THE UNIVERSITY OF TENNESSEE KNOXVILLE

AICIP RESEARCH

COSC 522 – Machine Learning

Lecture 8 – Gradient Descent

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Questions

- Need to have a big picture on the cost functions and the corresponding optimization approaches we've learned in this semester
- The analogy between Newton's method and Gradient Descent
- What is the geometrical interpretation of GD?
- What is the physical meaning of the learning rate?
- Why does the learning rate need to be very small?

General approach to learning

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- Specify a model (objective function) and estimate its parameters
- Learning algorithms
  - Maximum Posterior Probability (parametric and nonparametric)
  - Maximum Likelihood estimate
  - Fisher's linear discriminant
  - Principal component analysis
  - Kmeans and WTA
- Use optimization methods to find the parameters
  - Exhaustive search through the solution space
  - 1st derivative = 0, Newton-Raphson Method
  - Gradient descent

Newton-Raphson Method

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Used to find solution to equations

According to Taylor series : $f(x+\Delta x) \approx f(x) + \Delta x f'(x)$

$$f(x) + \Delta x f'(x) = 0 \Rightarrow \Delta x = -\frac{f(x)}{f'(x)}$$

$$\Rightarrow x^{k+1} = x^{k} - \frac{f(x)}{f'(x)}$$

Newton-Raphson method vs. Gradient Descent

$f(x) = x^2 - 5x - 4$
$f(x) = x\cos x$

- Newton-Raphson method
- Used to find roots
  - Find x for $f(x) = 0$
- The approach
  - Step 1: select initial x0
  - Step 2:

$$x^{k+1} = x^k - \frac{f(x^k)}{f'(x^k)}$$

  - Step 3: if $|x^{k+1} - x^k| < \varepsilon$, then stop; else $x^k = x^{k+1}$ and go back step 2.

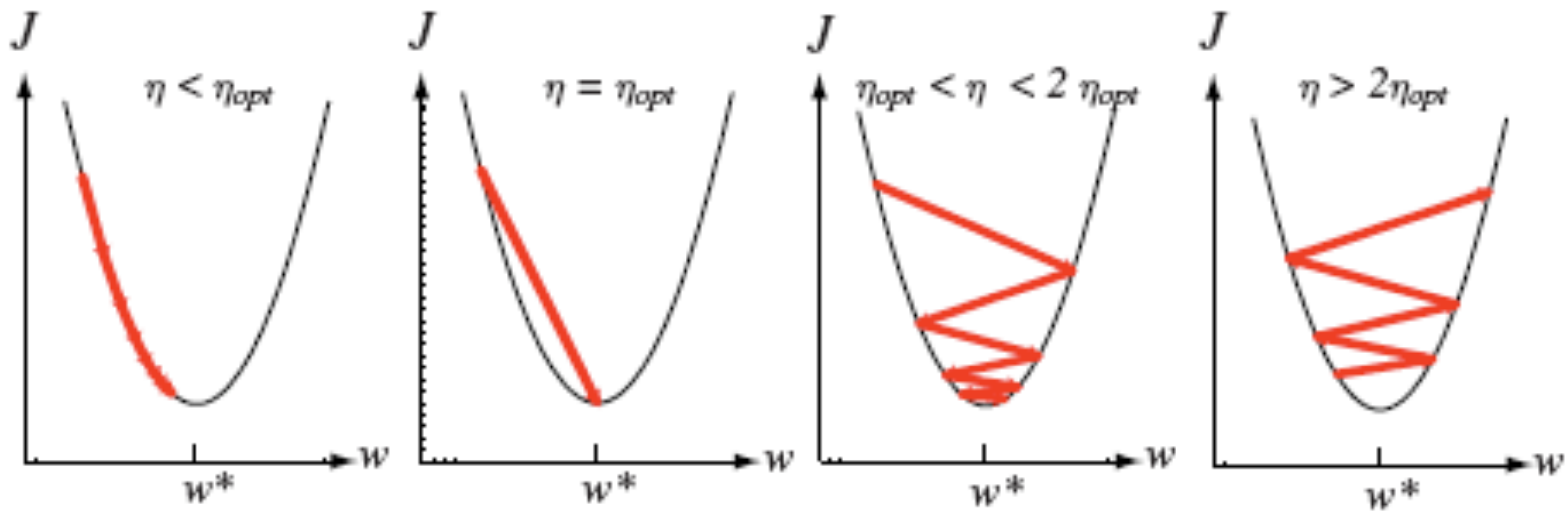
- Gradient descent
- Used to find optima, roots to derivatives
  - Find x* such that $f(x^*) < f(x)$
- The approach
  - Step 1: select initial x0
  - Step 2:

$$x^{k+1} = x^k - \frac{f'(x^k)}{f''(x^k)} = x^k - cf'(x^k)$$

  - Step 3: if $|x^{k+1} - x^k| < \varepsilon$, then stop; else $x^k = x^{k+1}$ and go back step 2.

On the learning rate

$$x^{k+1} = x^k - \frac{f'(x^k)}{f''(x^k)} = x^k - cf'(x^k)$$



Geometric interpretation

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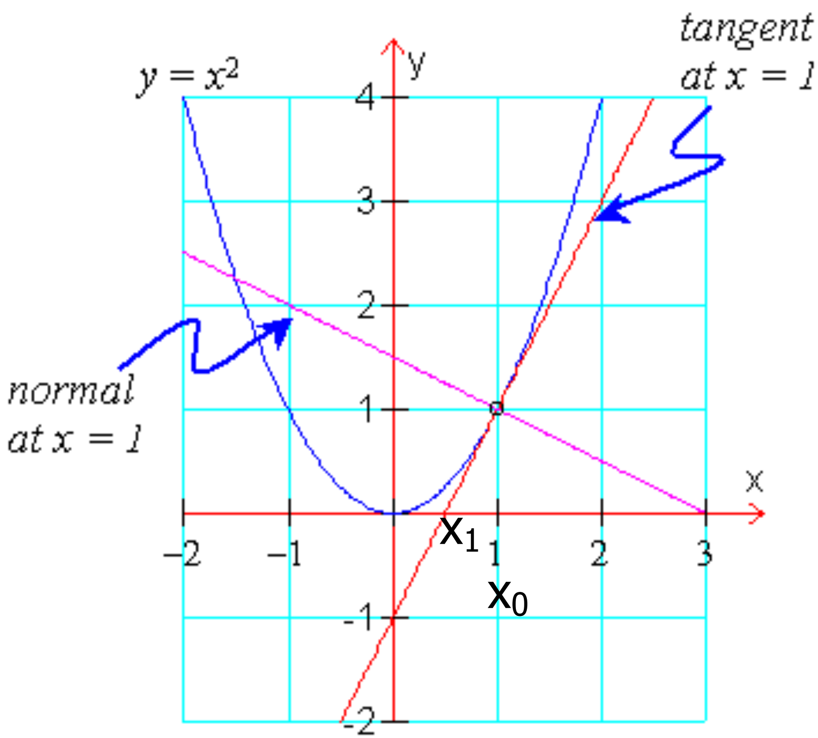

Gradient of tangent is 2

http://www.teacherschoice.com.au/Maths_Library/Calculus/tangents_and_normals.htm