ECE 472/572 - Digital Image Processing

Lecture 6 – Geometric and Radiometric Transformation

09/27/11

Roadmap

- Introduction
  - Image format (vector vs. bitmap)
  - IP vs. CV vs. CG
  - HLIP vs. LLIP
  - Image acquisition
- Perception
  - Structure of human eye
  - Brightness adaptation and Discrimination
  - Image resolution
- Image enhancement
  - Enhancement vs. restoration
  - Spatial domain methods
    - Point-based methods
      - Log trans. vs. Power-law
      - Contrast stretching vs. HE
      - Gray-level vs. Bit plane slicing
      - Image averaging (principle)
    - Mask-based methods - spatial filter
      - Smoothing vs. Sharpening filter
      - Linear vs. Non-linear filter
      - Smoothing (average vs. Gaussian vs. median)
      - Sharpening (UM vs. 1st vs. 2nd derivatives)
  - Frequency domain methods
    - Understanding Fourier transform
    - Implementation in the frequency domain
    - Low-pass filters vs. high-pass filters vs. homomorphic filter
- Geometric correction
  - Affine vs. Perspective transformation
    - Homogeneous coordinates
    - Inverse vs. forward transform
    - Composite transformation
  - General transformation
    - Model distortion with polynomial
    - Least square solution

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Questions

- Affine transformation vs. Perspective transformation
- Forward transformation vs. Inverse transformation
- Composite transformation vs. Sequential transformation
- Homogeneous coordinate
- General geometric transformations

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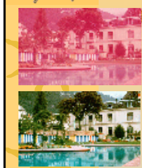
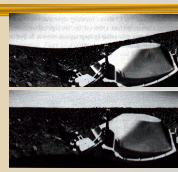
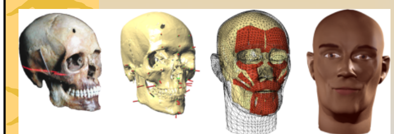
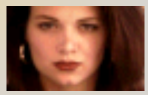
Usage



- Image correction
- Color interpolation
- Forensic analysis
- Entertainment effect







http://www.mpi-sb.mpg.de/resources/FAM/demos.html

http://w3.impa.br/~morph/

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Affine transformations

- Preserve lines and parallel lines
- Homogeneous coordinates
- General form $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Special matrices
  - R: rotation, S: scaling, T: translation, H: shear

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, S = \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix}, T = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix}, H = \begin{bmatrix} 1 & hx & 0 \\ hy & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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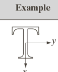
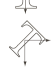
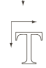
TABLE 2.2
Affine transformations based on Eq. (2.6.–23).

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\cos\theta + w\sin\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

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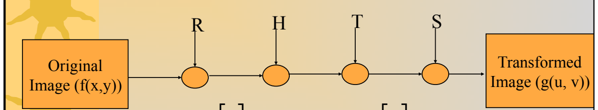
Composite vs. Sequential transformation



$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = S \cdot T \cdot H \cdot R \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$C = S \cdot T \cdot H \cdot R$$

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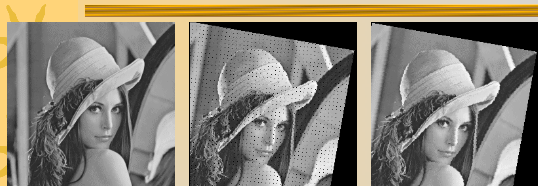
Forward vs. Inverse transforms



$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = C \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Forward transform

$$C^{-1} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Inverse transform

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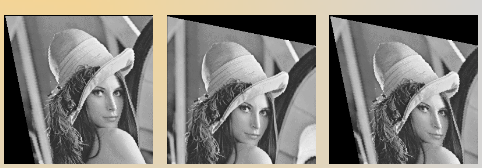
Examples - Shear



hy = 0.2 hx = 0.2 hx = hy = 0.2

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Examples – Translation + Rotation

theta = PI/4

theta = PI/4
tx = -140, ty = 60

Perspective transformation

- Preserve parallel lines only when they are parallel to the projection plane. Otherwise, lines converge to a vanishing point
- General form

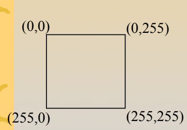
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \quad a_{31} \neq 0, a_{32} \neq 0$$

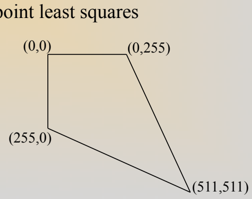
$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, u = \frac{u'}{w'}, v = \frac{v'}{w'}$$

Determine the coefficients

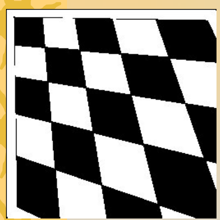
$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, u = \frac{u'}{w'}, v = \frac{v'}{w'}$$

8 unknowns, 4-point least squares





Example - PT



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General approaches

- Find tiepoints
- Spatial transformation

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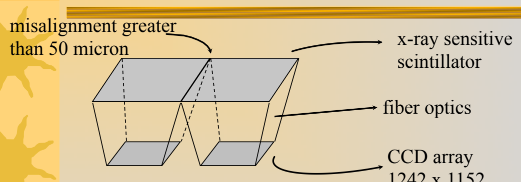
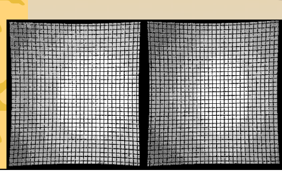
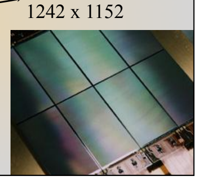
Example – CCD butting

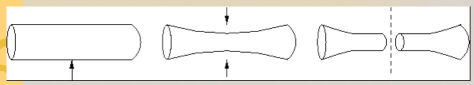






Sources of distortions

- defects in the production of fiber-optic tapers
- imperfect compression and cutting
- different light transfer efficiency across the whole surface



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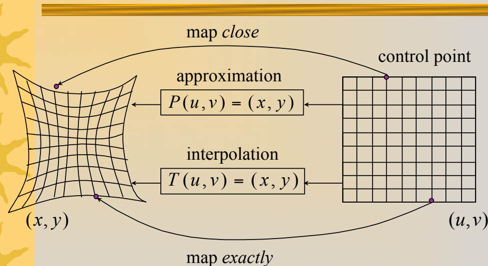
Geometric correction



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Spatial transformation

- Bilinear equation

$$\hat{x} = r(u,v) = a_1 u + a_2 v + a_3 uv + a_4$$

$$\hat{y} = s(u,v) = b_1 u + b_2 v + b_3 uv + b_4$$

- n-th degree polynomial

$$\begin{bmatrix} \hat{x}_i \\ \hat{y}_i \end{bmatrix} = \begin{bmatrix} P_x(u_i,v_i) \\ P_y(u_i,v_i) \end{bmatrix} = \sum_{k=0}^{d} \sum_{r+s=k} \begin{bmatrix} a_{krs} u_i^r v_i^s \\ b_{krs} u_i^r v_i^s \end{bmatrix}$$

- Use information from tiepoints to solve coefficients
  - Exact solution
  - Least square solution

$$\varepsilon = \min_{a,b} \sum_{i=0}^{m-1} [(x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2]$$

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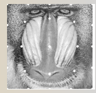
How is it applied?



- Step 1: Choose a set of tie points
  - (x_i, y_i): coordinates of tie points in the original (or distorted) image
  - (u_i, v_i): coordinates of tie points in the corrected image
- Step 2: Decide on which degree of polynomial to use to model the inverse of the distortion, e.g.,

$$\hat{x} = r(u,v) = a_1 u + a_2 v + a_3 uv + a_4$$

$$\hat{y} = s(u,v) = b_1 u + b_2 v + b_3 uv + b_4$$

- Step 3: Solve the coefficients of the polynomial using least-squares approach
- Step 4: Use the derived polynomial model to correct the entire original image

$$X = \begin{bmatrix} x_0 & x_1 & \cdots & x_{24} \end{bmatrix}^T$$

$$Y = \begin{bmatrix} y_0 & y_1 & \cdots & y_{24} \end{bmatrix}^T$$

$$W = \begin{bmatrix} u_0 & v_0 & u_0 v_0 & 1 \\ u_1 & v_1 & u_1 v_1 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ u_{24} & v_{24} & u_{24} v_{24} & 1 \end{bmatrix} \quad A = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix}^T \quad B = \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix}^T$$

$$A = W^{-1} X, B = W^{-1} Y$$

$$W^{-1} = (W^T W)^{-1} W^T$$

For each (u,v) in the corrected image, find the corresponding (x,y) in the original image and use its intensity as the intensity at (u,v).

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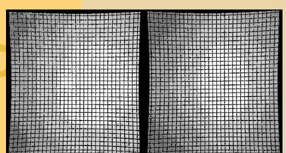
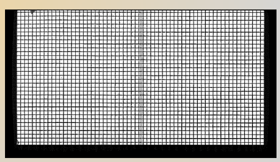
Example – Geometric correction

- Geometric correction of images from butted CCD arrays





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Example - Color correction

Tiepoints are colors (R, G, B), instead of spatial coordinates

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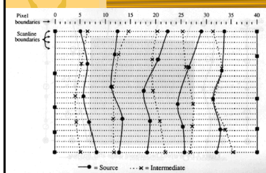
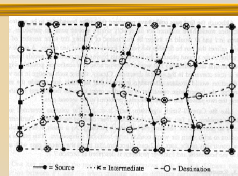
Example - Image warping





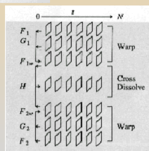
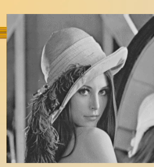
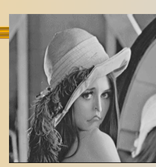
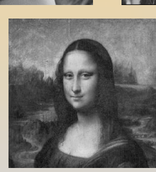


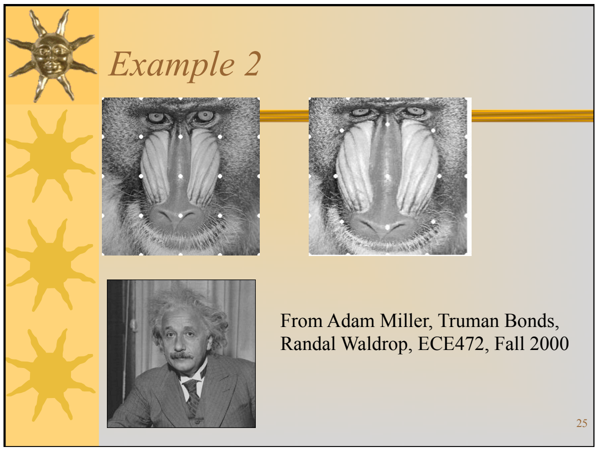
Image warping

- Two-pass mesh warping by Douglas Smythe
- Reference: G. Wolberg, Digital Image Warping, 1990

Example 1

From Joey Howell and Cory McKay, ECE472, Fall 2000


Example 2
From Adam Miller, Truman Bonds,
Randal Waldrop, ECE472, Fall 2000
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