Discrete Time Modeling of Power Electronics

Daniel Costinett
ECE 620
September 25th, 2019
Outline

• Introduction
  – Power converter analysis
  – Design space in power electronics

• Historical Development
  – Middlebrook: Design-oriented Analysis
  – Cuk: Averaged Analysis
  – Packard: Discrete Time Analysis

• Modern Converter Analysis
Analysis of Switching Converters
Analysis of Switching Converters

\[ v_o(t) - v(t) - i_L(t) \]

\[ s(t) \]

\[ V_g \]

\[ s(t) \]

\[ v_s(t) \]

\[ i_L(t) \]

\[ L \]

\[ C \]

\[ Rv(t) \]
Circuit Nonlinearities

\[ V_g \quad + \quad s(t) \quad - \quad C \quad + \quad R \]
Time Dependence
Semiconductor Large-Signal Nonlinearity
Circuit Parasitics
Passive Nonlinearity

\[ C_{ds} = f(v_{ds}) \]

\[ L = f(i(t), f_s) \]
Design Space
Example Design Sequence

Modeling: Motivation

Modeling: Motivation

Modeling: Motivation

Design Space Characteristics

• Highly non-convex
  – Many radically different solutions may achieve near-equal performance

• Minimal Differentiation
  – Losses account for <1% of power in some cases
    ▪ A 1% error in waveform prediction can result in >100% error in predicted loss!

• Vast range and form of potential design variables
HISTORICAL DEVELOPMENT OF MODELING
Tomas G. Wilson  
PhD, Harvard, 1953  
Duke Prof, 1959-1994  

Fred. C. Lee  
PhD, Duke, 1974  
VT Prof, 1977-2017  

John G. Kassakian  
PhD, MIT, 1973  
MIT Prof, 1973-present  

George C. Verghese  
PhD, Stanford, 1979  
MIT Prof, 1979-2017  

...Plus many others
# Top 100 authors in Engineering

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By The Way...

Top 100 authors in Engineering

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Historical Perspective on SMPS Modeling

Robert D Middlebrook (1929-2010)
  • Ph.D., Stanford 1955
  • First graduate student to work with transistors
  • CalTech Professor, 1955-1998
  • Wrote *An introduction to junction transistor theory*, 1957
  • Founded CalTech Power Electronics Group (1970)
  • Championed “Design-Oriented Analysis”

Timeframe:
1945   WWII Ends
1947   First transistor fabricated at Bell Labs
1957   Sputnik Launches

Kit Sum, “Remembering Dr. Middlebrook”, 2012
Design-Oriented Analysis

The Design Feedback Loop

Design-Oriented Analysis Tools

- “Low Entropy Expressions”
- Algebra-on-the-circuit
- Graphical Techniques
- Generalized Feedback Theorem

\[ G(s) = G_\infty \frac{T}{1 + T} + G_o \frac{1}{1 + T} \]

- Extra Element Theorem
Algebra on the Circuit

Nodal Analysis

\[ A = \frac{\beta R_B R_L}{(1 + \beta) r_E R_S + (1 + \beta) r_E R_B + R_S R_B} \]

Design-Oriented Analysis

\[ A = \frac{v_2}{v_1} = \frac{R_B}{R_S + R_B} \frac{\alpha R_L}{r_E + (R_S || R_B)/(1 + \beta)} \]

R. D. Middlebrook, “Low-Entropy Expressions: The Key to Design-Oriented Analysis”
Extra Elements

\[ A = \frac{v_2}{v_1} = \frac{R_B}{R_S + R_B} \frac{\alpha R_L}{r_E + (R_S \parallel R_B)/(1 + \beta)} \]
Middlebrook’s Extra Element Theorem

Object: find how addition of an element changes a transfer function \( G(s) \)

Original conditions:

Transfer function
\[
G(s) \bigg|_{Z(s) \to \infty}
\]

Addition of element \( Z(s) \):

Transfer function
\[
G(s)
\]

\[
\frac{v_{out}(s)}{v_{in}(s)} = \left( G(s) \bigg|_{Z(s) \to \infty} \right) \left( \frac{1 + \frac{Z_N(s)}{Z(s)}}{1 + \frac{Z_D(s)}{Z(s)}} \right)
\]
Finding $Z_D$

$Z_D(s)$ is the driving-point impedance (i.e., the Thevenin-equivalent impedance) at the port where the new element is connected. Formally, it is found by setting independent sources to zero, and injecting a current $i(s)$ at the port. $Z_D(s)$ is the ratio of $v(s)$ to $i(s)$. 
Finding $Z_N$

$Z_N$ is the impedance seen at the port when the output is nullled. In the presence of the input $v_{in}(s)$, a current $i(s)$ is injected at the port. This current is adjusted such that the output $v_{out}(s)$ is nullled to zero. Under these conditions, $Z_N(s)$ is the ratio of $v(s)$ to $i(s)$. **Note:** nulling is not the same as shorting.
Quick Example

\[ G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R_2}{R_1 + R_2} \]
Quick Example - Algebra

\[ G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R_2}{R_1 \parallel \frac{1}{sC} + R_2} \]

\[ = \frac{R_2}{\frac{1}{R_1} + sC + R_2} \]

\[ = \frac{R_2 \left( \frac{1}{R_1} + sC \right)}{1 + R_2 \left( \frac{1}{R_1} + sC \right)} \]

\[ = \left( \frac{R_2}{R_1} \right) \frac{1 + sCR_1}{1 + sC \left( \frac{R_2}{R_1} \right)} \]

\[ = \left( \frac{R_2}{R_1 + R_2} \right) \frac{1 + sCR_1}{1 + sC \left( R_1 \parallel R_2 \right)} \]
Quick Example - EET

\[ G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R_2}{R_1 + R_2} \left( \frac{1 + \frac{Z_N(s)}{Z(s)}}{1 + \frac{Z_D(s)}{Z(s)}} \right) \]
Finding $Z_D$

\[ G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R_2}{R_1 + R_2} \left( 1 + \frac{Z_N(s)}{Z(s)} \right) \left( 1 + \frac{Z_D(s)}{Z(s)} \right) \]

\[ Z_D(s) = R_1 || R_2 \]
Finding $Z_N$

\[ G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R_2}{R_1 + R_2} \left( 1 + \frac{Z_N(s)}{Z(s)} \right) \left( 1 + \frac{Z_D(s)}{Z(s)} \right) \]

\[ Z_D(s) = R_1 || R_2 \]

\[ Z_N(s) = R_1 \]
Quick Example - EET

\[ G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R_2}{R_1 + R_2} \left( \frac{1 + \frac{Z_N(s)}{Z(s)}}{1 + \frac{Z_D(s)}{Z(s)}} \right) \]

\[ Z_D(s) = R_1 || R_2 \]

\[ Z_N(s) = R_1 \]

\[ G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R_2}{R_1 + R_2} \left( \frac{1 + sCR_1}{1 + sC(R_1 || R_2)} \right) \]
Quick Example - EET

\[ G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R_2}{R_1 + R_2} \left( \frac{1 + \frac{Z_N(s)}{Z(s)}}{1 + \frac{Z_D(s)}{Z(s)}} \right) \]

\[ Z_D(s) = R_1 || R_2 \]

\[ Z_N(s) = R_1 \]

\[ G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R_2}{R_1 + R_2} \left( \frac{1 + sC R_1}{1 + sC (R_1 || R_2)} \right) \]

\[ = \left( \frac{R_2}{R_1 + R_2} \right) \frac{1 + sC R_1}{1 + sC (R_1 || R_2)} \]
Applications to Power Electronics

Transfer function
\[ G(s) \bigg|_{Z(s) \to \infty} \]

Linear circuit
Input
Output
Port

Open-circuit

\[ v_{in}(s) \quad + \quad - \]

\[ + \quad v_{out}(s) \quad - \]
Input Filter for SMPS

Control-to-output transfer function is

\[ G_{vd}(s) = \frac{\hat{v}(s)}{\hat{d}(s)} \bigg|_{\hat{v}_g(s) = 0} \]
Effect of Input Filter on Transfer Function

Effect of $L-C$ input filter on control-to-output transfer function $G_{vd}(s)$, buck converter example.

*Dashed lines:* original magnitude and phase

*Solid lines:* with addition of input filter
Early Days of Power Electronics

Fig. 6 Polar plot of the normalized control DF $H_c'$, showing the two phasor components at $\omega/\omega_s = 1/2, 1, 3/2, \cdots$, and the single phasor component at other frequencies.

Fig. 9 Experimental magnitude vs. frequency plot of $H_c'$ for voltage drive and for current drive.
AVERAGED MODELING
Historical Perspective on SMPS Modeling

Slobodan Cúk

• Ph.D., CalTech 1976
  • Averaged modeling of SMPS
  • Extension of Gene Wester (1972) thesis
• CalTech Professor, 1977-1999

• Inventor and namesake of the Cúk converter
• Still active in conferences in the field

Timeframe:
1968 First microprocessor
1970 First Power Electronics Specialist Conference
1976 Apple I
1985 Windows 1.0
Large Signal Modeling of SMPS
In switch position 1

\[
\begin{align*}
    v_g(t) - v_c(t) &= L \frac{di_L(t)}{dt} \\
    i_L(t) - \frac{v_c(t)}{R} &= C \frac{dv_c(t)}{dt}
\end{align*}
\]
Linear Circuit Modeling Using State Space

In switch position 1
\[
\begin{align*}
  v_g(t) - v_c(t) &= L \frac{di_L(t)}{dt} \\
  i_L(t) - \frac{v_c(t)}{R} &= C \frac{dv_c(t)}{dt}
\end{align*}
\]

Which can be written, in state space, form as
\[
\frac{d}{dt} \begin{bmatrix} i_L(t) \\ v_c(t) \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ L & -1 \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_c(t) \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} v_g(t)
\]

Or, generally,
\[
\dot{x}(t) = A_1 x(t) + B_1 u(t)
\]

In the second switch position, we will have a new (linear) circuit with
\[
\dot{x}(t) = A_2 x(t) + B_2 u(t)
\]
Switching Signal

In a PWM converter with two switch positions, the two linear circuits combine according to a switching function \( s(t) \)

\[
\dot{x}(t) = [A_1 s(t) + A_2 s'(t)]x(t) + [B_1 s(t) + B_2 s'(t)]u(t)
\]

where

\[
s(t) = \begin{cases} 
1, & \text{if } nT_s < t < (n + D)T_s \\
0, & \text{if } (n + D)T_s < t < (n + 1)T_s 
\end{cases}
\]

\[
s'(t) = 1 - s(t)
\]
SMPS State Space

In traditional state space modeling of LTI systems

\[ \dot{x}(t) = Ax(t) + Bu(t) \]

with \( u(t) \) containing a control input. When \( A \) and \( B \) are constant, this is a linear system. However, we have

\[ \dot{x}(t) = [A_1 s(t) + A_2 s'(t)]x(t) + [B_1 s(t) + B_2 s'(t)]u(t) \]

or, equivalently

\[ \dot{x}(t) = A(t)x(t) + B(t)u(t) \]

which is time-varying: how do we deal with it?
Converting to Linear System

Assume that our system model

\[ \dot{x}(t) = [A_1 s(t) + A_2 s'(t)]x(t) + [B_1 s(t) + B_2 s'(t)]u(t) \]

can be approximated by some linear system

\[ \dot{x}(t) = Ax(t) + Bu(t) \]

which removes the nonlinearity of the system

- Nonlinearities came from switching
- Expect that switching dynamics will be lost

Note: This system is now LTI in \( x(t) \) and \( u(t) \), but not in our control signal, \( s(t) \)
Approximate Steady State Waveforms

\[ x(t) \quad x(0) \quad x(T_s) \]

\[ (n)T_s \quad (n+1)T_s \]

\[ A_1 x(t) + B_1 u(t) \]

\[ A_2 x(t) + B_2 u(t) \]

\[ x(NT_s) \]
Approximate Steady State Waveforms

\[ x(t) = \frac{1}{T_s} \int_0^{T_s} x(t) dt \]
Approximate Steady State Waveforms
The Averaging Approximation

If waveforms can be approximated as linear

\[
\dot{x}(t) = \begin{cases} 
A_1\langle x(t) \rangle + B_1\langle u(t) \rangle, & \text{if } nT_s < t < (n + D)T_s \\
A_2\langle x(t) \rangle + B_2\langle u(t) \rangle, & \text{if } (n + D)T_s < t < (n + 1)T_s
\end{cases}
\]

so the average slope is

\[
\langle \dot{x}(t) \rangle = \frac{1}{T_s} (A_1\langle x(t) \rangle + B_1\langle u(t) \rangle)DT_s + (A_2\langle x(t) \rangle + B_2\langle u(t) \rangle)(1 - D)T_s
\]

or, rearranging

\[
\langle \dot{x}(t) \rangle = (DA_1 + D'A_2)\langle x(t) \rangle + (DB_1 + D'B_2)\langle u(t) \rangle
\]
The Averaged System

This equation is now the model of a new, equivalent linear system

\[ \dot{x}(t) = Ax(t) + Bu(t) \]

where

\[ A = DA_1 + D'A_2 \]
\[ B = DB_1 + D'B_2 \]

which has averaged behavior over one switching period.

This approximation is *perhaps* valid, if

- State waveforms are dominantly linear
- Dynamics of interest are at \( f_{bw} \ll f_s \)
Averaged Dynamic Modeling

• System is linear with respect to inputs $U$
• Still, the control input $(D)$ is hidden in the state matrices
• Variations in $D$ will result in time-varying nonlinear state matrices
• Find dynamic model through small-signal analysis
  − Linearize around a steady-state operating point
Response to State Perturbations
Response to State Perturbations

\[
\dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{u}(t) + \hat{d}(t)
\]
Response to Control Perturbations
Response to Control Perturbations
Response to Control Perturbations

\[ (A_1 \langle x(t) \rangle + B_1 \langle u(t) \rangle) \hat{d}T_s - (A_2 \langle x(t) \rangle + B_2 \langle u(t) \rangle) \hat{d}T_s \]
Complete Small Signal Model

So, the complete small signal system is

$$\dot{x}(t) = A\dot{x}(t) + B\dot{u}(t) + \{(A_1 - A_2)X + (B_1 - B_2)U\}\hat{d}(t)$$

Where $X$ and $U$ are the steady-state operating point
Return to Design-Oriented Analysis

Rather than averaging state space, can average circuit or switches to arrive at the same result

Full Switching Model

Averaged, Nonlinear

Linearized Small-Signal Model
Fig. 2.1 Flowchart of averaging approaches in modeling switching dc-to-dc converters, leading to the canonical circuit model (block 5). Path a: general state-space modeling; Path b: circuit transformation method.
Using same approach, equivalent circuit structure is the same for every converter with:

- CCM, PWM operation
- One inductor and one capacitor
- Cuk thesis proposes computer-aided design from the canonical model
Example Averaged Model

Full Switching Model

Averaged, Nonlinear

DC Averaged Model

Linearized Small-Signal Model
Remaining Questions

• How valid are the averaging approximations made in the derivation?
• Under what conditions do these apply?
CCM Buck Converter

[Diagram of CCM Buck Converter]
ZVS-QSW Buck Converter
QR Push Pull Converter

![Diagram of QR Push Pull Converter]

- $v_{ds1}$ = 20 V / div, $v_{ds2}$ = 20 V / div, $i_{lp1}$ = 50 A / div
- $i_{lp2}$ = 50 A / div, time = 2 µs / div
Switched Capacitor Converters
Averaging: Discussion

\[ \langle x(t) \rangle_{T_s} = \frac{1}{T_s} \int_{t-T_s/2}^{t+T_s/2} x(\tau) d\tau \]

\[ G_{av}(j\omega) = \frac{e^{j\omega T_s/2} - e^{-j\omega T_s/2}}{j\omega T_s} = \frac{\sin(\omega T_s/2)}{\omega T_s/2} \]

Averaging removes switching frequency ripple and harmonics.
Discrete Time Nature of PWM
Discrete Time Nature of PWM

---

Diagram showing the discrete time nature of PWM with waveforms and timing intervals. The diagram illustrates the relationship between the control voltage $v_c(t)$, the modulation signal $v_m(t)$, the PWM output $s(t)$, and the time intervals $(n)T_s$, $(n+D)T_s$, and $(n+1)T_s$. The graphical representation is a visual explanation of how PWM operates in discrete time.
Discrete Time Nature of PWM
DISCRETE TIME MODELING
Historical Perspective on SMPS Modeling

Dennis John Packard (?)
• Ph.D., CalTech 1976
  • Discrete Time Modeling of SMPS
• Post-graduation unknown
• Dissertation appears to be his only publication

• Model without averaging
• Method accurate across entire frequency range
• In practice, still used linearization of dynamics
  – Applicable to converters under study at the time
• Developed techniques contemporary with Cúk
Large Signal Modeling of SMPS

Original converter:

- Switch in position 1

- Switch in position 2
Discrete Time Modeling

• Every subcircuit is a passive, linear circuit
• Passive, linear circuits can be solved in closed-form
  − Can model states at discrete times without averaging
• Only assumptions required
  − Independent inputs are DC or slowly varying
  − Small-signal perturbations
Solution to State Space Equation

Closed form solution to state space equation

$$\dot{x}(t) = Ax(t) + Bu(t)$$

Multiply both sides by $e^{-At}$

$$e^{-At}\dot{x}(t) - e^{-At}Ax(t) = e^{-At}Bu(t)$$

Left-hand side is

$$\frac{d}{dt}(e^{-At}x(t)) = e^{-At}Bu(t)$$
Solution to State Space Equation

\[
\frac{d}{dt} \left( e^{-At}x(t) \right) = e^{-At}Bu(t)
\]

Can now be solved by direct integration

\[
e^{-At}x(t) - x(0) = \int_{0}^{t} e^{-A\tau}Bu(\tau) \, d\tau
\]

Rearranging

\[
x(t) = e^{At}x(0) + \int_{0}^{t} e^{-A(t-\tau)}Bu(\tau) \, d\tau
\]
Matrix Exponential

Matrix exponential defined by Taylor series expansion

\[ e^{At} = I + At + \frac{(At)^2}{2!} + \cdots + \frac{(At)^N}{N!} = \sum_{k=0}^{N} \frac{(At)^k}{k!} \]

Well-known issue with convergence in many cases

Properties of the Matrix Exponential

• Matrix exponential always exists
  - i.e. summation will always converge

• Exponential of any matrix is always invertible, with

\[ e^A e^{-A} = I \]
First Order Taylor Series Expansion

Linear ripple approximation

\[ e^{At} \approx I + At \]

Valid only if switching frequency much faster than system modes
Simplification for Slow-Varying Inputs

\[ x(t) = e^{At}x(0) + \int_{0}^{t} e^{-A(t-\tau)} Bu(\tau) \, d\tau \]

If A is invertible and \( u(\tau) \approx U \)

\[ x(t) = e^{At}x(0) + A^{-1}(e^{At} - I)BU \]
Application to Switching Converter
Application to Switching Converter

\[ x(t) \]

\[ x(0) \]

\[ x(T_s) \]

\[ x(DT_s) \]

\[ A_1 x(t) + B_1 u(t) \]

\[ A_2 x(t) + B_2 u(t) \]
Application to Switching Converter

\[ x(DT_s) = e^{A_1 DT_s}x(0) + A_1^{-1}(e^{A_1 DT_s} - I)B_1 U \]
Application to Switching Converter

\[ x(DT_s) = e^{A_1 DT_s} x(0) + A_1^{-1} (e^{A_1 DT_s} - I) B_1 U \]

\[ x(T_s) = e^{A_2 D'T_s} x(DT_s) + A_2^{-1} (e^{A_2 D'T_s} - I) B_2 U \]
Application to Switching Converter

\[ x(DT_s) = e^{A_1DT_s}x(0) + A_1^{-1}(e^{A_1DT_s} - I)B_1U \]

\[ x(T_s) = e^{A_2D'T_s}x(DT_s) + A_2^{-1}(e^{A_2D'T_s} - I)B_2U \]

\[ x(T_s) = e^{A_2D'T_s}e^{A_1DT_s}x(0) + A_2^{-1}(e^{A_2D'T_s} - I)B_2U + e^{A_2D'T_s}A_1^{-1}(e^{A_1DT_s} - I)B_1U \]
General Form

Generally, for \( n_{sw} \) separate switching positions

\[
x(T_s) = \left( \prod_{i=n_{sw}}^{1} e^{A_i t_i} \right) x(0) + \sum_{i=1}^{n_{sw}} \left\{ \left( \prod_{k=n_{sw}}^{i+1} e^{A_k t_k} \right) A_i^{-1} (e^{A_i t_i} - I) B_i \right\} U
\]

Equation is in the form of a discrete-time system with

\[
x[n + 1] = \Phi x[n] + \Psi U[n]
\]

Again, the effect of changing modulation (i.e. \( t_i \)) is hidden in nonlinear terms

\[
\hat{x}[n + 1] = \Phi \hat{x}[n] + \Psi \hat{u}[n] + \Gamma \hat{d}[n]
\]

Find \( \Gamma \) by small-signal modeling
Small Signal Modeling
Small Signal Modeling

![Graph of small signal modeling with points labeled as follows: x(t), x(0), x(DT), x(T) at times (n)T_s, (n+1)T_s, (n+2)T_s. The graph illustrates the behavior of a signal over time, showing an initial condition x(0) and a perturbed signal x(DT) with a small time increment ΔT_s.]
Small Signal Modeling

\[ \hat{x}_d = (A_1 x(DT_s) + B_1 U) \Delta T_s - (A_2 x(DT_s) + B_2 U) \Delta T_s \]
Complete Small Signal Model

This completes the small-signal model

\[ \hat{x}[n + 1] = \Phi \hat{x}[n] + \Psi \hat{u}[n] + \Gamma \hat{d}[n] \]

where

\[ \Gamma = e^{A_2 D' T_s} \left( (A_1 - A_2) X_D + (B_1 - B_2) U \right) T_s \]

with \( X_D = x(DT_s) \) in steady-state.
Example Results

* Includes $t_d=760\text{ns}$ of delay in feedback loop

L. Corradini et. al. Digital Control of High Frequency Switched-Mode Power Converters, Section 3.2
Inclusion of Delay

\[ G_{vu}^\dagger(s) = G_{vu}(s)e^{-st_d} \]
Current Control

Averaged Model

DT Model

L. Corradini et. al. Digital Control of High Frequency Switched-Mode Power Converters, Section 3.2
Natural Response Comparison

Averaged model natural response

\[ \dot{\hat{x}}(t) = A\hat{x}(t) = (DA_1 + D'A_2)\hat{x}(t) \]

And discrete time natural response

\[ \hat{x}[n + 1] = \Phi\hat{x}[n] = e^{A_2D'T_s}e^{A_1D'T_s}\hat{x}[n] \]

Use Tustin interpolation of difference equation to convert discrete time to continuous time system

\[ \hat{x}_{DT}(t) = \frac{\hat{x}[n + 1] - \hat{x}[n]}{T_s} \]

\[ = \frac{(e^{A_2D'T_s}e^{A_1D'T_s} - I)\hat{x}[n]}{T_s} \]
Natural Response Comparison

\[
\dot{\hat{x}}_{DT}(t) = \frac{(e^{A_2D'T_s}e^{A_1DT_s} - I)\hat{x}[n]}{T_s}
\]

Taylor series to 1\textsuperscript{st} order (linear ripple approx.)

\[
\dot{\hat{x}}_{DT}(t) \approx \frac{((I + A_2D'T_s)(I + A_1DT_s) - I)\hat{x}[n]}{T_s}
\]

\[
\approx \frac{(I + A_1DT_s + A_2D'T_s + A_1A_2DD'T_s^2 - I)\hat{x}[n]}{T_s}
\]

\[
\approx (DA_1 + D'A_2)\hat{x}[n]
\]
Comparison: Discussion

- Averaged and discrete time formulations are equivalent if
  - Ripple in states is
    1. Small, such that $\hat{x}[n] \approx \hat{x}(t)$
    2. Linear, so $e^{A_i t_i} \approx (I + A_i t_i)$
    3. Low frequency, such that $T_s^2 \ll \|A_1 A_2\|$
The Rest is History...

Averaged Modeling

\[
x(T_s) = \left( \prod_{i=n_{sw}}^{1} e^{A_i t_i} \right) x(0) + \sum_{i=1}^{n_{sw}} \left\{ \left( \prod_{k=n_{sw}}^{i+1} e^{A_k t_k} \right) A_i^{-1}(e^{A_i t_i} - I)B_i \right\} U
\]
Comparison of Modeling Approaches

“State-space averaging, while possessing a very convenient continuous, time-invariant form, and having been successful in many applications, is inaccurate when the frequencies of interest approach one-half the fundamental switching frequency of the converter.

The discrete modeling technique, while very accurate, requires the abandonment of the usual continuous time model in favor of difference equations, which are unfamiliar to the circuit designer and do not reflect the continuous nature of the converter waveforms.”

Sampled Data Modeling: Intermediary

- Approximate sampling and delay effects while retaining averaged model
- Sufficiently accurate when linear ripple approximation applies

The Dual Active Bridge (DAB)
Example Waveforms

D Costinett, H Nguyen, R Zane, and D Maksimovic, “GaN-FET Based Dual Active Bridge DC-DC Converter,” APEC 2011
Linear Averaged Modeling of DAB

- Assume squarewave outputs of both bridges
- Average output current solved algebraically

\[
\langle i_o \rangle \bigg|_{T_S} = \frac{1}{T_S} \int_0^{T_S} i_o(t) dt = \frac{v_g(t)}{n_t L_i T_S} \left( T_s t_\varphi - 2t_\varphi^2 \right)
\]

\[
G_{v_\varphi}(s) = \frac{\hat{v}_{out}}{\hat{t}_\varphi} = \frac{i_o}{\hat{t}_\varphi} \cdot \frac{1}{sC_{out}}
\]

- Simple, one-state averaged system
Experimental Verification $G_{v\phi}$

- Unmodeled ZVS behaviors significantly alter converter dynamics
- HF behaviors aliased down to low-frequency

D Costinett, “Reduced Order Discrete Time Modeling of ZVS Transition Dynamics in the Dual Active Bridge Converter”, APEC 2015
State Space Modeling

- Three-state system
  \[ x = [v_p \ i_l \ v_{out}]^T \]
- In each switching subinterval
  \[ \dot{x}(t) = A_i x(t) + B_i u(t) \]
  \[ y(t) = C_i x(t) \]
DAB Waveforms

\[ e^{A_i t_i} \]
Linear Waveform Approximation

\[ e^{A_i t_i} \approx I + A_i t_i \]
Second Order Approximation

- Possible to empirically select only dominant terms

\[ e^{\mathbf{A}t_i} \approx I + \mathbf{A}_i t_i \approx I + \mathbf{A}_i t_i + \frac{(\mathbf{A}_i t_i)^2}{2!} \]

DT Modeling

\[ \Phi = e^{A_6 t_6} e^{A_5 t_5} e^{A_4 t_4} e^{A_3 t_3} e^{A_2 t_2} e^{A_1 t_1} \]

\[ \Gamma = e^{A_6 t_6} e^{A_5 t_5} e^{A_4 t_4} e^{A_3 t_3} (A_2 - A_3) X_p \frac{T_s}{2\pi} - e^{A_6 t_6} (A_5 - A_6) X_p \frac{T_s}{2\pi} \]
DT Modeling – Half-Cycle

\[
\Phi = e^{A_3 t_3} e^{A_2 t_2} e^{A_1 t_1} I_{HC} \frac{T_s}{2\pi}
\]

\[
\Gamma = e^{A_3 t_3} (A_2 - A_3) X_{p1} \frac{T_s}{2\pi}
\]

\[
I_{HC} = \begin{bmatrix}
-1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & +1
\end{bmatrix}
\]
Discrete Time Model Accuracy

- Discrete time model matches experimental results
- ZVS dynamics indeed responsible for averaged model error
Order of System

- 3rd-order state space model, $x = [v_p \ i_l \ v_{out}]^T$
- Resulting transfer function is of the form
  \[
  G_{v\phi}(z) = G_{v\phi0} \frac{1 - q_1 z^{-1}}{(1 - p_1 z^{-1})(1 - p_2 z^{-1})}
  \]
  which is second order.
- Outside of resonant transition $|v_p| = V_g$ is constant, resulting in reduced order.
- **Goal:** eliminate constant state, while maintaining effect of resonant transition on $i_l(t)$, $v_{out}(t)$.
New Approach: Model switching as Disturbance

- Consider how model takes transition dynamics into account:

\[
\Phi = e^{A_3 t_3} e^{A_2 t_2} e^{A_1 t_1} I_{HC}
\]

- Red term models relation between states at start/end of ZVS subinterval

\[
\hat{x}(t_1) = e^{A_1 t_1} \hat{x}(t_0)
\]

with \( A_1 \in \mathbb{R}^{3x3} \)

- Using circuit analysis, solve new matrix \( A_{res} \in \mathbb{R}^{2x2} \) which models how ZVS transition affects states at \( t=t_1 \)
Resonant Transition Matrix

• Maintaining second order, \( x = [i_l \quad v_{out}]^T \)

• New matrix takes the linear form

\[
A_{res} = \begin{bmatrix}
\frac{\partial v_{out}(t_1)}{\partial v_{out}(0)} & \frac{\partial v_{out}(t_1)}{\partial i_l(0)} \\
\frac{\partial i_l(t_1)}{\partial v_{out}(0)} & \frac{\partial i_l(t_1)}{\partial i_l(0)}
\end{bmatrix}
\]

• Linearized with respect to \( x(0) \), rather than time
Resonant Interval Solution

- Linearized relations solved using circuit analysis
  \[
  \frac{\Delta \lambda}{\Delta I} = L_l - L_l \sqrt{1 - \left(\frac{2V_g}{R_0 I_0}\right)^2},
  \]
  \[
  \frac{\Delta Q}{\Delta I} = \frac{2V_g C_p}{I_0},
  \]

- Linearized with respect to state, not time!

Linearized Result

- Assuming balanced operation and small ripple on $V_{out}$

$$\mathbf{A}_{res} = \begin{bmatrix} \frac{\partial v_{out}(t_1)}{\partial v_{out}(0)} & \frac{\partial v_{out}(t_1)}{\partial i_l(0)} \\ \frac{\partial i_l(t_1)}{\partial v_{out}(0)} & \frac{\partial i_l(t_1)}{\partial i_l(0)} \end{bmatrix} = \begin{bmatrix} 1 & \frac{\Delta Q}{\Delta I} & \frac{1}{C_{out}} \\ 0 & 1 - \frac{\Delta \lambda}{\Delta I} & \frac{1}{L_l} \end{bmatrix}$$

- Resulting model is now

$$\mathbf{\Phi} = e^{A_3 t_3} e^{A_2 t_2} \mathbf{A}_{res} \mathbf{I}_{HC}$$

$$\mathbf{\Gamma} = e^{A_3 t_3} (A_2 - A_3) \mathbf{X}_0$$

where $\mathbf{\Phi} \in \mathbb{R}^{2 \times 2}$ and $\mathbf{\Gamma} \in \mathbb{R}^{2 \times 1}$
Model Accuracy

- Very little deviation from full-order model
Resulting Model

\[
\Phi = 1 - \frac{\omega_f^2 t\zeta^2}{2n_t^2} \quad \frac{\Delta Q}{\Delta I} \left( L_n n_t - \frac{t\zeta^2}{2C_{out} n_t} \right) + t\zeta \left( \frac{\Delta \lambda}{\Delta I} - L_n \right) \\
\frac{t\zeta}{L_n n_t} \quad -1 + \frac{\Delta \lambda}{\Delta I} \left( C_{out} n_t - \frac{t\zeta^2}{2L_n n_t} \right) - t\zeta \frac{\Delta Q}{\Delta I} + \frac{t\zeta^2}{2n_t} \frac{w_f^2}{n_t}
\]

• Previously, an accurate, closed-form expression for \( \Phi \) was intractable
• Model now of suitable complexity for design-oriented analysis
OPPORTUNITIES IN CONVERTER DESIGN
Time Dependence

\[ V_g \]

\[ s(t) \]

\[ (n)T_s \quad (n+D)T_s \quad (n+1)T_s \]

\[ L \quad C \quad R \]
Steady-State Large-Signal Analysis

\[
x(T_s) = \left( \prod_{i=n_{sw}}^{1} e^{A_i t_i} \right) x(0) + \sum_{i=1}^{n_{sw}} \left\{ \left( \prod_{k=n_{sw}}^{i+1} e^{A_{k} t_{k}} \right) A_i^{-1}(e^{A_i t_i} - I) B_i \right\} U
\]

In steady-state, \( x(T_s) = x(0) \)

\[
x(T_s) = \left( I - \prod_{i=n_{sw}}^{1} e^{A_i t_i} \right)^{-1} \sum_{i=1}^{n_{sw}} \left\{ \left( \prod_{k=n_{sw}}^{i+1} e^{A_{k} t_{k}} \right) A_i^{-1}(e^{A_i t_i} - I) B_i \right\} U
\]

Gives explicit solution for steady-state operation of any switching circuit
Circuit Parasitics

$V_g$ + $s(t)$ $-$ $C_{ds}$ $L$ $C$ $R$

$V_{gs}$ 2 V/div
$i_d$ 10 A/div
$V_{ds}$ 100 V/div

$t_{d,on}$ $t_{cr}$ $t_{vf}$

Time (ns)
Passive Nonlinearity

\[ L = f(i(t), f_s) \]

\[ C_{ds} = f(v_{ds}) \]
Net Impact on Main States

- Extend perturbation analysis to large-signal behavior
- Can accommodate empirical data
Semiconductor Large-Signal Nonlinearity
Issue with Equivalent Circuit

- Still relies on small-signal model of semiconductors within interval
Conclusions

• Historically, simplified design-oriented analysis tends to win out

• Averaged modeling of power electronics
  + Simple, fast, and intuitive
  − Inaccurate with many modern topologies

• Discrete time modeling of power electronics
  + Accurate for any topology
  − Complex, often only suited to numerical calculation
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