V_1 = 50\angle 0$; $\underline{V_2 = -2V_1 = -100\angle 0}$

Since V_2 is positive, down, Fig 17,21,
It must be have the source positive, up.

$$I_2 = \frac{50\angle 40 + 100}{60 - j80} = 1.42\angle 66.2 \text{ A, rms}$$

Checks with circuit analysis.

Example 17.7

Consider the stero amplifier and speaker configuration shown below.

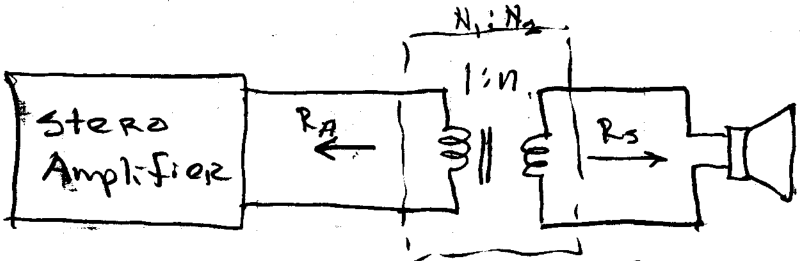


Figure 17.22: Circuit for Example 17.7.

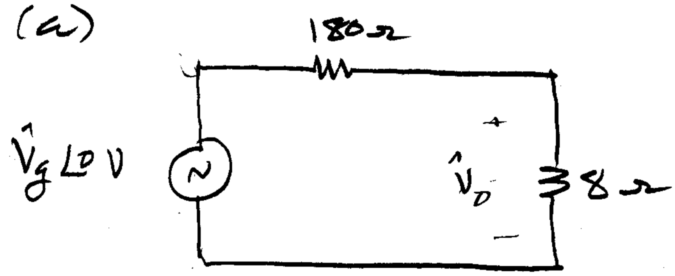
Assume that the resistance of the amplifier is 180 Ω. Assume that the open-circuit voltage of the amplifier is V_g volts, rms. Assume that the speaker has 8 Ω.

(a) Without using a transformer, determine the power delivered to the speaker. This will be expressed in terms of V_g.

(b) Determine the value of n for the transformer so that maximum power is delivered to the speaker.

(c) Determine the amount of power delivered to the speaker with the transformer in place.

(a)



The voltage across the speaker is

$$V_o = \frac{8 \times V_g}{8 + 180}$$

$$P_o = \frac{V_o^2}{8} = \left[\frac{64 V_g^2}{35344}\right] \times \frac{1}{8}$$

$$P_o = 0.22635 V_g^2 \text{ mW}$$

(b) The resistance reflected is

$$R_{Ref} = \frac{8}{n^2}$$

We want this to equal 180

$$\frac{8}{n^2} = 180$$

$$n = \sqrt{\frac{8}{180}} = 0.2108$$

If $N_2 = 5$, $N_1 = \frac{5}{.2108} = 23.7$ turns

(c) With the impedance match

$$P_o = \frac{V_o^2}{180} = \frac{1}{180}\left[\frac{180 V_g}{180+180}\right]^2 = \frac{.25 V_g^2}{180}$$

$$P_o = 1.4 \, V_g^2 \text{ mW}$$

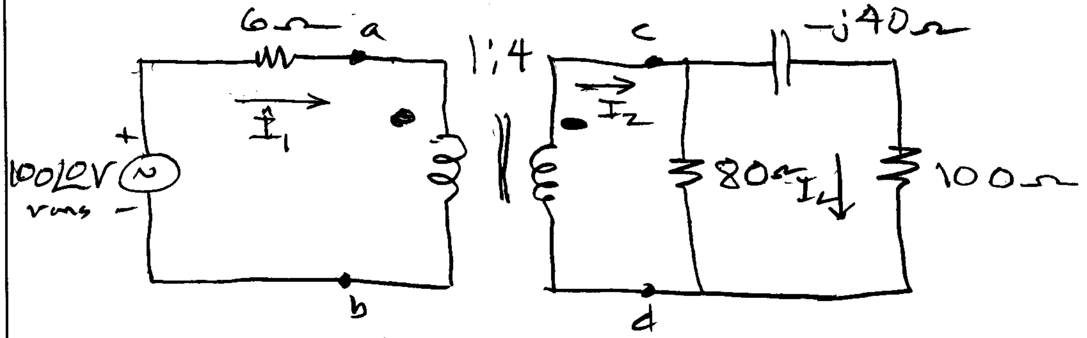
compared to

$$P_o = 0.22635 \, V_g^2 \text{ mW}$$

6.2 times more power out.

Example 17.8

You are given the ideal transformer circuit shown below.



(a) Reflect the secondary impedance to the primary and find $\hat{I}_1$

(b) After finding $\hat{I}_1$, find $\hat{I}_2$.

(c) Find the average power delivered to the 10Ω load resistor.

(a) The impedance to the right of c-d is

$$Z_{cd} = \frac{80(100-j40)}{180-j40} = 46.7\angle{-9.3^\circ}\ \Omega$$

$$Z_{reflected} = Z_R = \frac{46.7\angle{-9.3}}{4^2}$$

$$Z_R = 2.92\angle{-9.3}\ \Omega$$

$$\hat{I}_1 = \frac{100\angle 0}{6+2.92\angle{-9.3}} = 11.24\angle{3.04^\circ}\ A, \text{rms}$$

$$\underline{\hat{I}_1 = 11.24\angle{3.04^\circ}\ A, \text{rms}}$$

Example 17.8 (cont.)

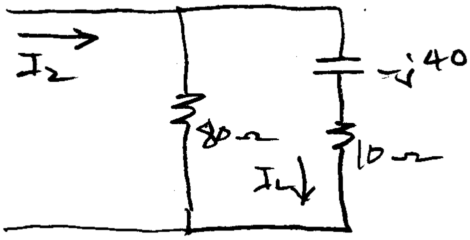
(b)

$$\frac{I_2}{I_1} = \frac{1}{n} = \frac{1}{4}$$

$$I_2 = \frac{I_1}{4} = \frac{11.24\angle 3.04}{4} = 2.81\angle 3.04 \text{ A, rms}$$

$$\boxed{I_2 = 2.81\angle 3.04 \text{ A, rms}}$$

(c) We use the current division rule



$$I_L = \frac{I_2 \times 80}{80 + 10 - j40} = \frac{(2.81\angle 3.04)80}{90 - j40}$$

$$I_L = 2.28\angle 27.0 \text{ A, rms}$$

$$P_{10} = |I_L|^2 10 = (2.28)^2 \times 10$$

$$\boxed{P_{10} = 52 \text{ W}}$$

Example 17.9

Find the current $\hat{I}_1$ in the circuit below using reflected impedance.

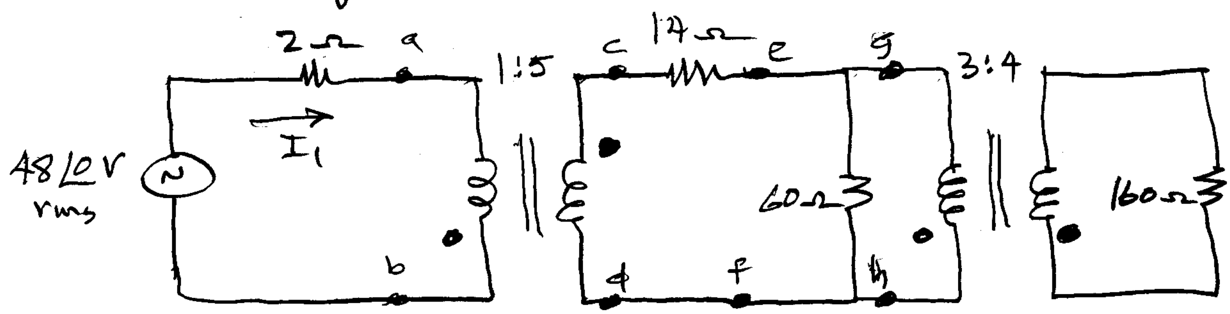


Figure 17.23 Circuit for example 17.9

The dots on the transformers make no difference in reflecting the resistance starting at the right we have

$$Z_{gh} = \frac{160}{(4/3)^2} = \frac{160 \times 9}{16} = 90\,\Omega$$

$$Z_{ef} = \frac{60 \times 90}{60 + 90} = 36\,\Omega$$

$$Z_{cd} = 14 + 36 = 50\,\Omega$$

$$Z_{ab} = \frac{50}{5^2} = 2\,\Omega$$

$\therefore$

$$\hat{I}_1 = \frac{48\angle 0}{2+2} = 12\text{ A, rms}$$

$$\boxed{\hat{I}_1 = 12\text{ A, rms}}$$

Example 17.10

You are given the circuit below that contains an ideal transformer. Determine the power delivered to the 10 Ω resistor.

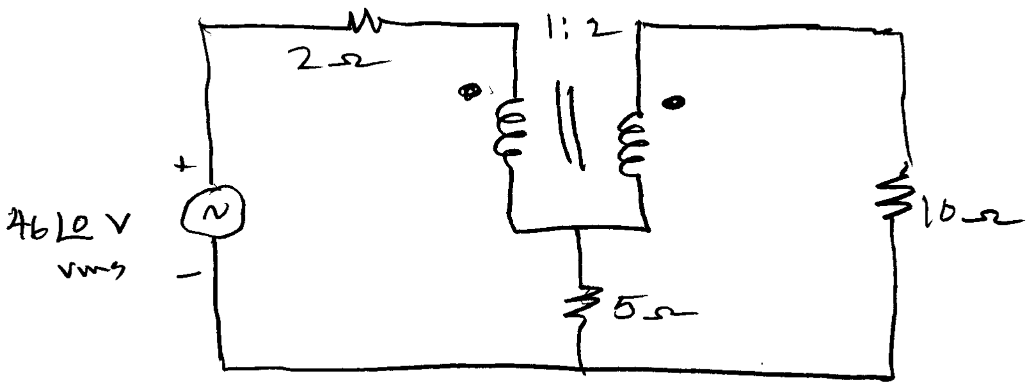
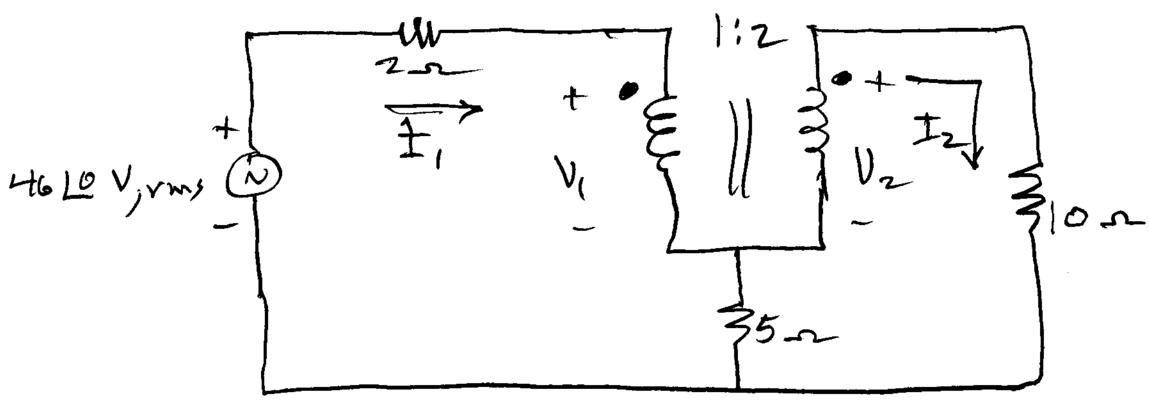


Figure 17.24: Circuit for Example 17.10.

We are given the choice in assigning $\hat{I}_1$, $\hat{I}_2$, $\hat{V}_1$ and $\hat{V}_2$ for the transformer. My choices are as shown in the following circuit.



As the circuit stands, we have 4 unknowns: $\hat{I}_1$, $\hat{I}_2$, $\hat{V}_1$ and $\hat{V}_2$. We therefore need 4 equations.

We write the equations as follows:

$$\frac{I_2}{I_1} = +\frac{1}{2} \Rightarrow \boxed{I_1 - 2I_2 = 0}$$

$$\frac{V_2}{V_1} = 2 \Rightarrow \boxed{2V_1 - V_2 = 0}$$

$$2I_1 + V_1 + 5(I_1 - I_2) = 46\angle 0$$

$$\boxed{V_1 + 7I_1 - 5I_2 = 46\angle 0}$$

$$10I_2 + 5(I_2 - I_1) - V_2 = 0$$

$$\boxed{-V_2 - 5I_1 + 15I_2 = 0}$$

$$\begin{array}{cccc} V_1 & V_2 & I_1 & I_2 \\ \end{array}$$

$$\begin{bmatrix} 0 & 0 & 1 & -2 \\ 2 & -1 & 0 & 0 \\ 1 & 0 & 7 & -5 \\ 0 & -1 & -5 & 15 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 46 \\ 0 \end{bmatrix}$$

$I_2 = 4$ A, rms

$$P_{10} = I_2^2 \times 10 = 160 \text{ W}$$

$$\boxed{P_{10} = 160 \text{ W}}$$