Review 2 - Time-Domain System Analysis

1. All signals are “mixture” that can be decomposed into linear combination of some basis signals. There are two classes of basis signals,
   (a) delayed impulses \( \Rightarrow \) time-domain system analysis or convolution
   (b) complex exponentials \( \Rightarrow \) frequency-domain system analysis

2. For discrete-time or continuous-time LTI systems
   (a) the impulse response uniquely or completely characterizes a system where the impulse response \( h \) is the output of the system when the input is an impulse \( \delta \)
      \[
      \delta[n] \rightarrow h[n] \quad DT \\
      \delta(t) \rightarrow h(t) \quad CT
      \]
   (b) representing an arbitrary input signal as weighted sum of delayed impulses
      \[
      x[n] = \sum_{k=-\infty}^{\infty} x[k] \delta[n-k] \quad DT \\
      x(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t-\tau) d\tau \quad CT
      \]
   (c) the convolution
      \[
      y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] \quad \text{convolution sum} \\
      y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau \quad \text{convolution integral}
      \]

3. Properties of LTI systems
   (a) commutative: \( x * h = h * x \)
   (b) distributive: \( x * (h_1 + h_2) = x * h_1 + x * h_2 \)
   (c) associative: \( x * (h_1 * h_2) = (x * h_1) * h_2 = x * h_1 * h_2 \)
   (d) invertible: \( h * h_1 = \delta \) where \( h_1 \) is the impulse response of the inverse system
   (e) BIBO stable: when \( h[n] \) is absolutely summable, i.e., \( \sum_{n=-\infty}^{\infty} |h[n]| < \infty \)
   (f) causal: \( h[n] = 0 \) for \( n < 0 \). For LTI systems, causality is equivalent to the initial rest condition (i.e., if the input to a causal system is 0 up to some point in time, then the output must also be 0 up to that time)

4. How to find \( h \)?
(a) By functionality: For lowpass filters, \( \sum h[n] = 1 \); for highpass filters, \( \sum h[n] = 0 \)

(b) By solving the difference or differential equations (LCDE: Linear Constant-coefficient Differential/Difference Equation). Note that the LCDE itself does not completely characterize a system, LCDE plus the auxiliary condition does. For causal LTI systems, the auxiliary condition is the condition of initial rest.
   
i. Use the method of “forced response,” where \( y(t) = y_p(t) + y_h(t) \)
   
ii. Use the method of “recursion”