Spring 2019

ECE471/571 – Homework #2 (Due 02/12/19)

Note: UG: 100+10, G: 100

Data set used in this homework:

X	Y	Class
0.8	1.2	1
0.9	1.4	1
1.2	1.4	1
1.1	1.5	1
0.8	1.1	2
0.6	1	2
0.65	1.1	2
0.75	0.9	2

1) (85/70) Mahalanobis distance vs. Euclidean distance.

   a. (15/15) Manually calculate the mean and covariance of the two classes of training samples. You can use calculator for intermediate calculations. However, you need to show details.

   b. (15/15) Based on the means and covariance matrix, plot the contour maps of the two multi-variate Gaussian distribution for the two classes.

   c. (10/10) Write the equations to calculate these two distances. (Note: ONLY the equation.)

   d. (15/15) Explain intuitively (in no more than three sentences) the differences between the two distances.

   e. (30/15) Use the following example to understand the differences these two distances make in classification. Here, the minimum distance classifier (i.e., Case

I) is used.

i. Plot the above data set on the same figure.

ii. Given a test sample x = $[0.85\ 1.15]^T$, calculate the Euclidean distance to the two class means. Based on the distances, which class should x belong to?

iii. Use the same test sample, calculate the Mahalanobis distance to the two classes. Based on this pair of distances, which class should x belong to?

iv. Plot the test sample x on the same figure as the data set. Just by observing the plot, which decision do you think makes more sense?

2) (15/15) Plot the 2-D Gaussian with the following covariance characteristics. From the plots, elaborate on the physical meaning of each element in the covariance matrix.

a. The off-diagonal elements are zero and the diagonal elements are equal to each other

b. The off-diagonal elements are zero and the diagonal elements are not equal to each other

c. The off-diagonal elements are positive and the diagonal elements are not equal to each other

d. The off-diagonal elements are negative and the diagonal elements are not equal to each other

3) (+10/15) Using maximum likelihood method to derive the equation for mean and variance assuming the pdf (or likelihood) is modeled by 1-D Gaussian.