

# COSC 522 – Machine Learning

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## Lecture 9 – Classifier Fusion

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Hairong Qi, Gonzalez Family Professor  
Electrical Engineering and Computer Science  
University of Tennessee, Knoxville

<https://www.eecs.utk.edu/people/hairong-qi/>

Email: [hqi@utk.edu](mailto:hqi@utk.edu)

Course Website: <http://web.eecs.utk.edu/~hqi/cosc522/>

# Roadmap

- Supervised learning
  - Maximum Posterior Probability (MPP):  
For a given  $x$ , if  $P(w_1|x) > P(w_2|x)$ , then  $x$  belongs to class 1, otherwise 2.
    - Parametric Learning
      - Case 1: Minimum Euclidean Distance (Linear Machine),  $\Sigma_i = \sigma^2 I$
      - Case 2: Minimum Mahalanobis Distance (Linear Machine),  $\Sigma_i = \Sigma$
      - Case 3: Quadratic classifier,  $\Sigma_i =$  arbitrary
      - Estimate Gaussian parameters using MLE
    - Nonparametric Learning
      - Parzon window (fixed window size)
      - K-Nearest Neighbor (variable window size)
- Unsupervised learning
  - Non-probabilistic approaches
    - kmeans, wta
  - Hierarchical approaches
    - Agglomerative clustering
- Supporting preprocessing techniques
  - Dimensionality Reduction
    - Supervised linear (FLD)
    - Unsupervised linear (PCA)
    - Unsupervised nonlinear (t-SNE)
- Supporting postprocessing techniques
  - Classifier Fusion

# Questions

- Rationale with fusion?
- Different flavors of fusion?
- The fusion hierarchy
- What is the cost function for Naïve Bayes?
- What is the procedure for Naïve Bayes?
- What is the limitation of Naïve Bayes?
- What is the procedure of Behavior-Knowledge-Space (BKS)?
- How does it resolve issues with NB?
- What is Boosting and what is its difference to committee-based fusion approaches?
- What is AdaBoost?

# Motivation

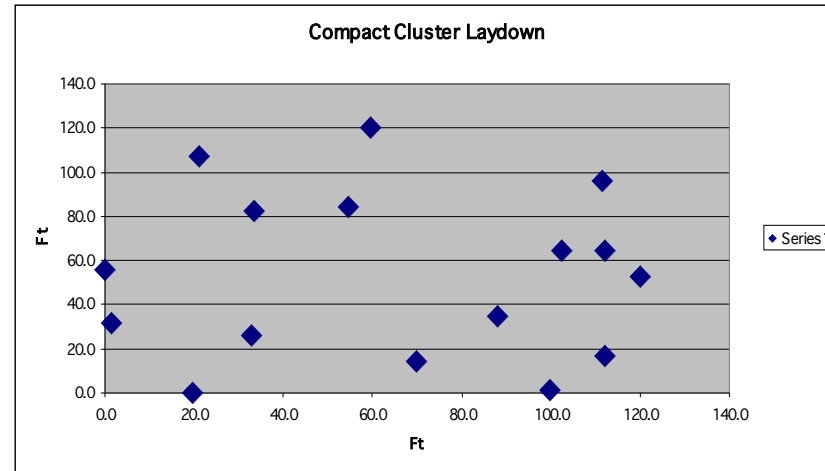
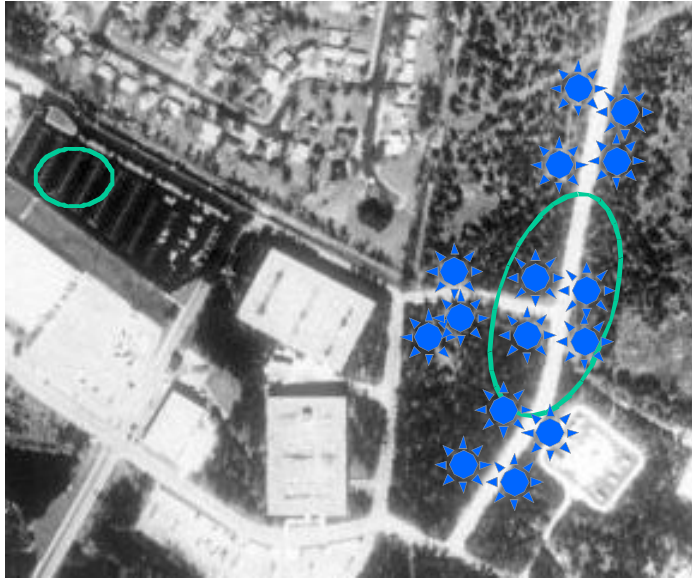
Three heads are better than one.

- Combining classifiers to achieve higher accuracy
  - Combination of multiple classifiers
  - Classifier fusion
  - Mixture of experts
  - Committees of neural networks
  - Consensus aggregation
  - ...
- Reference:
  - L. I. Kuncheva, J. C. Bezdek, R. P. W. Duin, “Decision templates for multiple classifier fusion: an experimental comparison,” *Pattern Recognition*, 34: 299-314, 2001.
  - Y. S. Huang and C. Y. Suen, “A method of combining multiple experts for the recognition of unconstrained handwritten numerals,” *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 17, no. 1, pp. 90–94, Jan. 1995.

# Popular Approaches

- ◆ Data-based fusion (early fusion)
- ◆ Feature-based fusion (middle fusion)
- ◆ Decision-based fusion (late fusion)
  
- ◆ Approaches
  - ◆ Committee-based
    - ◆ Majority voting
    - ◆ Bootstrap aggregation (Bagging) [Breiman, 1996]
  - ◆ Bayesian-based
    - ◆ Naïve Bayes combination (NB)
    - ◆ Behavior-knowledge space (BKS) [Huang and Suen, 1995]
  - ◆ Boosting
    - ◆ Adaptive boosting (AdaBoost) [Freund and Schapire, 1996]
  - ◆ Interval-based integration

# Application Example – Civilian Target Recognition



Ford 250

Harley Motorcycle

Ford 350

Suzuki Vitara



# Consensus Patterns

- Unanimity (100%)
- Simple majority (50%+1)
- Plurality (most votes)

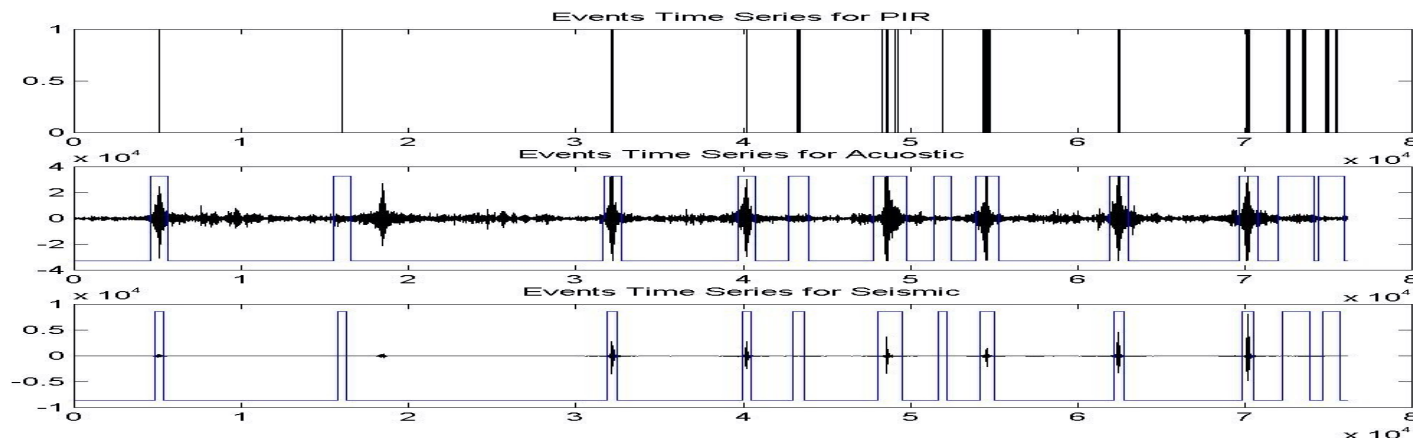
# Example of Majority Voting - Temporal Fusion

- ◆ Fuse all the 1-sec sub-interval local processing results corresponding to the same event (usually lasts about 10-sec)
- ◆ Majority voting

$$\overline{\varphi}_i^j = \arg \max_c \omega_c, \quad c \in [1, C]$$

number of local  
output c occurrence

number of possible local  
processing results





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# Naïve Bayes (the independence assumption)

The real class is DW, the classifier says it's HMV

Confusion matrix

k

C1	AAV	DW	HMV
AAV	894	329	143
DW	99	411	274
HMV	98	42	713

C2	AAV	DW	HMV
AAV	1304	156	77
DW	114	437	83
HMV	13	107	450

i = 1, 2 (classifiers)

L1	AAV	DW	HMV
AAV			
DW			
HMV			

L2	AAV	DW	HMV
AAV			
DW			
HMV			

Probability that the true class is k given that C<sub>i</sub> assigns it to s

Probability multiplication

# NB – Derivation

- Assume the classifiers are **mutually independent**
- Bayes combination - Naïve Bayes, simple Bayes, idiot's Bayes
- Assume
  - L classifiers,  $i=1,\dots,L$
  - c classes,  $k=1,\dots,c$
  - $s_i$ : class label given by the  $i^{\text{th}}$  classifier,  $i=1,\dots,L$ ,  $\mathbf{s}=\{s_1,\dots,s_L\}$

$$P(\omega_k|\mathbf{s}) = \frac{p(\mathbf{s}|\omega_k)P(\omega_k)}{p(\mathbf{s})} = \frac{P(\omega_k) \prod_{i=1}^L p(s_i|\omega_k)}{p(\mathbf{s})}$$

$$P(\omega_k) = N_k/N$$

$$p(s_i|\omega_k) = cm_{k,s_i}/N_k$$

$$P(\omega_k|\mathbf{s}) \approx \frac{1}{N_k^{L-1}} \prod_{i=1}^L cm_{k,s_i}$$

# BKS

- Majority voting won't work
- Behavior-Knowledge Space algorithm (Huang&Suen)

Assumption:

- 2 classifiers
- 3 classes
- 100 samples in the training set

Then:

- 9 possible classification combinations

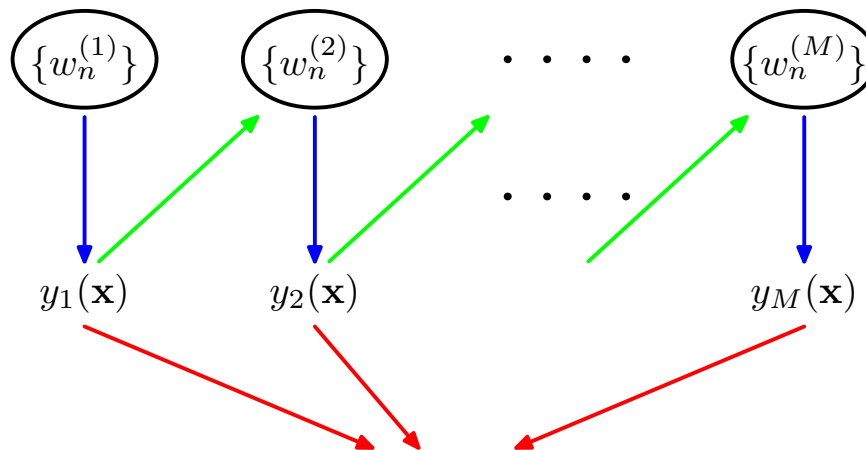
$C_1, C_2$	samples from each class	fused result
1,1	10/3/3	1
1,2	3/0/6	3
1,3	5/4/5	1,3
	...	
3,3	0/0/6	3

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# Boosting

- Base classifiers are trained **in sequence!**
- Base classifiers as **weak learners**
- Weighted majority voting to combine classifiers



$$Y_M(\mathbf{x}) = \text{sign} \left( \sum_m^M \alpha_m y_m(\mathbf{x}) \right)$$

# AdaBoost

- Step 1: Initialize the data weighting coefficients  $\{w_n\}$  by setting  $w_n^{(1)} = 1/N$ , where  $N$  is the # of samples
- Step 2: for each classifier  $y_m(\mathbf{x})$

- (a) Fit a classifier  $y_m(\mathbf{x})$  to the training data by minimizing the weighted error function

$$J_m = \sum_{n=1}^N w_n^{(m)} I(y_m(\mathbf{x}_n) \neq t_n)$$

- (b) Evaluate the quantities

$$\epsilon_m = \frac{\sum_{n=1}^N w_n^{(m)} I(y_m(\mathbf{x}_n) \neq t_n)}{\sum_{n=1}^N w_n^{(m)}}$$

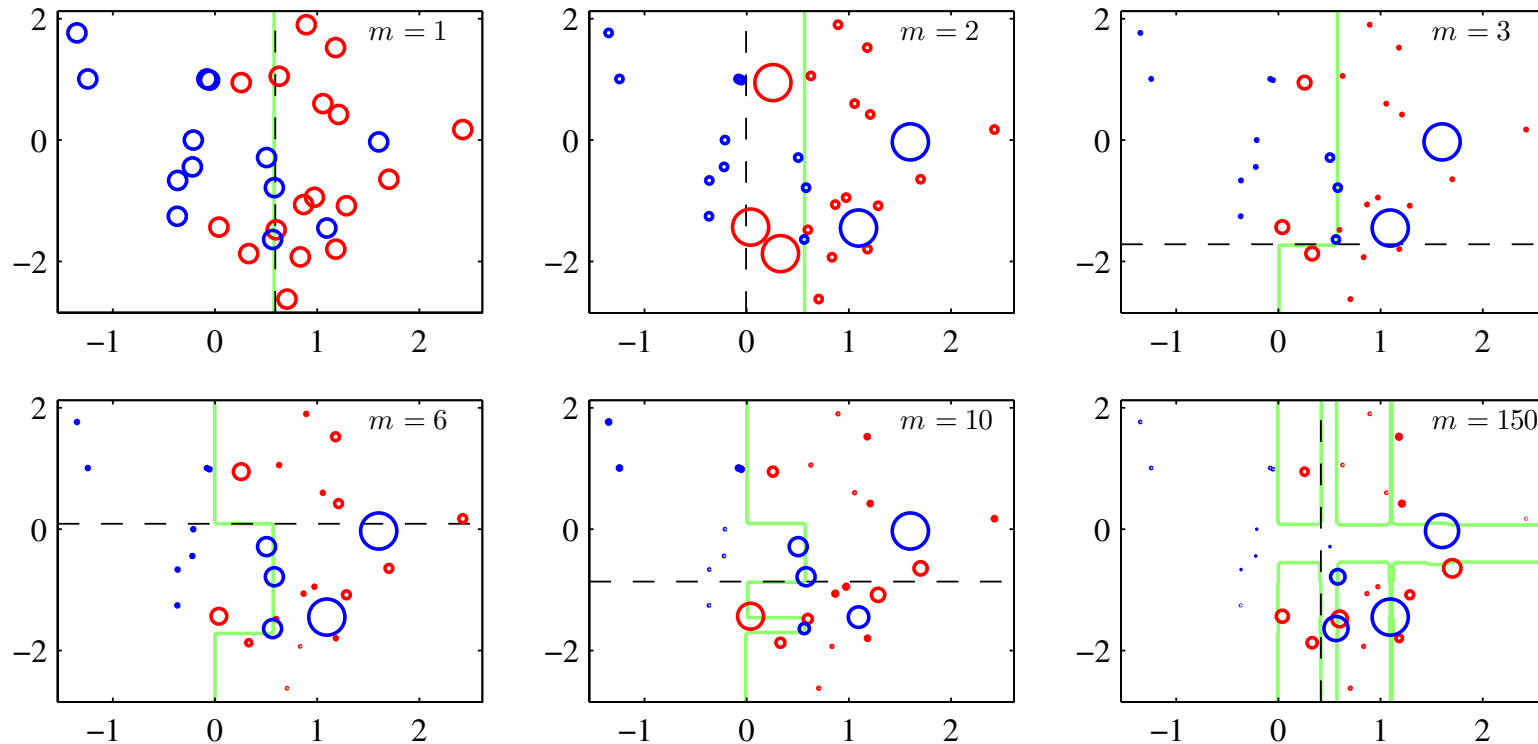
$$\alpha_m = \ln \left\{ \frac{1 - \epsilon_m}{\epsilon_m} \right\}$$

- (c) Update the data weighting coefficients

$$w_n^{(m+1)} = w_n^{(m)} \exp \{ \alpha_m I(y_m(\mathbf{x}_n) \neq t_n) \}$$

- Step 3: Make predictions using the final model

$$Y_M(\mathbf{x}) = \text{sign} \left( \sum_{m=1}^M \alpha_m y_m(\mathbf{x}) \right)$$



**Figure 14.2** Illustration of boosting in which the base learners consist of simple thresholds applied to one or other of the axes. Each figure shows the number  $m$  of base learners trained so far, along with the decision boundary of the most recent base learner (dashed black line) and the combined decision boundary of the ensemble (solid green line). Each data point is depicted by a circle whose radius indicates the weight assigned to that data point when training the most recently added base learner. Thus, for instance, we see that points that are misclassified by the  $m = 1$  base learner are given greater weight when training the  $m = 2$  base learner.

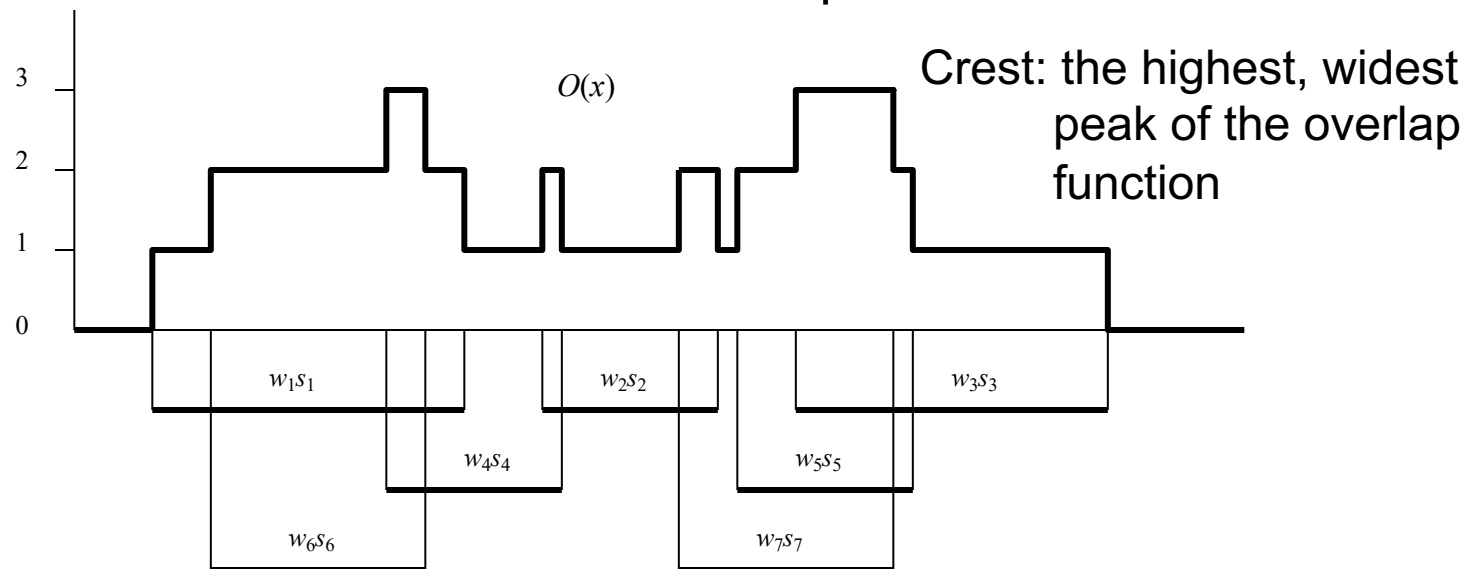


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- **Interval-based fusion**

# Value-based vs. Interval-based Fusion

- Interval-based fusion can provide fault tolerance
- Interval integration – overlap function
  - Assume each sensor in a cluster measures the same parameters, the integration algorithm is to construct a simple function (overlap function) from the outputs of the sensors in a cluster and can resolve it at different resolutions as required



# A Variant of kNN

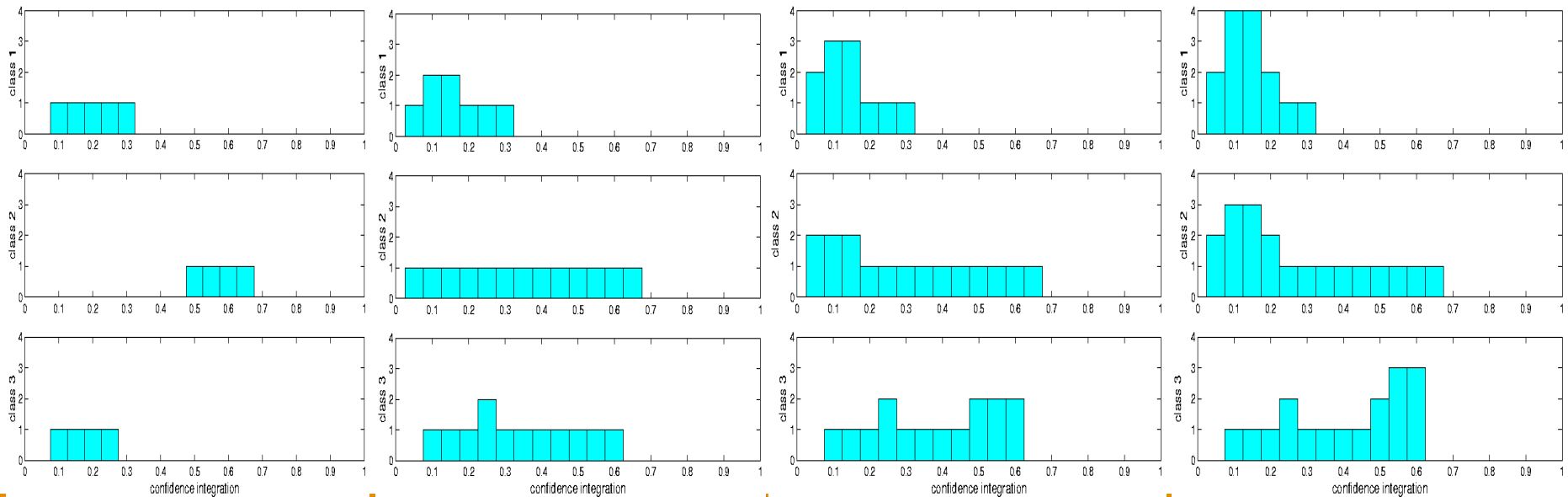
- Generation of local confidence ranges (For example, at each node  $i$ , use kNN for each  $k \in \{5, \dots, 15\}$ )

	Class 1	Class 2	...	Class n	
<b>k=5</b>	3/5	2/5	...	0	← confidence level
<b>k=6</b>	2/6	3/6	...	1/6	
...	...	...	...	...	
<b>k=15</b>	10/15	4/15	...	1/15	
	<b>{2/6, 10/15}</b>	<b>{4/15, 3/6}</b>	...	<b>{0, 1/6}</b>	← confidence range
	smallest	largest in this column			

- Apply the integration algorithm on the confidence ranges generated from each node to construct an overlapping function

# Example of Interval-based Fusion

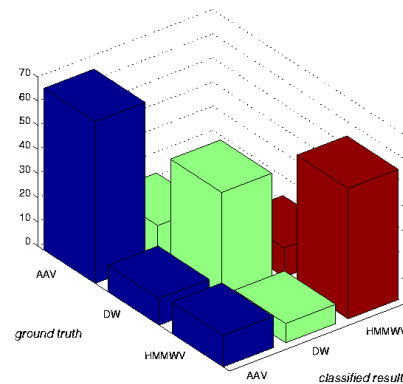
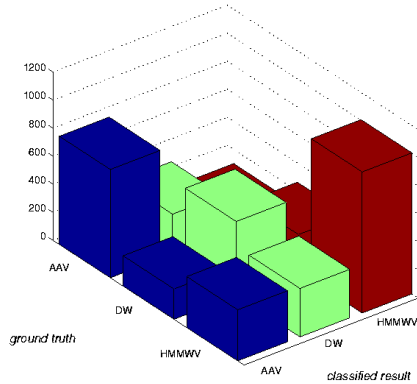
	stop 1		stop 2		stop 3		stop 4	
	c	acc	c	acc	c	acc	c	acc
class 1	1	0.2	0.5	0.125	0.75	0.125	1	0.125
class 2	2.3	0.575	4.55	0.35	0.6	0.1	0.75	0.125
class 3	0.7	0.175	0.5	0.25	3.3	0.55	3.45	0.575



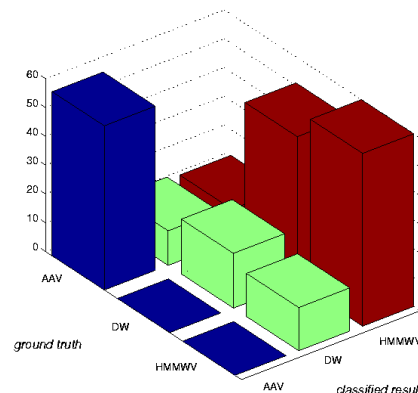
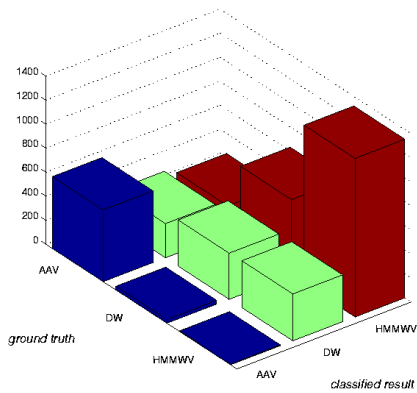
# An example

# Confusion Matrices of Classification on Military Targets

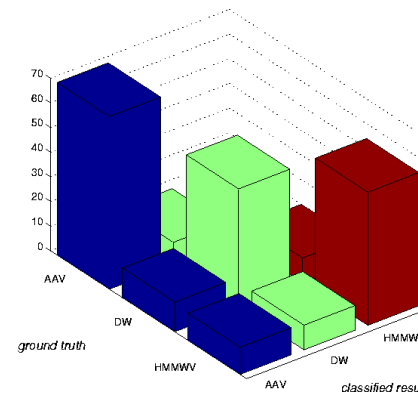
	AAV	DW	HMV
AAV	29	2	1
DW	0	18	8
HMV	0	2	23



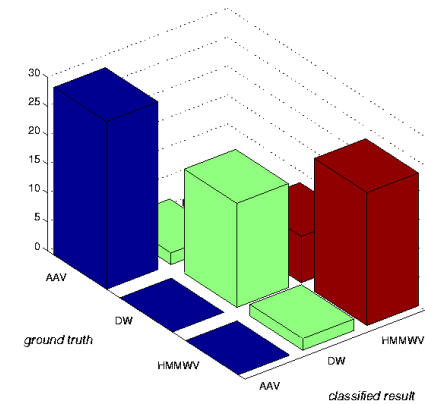
Acoustic (75.47%, 81.78%)



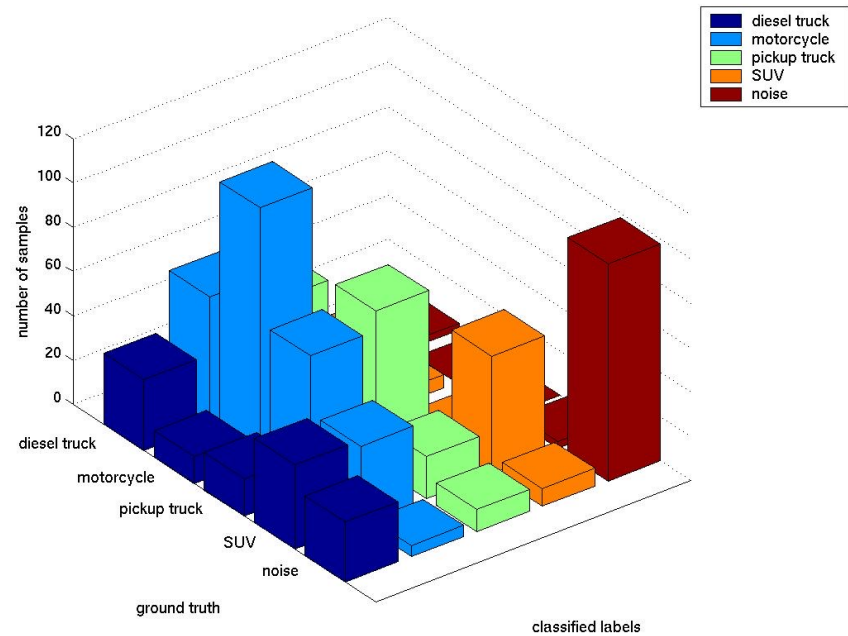
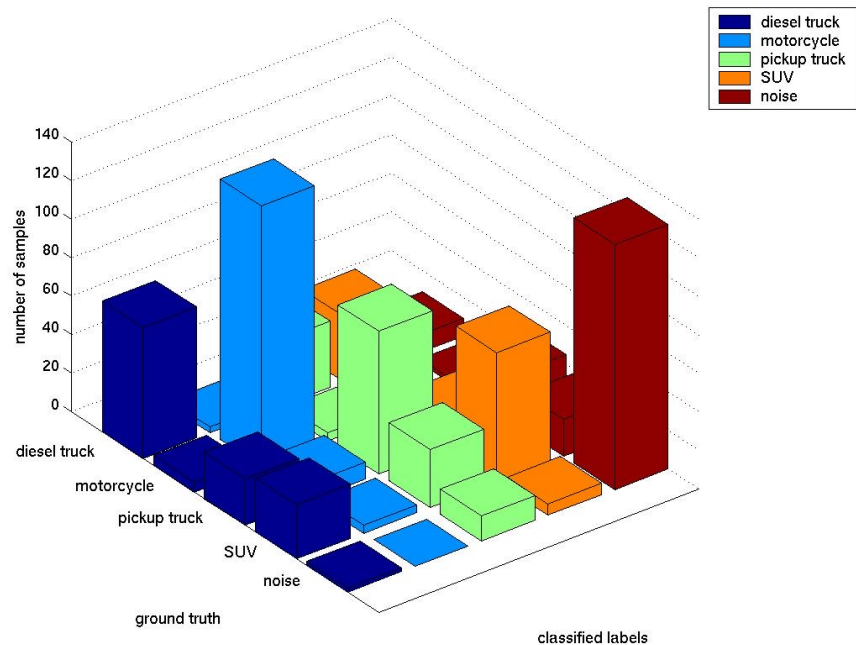
Seismic (85.37%, 89.44%)



Multi-modality fusion (84.34%)

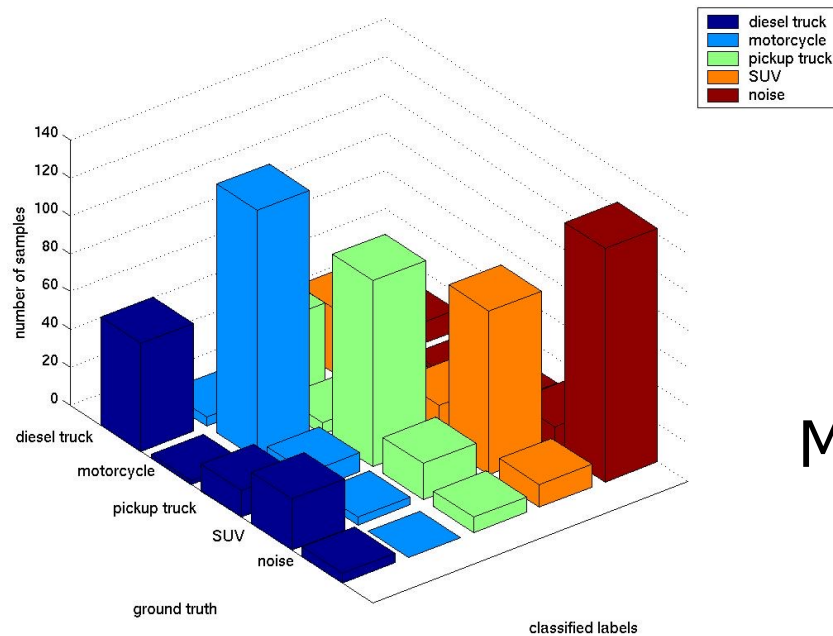


Multi-sensor fusion (96.44%)

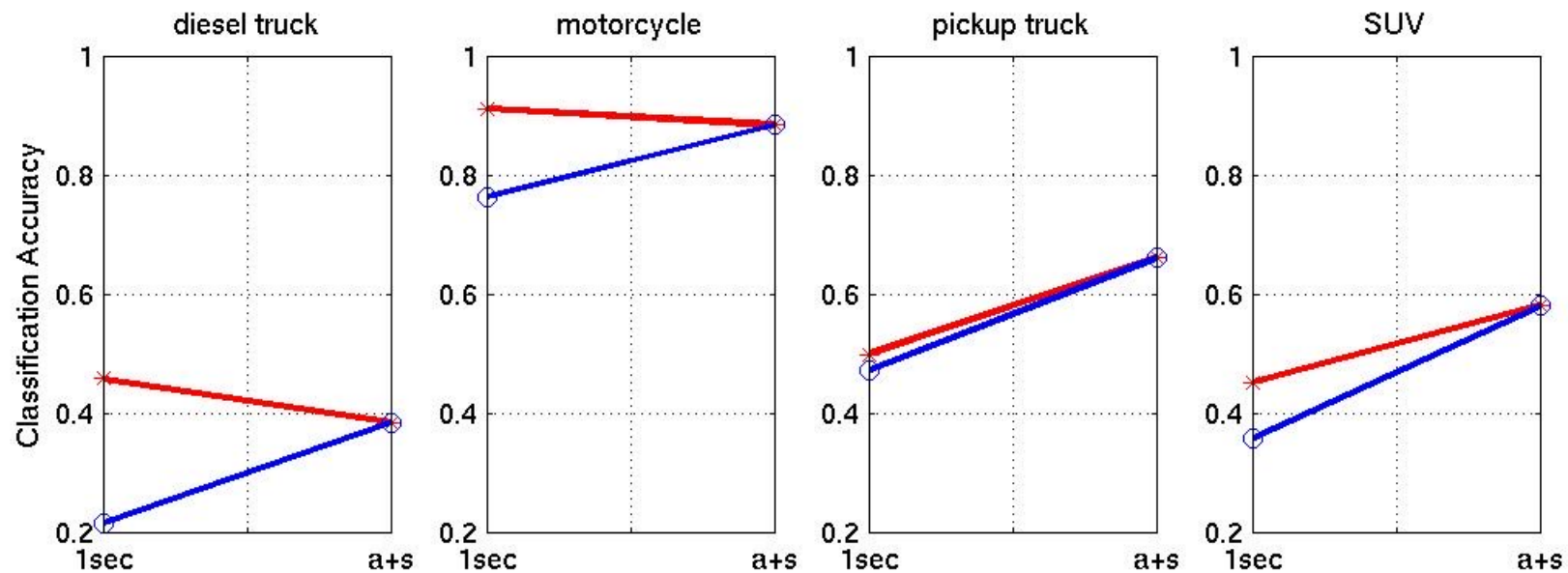


Acoustic

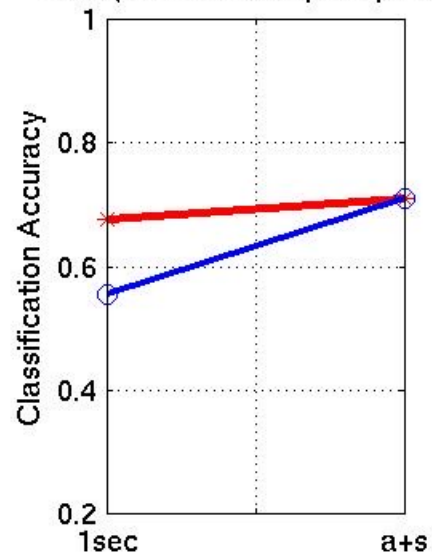
Seismic



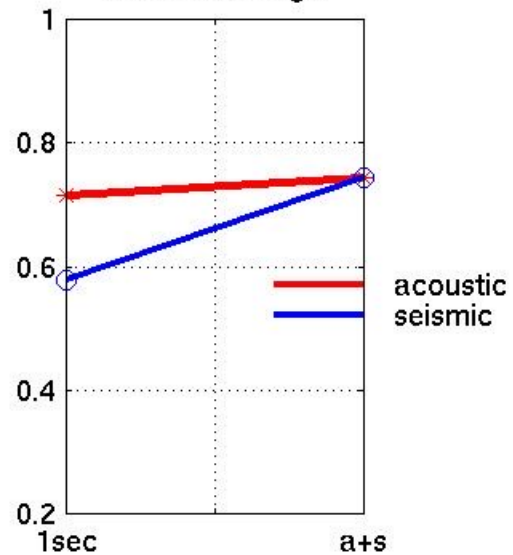
Multi-modal



Truck (diesel truck + pickup truck)



Overall Average





# Reference

- For details regarding majority voting and Naïve Bayes, see

[http://www.cs.rit.edu/~nan2563/combining\\_classifiers\\_notes.pdf](http://www.cs.rit.edu/~nan2563/combining_classifiers_notes.pdf)