ECE471/571 – Homework #1 (Due 01/22)

Note: UG: 100+10, G: 100

Problem 1 (100/70): In a 1-D, 2-class problem, the density functions of both classes are adequately represented by univariate Gaussians, with $\mu_1=4$, $\sigma_1=2$, $\mu_2=6$, $\sigma_2=3$.

(1) (15/10) Sketch the two density functions on the same figure using pencil and paper (i.e., without MATLAB or any other software package). Assume equal prior probability, predict how many decision regions there would be.

(2) (55/30) Assume equal prior probability,

a. (10/5) If x=4.7, which class does x belong to? Use the MAP method. Show detailed steps.

b. (15/10) Find the decision boundary using analytical methods instead of the sketch.

c. (10/5) Write an expression for the probability of error $p(error \mid \omega_1)$ that an error occurs given that the truth is class 1.

d. (20/10) Solve for the overall probability of error.

(3) (30/20) Assume that $P(\omega_1) = 0.6$, $P(\omega_2) = 0.4$.

a. (20/10) Use MATLAB to draw the pdf and the posteriori probability. Comment on the difference.

b. (5/5) Redo question 2 (a)

c. (5/5) Under what condition that there would just be one decision region and two decision regions?

Problem 2 (+10/30): The probability densities representing a two-class pattern are

$$p(y \mid \omega_1) = \begin{cases} \exp(y-2) when (y \le 2) \\ 0 when (y > 2) \end{cases}$$

$$p(y \mid \omega_2) = \begin{cases} \exp(-(y-b)) when (y > b) \\ 0 otherwise \end{cases}$$

The prior probabilities are $P(\omega_1) = P(\omega_2) = 0.5$

(a) (+10/15) Sketch the two densities on the same figure for $b<2$. Show the regions corresponding to the decision rule that minimizes the probability of error.

(b) (0/10) What is $P(error \mid \omega_1)$ in terms of b? (Consider all values of b from -inf to +inf)

(c) (0/5) What is the value of b that maximizes $P(error \mid \omega_1)$?