Homework 3 - Sampling and Understanding of Relationships among the z-, n-, and ω-Domains

ECE505, Spring 2020
EECS, University of Tennessee
(Due 02/18)

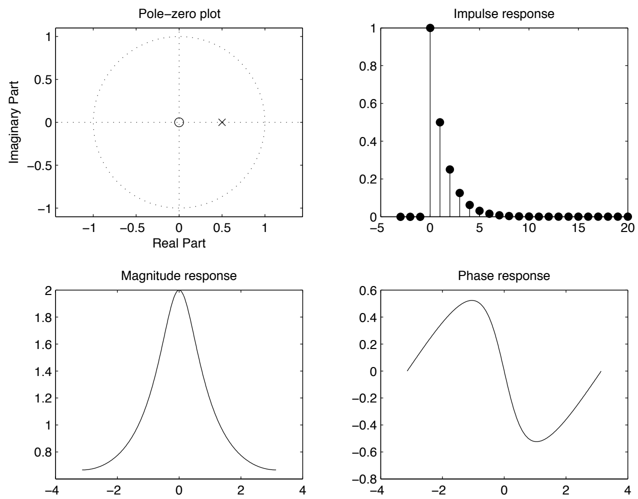
1. Characterize Relationships between the Three Domains

   (a) Write a function, `function nwz(Z, P)` that generates a figure with 2×2 subplots, including the pole-zero plot (221), the impulse response (222), the magnitude response (223), and the phase response (224). The inputs to the function are two vectors with `Z` specifying the zero and `P` the pole locations, respectively. Check your code using the following inputs and compare the output with the following figure.

   ```
   nwz([0], [0.5]);
   ```

   (b) Real poles. Observe and summarize the effect of the location of the real pole to, including but not limited to, the oscillation period and rate of decay of $h(n)$, peak location and width of $H(e^{j\omega})$. Conclude your summary based on plots with the real pole moving from -2, -1, -0.9, -0.5, -0.2, 0, 0.2, 0.5, 0.9, 1, 2. Provide in the report a few representative plots to support your conclusion.

   (c) Complex poles. If the polynomial coefficients are real and the denominator polynomial, $A(z)$, has complex roots, then the roots must be conjugate pairs. Observe and summarize the effect of the location of the complex poles to, including but not limited to, the oscillation period and rate of decay of $h(n)$, peak location and width of $H(e^{j\omega})$. Conclude your summary based on plots with the complex poles sitting on the unit circle but with different angles of the pole and different magnitude of the pole. For example, two zeros at the origin, poles with magnitude 0.75 at angles of $\pm 45°$, $\pm 90°$, and $\pm 135°$; then increase the magnitude at the three angle locations from 0.75 to 0.9, 0.95, 1, and 2. Provide in the report a few representative plots to support your conclusion.



2. (15 pts) Sampling and Aliasing. Consider the cosine wave

$$x(t) = \cos(1000\pi t + \pi/3)$$

Suppose we obtain a sequence of numbers, $x[n]$, by sampling the waveform at equally spaced time instants nT, where T is the time interval between two samples,

$$x[n] = \cos(1000\pi nT + \pi/3)$$

(a) What is the minimum sampling rate (or the highest sampling interval) to guarantee a perfect reconstruction if using an ideal D-to-C converter?

(b) Suppose $T = 0.0001$, plot the analog signal and highlight the location of samples. When reconstructed using an ideal D-to-C converter, what is the frequency of the reconstructed signal?

(c) Suppose $T = 0.00125$, and $y_1[n]$ is generated. On the same figure, plot the analog signal reconstructed from the samples, $y_1[n]$, assuming an ideal D-to-C converter. What is the frequency of the reconstructed signal? Show the mathematical formulation of the family of folding frequencies.

(d) Suppose $T = 0.0025$, and $y_2[n]$ is generated. On the same figure, plot the analog signal reconstructed from the samples, $y_2[n]$, assuming an ideal D-to-C converter. What is the frequency of the reconstructed signal? Show the mathematical formulation of the family of alias frequencies.

3. Sampling and Aliasing. In the rotating disk and strobe demo, we observed that different flashing rates of the strobe light would make the spot on the disk stand still or move in different directions. Assume the spot is at the top when $n = 0$ (the first flash)

   (a) Assume that the disk is rotating clockwise at a constant speed of 13 rev/sec. If the flashing rate is 15 times per second, express the movement of the spot on the disk as a complex phasor, $p[n]$, that gives the position of the spot at the nth flash.

   (b) For the conditions in (a), determine the apparent speed (in revolutions per second) and direction of movement of the "strobed" spot.

   (c) Now assume the rotation speed of the disk is unknown. If the flashing rate is 13 times per second, and the spot on the disk moves counter-clockwise by 15 degrees with each flash, determine the rotation speed of the disk (in rev/sec). If the answer is not unique, give all possible rotation speeds.

   (d) If the strobe light can be flashed at a rate of n flashes per second where n is an integer greater than zero, determine all possible flashing rates such that the disk can be made to stand still.

Acknowledgement

This homework is designed in part based on labs provided in McClellan, Schafer, and Yoder's *DSP FIRST: A Multimedia Approach*.