

Note: UG: 100, G: 100

Problem 1: (25/15) On Gradient Descent (GD). Find the global minimum of $f(t) = 50 * \sin(t) + t^2$ over $-10 \leq t \leq 10$. This problem intends to give you a hands-on experience on how gradient descent works and how it can get trapped at the local minima.

- a) (5/3) Use MATLAB (or whatever toolbox) to plot this function. Visualize the multiple local minima and the global minimum.
- b) Use gradient descent to find the global minimum. (you can use the code provided in the lecture note)
 - i) (8/4) Pick a starting point at $t=7$. What's the minimum? Show the convergence path. Experimenting with different learning rate.
 - ii) (7/3) Pick a starting point at $t=1$. What's the minimum? Show the convergence path. Experimenting with different learning rate.
 - iii) (5/5) Comment on the results from the above experiments

Problem 2: (10/10) On Perceptron. Use Perceptron to implement the OR logic and the XOR logic. Show output from each iteration with the maximum number of iterations being 10. You can use either MATLAB or C/C++.

Problem 3: (25/25) Comparison between FLD, PCA, and Perceptron.

Note that FLD and PCA are dimensionality reduction methods that only output a projection direction. Additional classification methods need to be applied to find the decision boundary. Suppose the minimum (Euclidean) distance (MD) classifier is used. On the other hand, Perceptron is a linear classifier that outputs the decision boundary directly. On the same figure, plot the four samples of the AND gate, and compare the decision boundary from FLD+MD, PCA+MD, and Perceptron. You can use whichever language that you feel comfortable (pencil & paper, MATLAB, C/C++).

Problem 4: (20/20) On decision tree. Assume 100 samples are classified to node N, among which 90 samples actually belong to class 1 and 10 samples actually belong to class 2. For the following two split candidates, which one is better according to the Gini impurity?

- Option 1: 70 class 1 samples go to “left”, 20 class 1 samples and 10 class 2 samples go to “right”

- Option 2: 80 class 1 samples go to “left”, 10 class 1 samples and 10 class 2 samples go to “right”

Problem 5: (20/30) For a data set of four samples, divided into two categories as follows: Note that for 571 students, you need to review some materials related to SVM before the lecture.

| x1 | x2 | label |
|----|----|-------|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

- (a) (0/5) Three classification algorithms have been applied, minimum distance, perceptron, and SVM. The figure showed the decision boundary for minimum distance and perceptron. Show the decision boundary for SVM.
- (b) (0/5) In a heuristic manner, describe how SVM achieves the best generalization capability.
- (c) (20/20) If moving the second sample from (0,1) to (0,0.9), use clustering approaches to classify the samples into two classes using dmin and dmax, respectively.

