Lecture 3

Recap

TI System

Other system properties

LCDE

Frequency

FΤ

Propertie

Digital Signal Processing Lecture 3 - Discrete-Time Systems

Electrical Engineering and Computer Science University of Tennessee, Knoxville

Overview

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FS Expansio

FT

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- 2 LTI Systems
- 3 Other system properties
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- 5 Frequency response
- 6 FS Expansion
- 7 FT
- 8 Properties

Recap

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- Introduction
 - Three components of DSP
 - Clarifications: Discrete-time vs. Digital
- Discrete-time signals
 - unit sample (impulse), unit step, exponential sequences
 - relationships
 - periodicity of sinusoidal sequences

A general system and an LTI system

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Recap

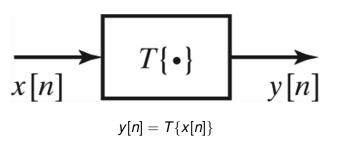
LTI Systems

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Linearity

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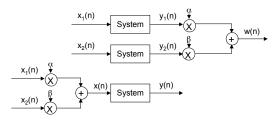
FT

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A system is linear iff

$$T\{ax_1[n] + bx_2[n]\} = aT\{x_1[n]\} + bT\{x_2[n]\}$$

- Homogeneity or scaling or multiplicative property
- Additivity property



 \blacksquare w[n] needs to be equal to y[n]

Time-invariance (or shift-invariance)

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Frequency response

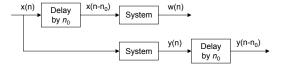
FS Expansion

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For time-invariant systems, the input-output characteristics do not change over time, i.e.,

$$x[n] \longrightarrow y[n]$$
 implies $x[n-n_0] \longrightarrow y[n-n_0]$



• w[n] needs to be equal to $y[n-n_0]$

The convolution sum

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The unit sample response uniquely characterizes the system.

■ If $\delta[n] \to h[n]$, then $\delta[n-k] \to h[n-k]$.

$$x[n] = \sum x[k]\delta[n-k]$$

$$y[n] = \sum x[k]h[n-k] = x[n] * h[n]$$

Convolution sum is commutative.

$$y[n] = \sum_{r} x[n-r]h[r] = h[n] * x[n]$$

Stability

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A system is said to be bounded-input bounded-output (BIBO) stable iff every bounded input produces a bounded output. i.e., there exist some finite numbers, B_x and B_y, s.t.

$$|x[n]| \leq B_x < \infty, |y[n]| \leq B_y < \infty$$

for all *n*.

■ Abosolutely summable: For LTI, if $|x[n]| < \infty$, to have $|y[n]| < \infty$, we need to have

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

Causality

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Properties

The output of a causal system at any time n depends only on present and past inputs, i.e., x[n], x[n-1], x[n-2], · · · , but not on future inputs, i.e., x[n+1], x[n+2], · · ·

Static (or memoryless) vs. Dynamic (or with memory) systems

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Properties

For a static system, at any instant n, the output of the system only depends, at most, on the input sample at the same time: y[n] = T(x[n])

Exercises

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$$y[n] = x[2n]$$

$$y[n] = x[-n]$$

$$y[n] = ax[n] + b$$

$$y[n] = e^{x[n]}$$

$$y[n] = x[n] \cos \omega_0 n,$$

$$h[n] = 2^n u[-n]$$

$$h[n] = 2^n u[n]$$

$$h[n] = (\frac{1}{2})^n u[n]$$

Linear constant-coefficient difference equation (LCDE)

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Nth order LCDE

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{m=0}^{M} b_m x[n-m]$$

■ When N = 0, $a_0 = 1$, i.e., 0-th order LCDE

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

That is,

$$h[n] = \begin{cases} b_n, & n = 0, 1, \dots, M \\ 0, & otherwise \end{cases} = \sum_{k=0}^{M} b_k \delta[n-k]$$

■ When $N \neq 0$, $a_0 = 1$

$$y[n] = \sum_{m=0}^{M} b_m x[n-m] - \sum_{k=1}^{N} a_k y[n-k]$$

Initial conditions

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Properties

■ The homogeneous solution, $y_h[n]$

$$y[n] = y_p[n] + y_h[n]$$

$$\sum_{k=0}^{N} a_k y_h[n-k] = 0 \to y_h[n] = \sum_{m=1}^{N} A_m z_m^n$$

- If a system is characterized by an LCDE and is further specified to be linear, time-invariant, and causal, then the solution is unique. These auxiliary conditions are referred to as the initial-rest conditions. That is, if x[n] = 0 for $n < n_0$, then y[n] = 0 for $n < n_0$.
- Note that not for every set of boundary conditions that LCDE would correspond to an LTI system.

Exercises

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$$y[n] - ay[n-1] = x[n]$$

Suppose $x[n] = \delta[n]$

- **assume** y[n] = 0, n < 0
- assume y[n] = 0, n > 0

Eigenfunctions for LTI systems

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Complex exponentials are eigenfunctions of LTI systems and the frequency response is the eigenvalue of LTI systems.

Let $x[n] = e^{j\omega n}$, then

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=-\infty}^{\infty} h[k]e^{j\omega(n-k)} = e^{j\omega n} \sum_{k} h[k]$$

Let $H(e^{j\omega}) = \sum_{k} h[k]e^{-j\omega k}$, then

$$y[n] = H(e^{j\omega})e^{j\omega n}$$

■ Frequency response is the eigenvalue of the system:

$$H(e^{j\omega}) = \sum_{k} h[k]e^{-j\omega k}$$

- Properties of frequency response:
 - lacksquare Function of continuous variable ω
 - Periodic function of ω with period of 2π

Exercises

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Frequency response

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- Find the output to $x[n] = A\cos(\omega_0 n + \phi)$
- Find the frequency response to y[n] ay[n-1] = x[n]
- Find the output to $x[n] = e^{j\omega n}u[n]$ (suddenly applied exponential input)

Fourier-series expansion

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- $H(e^{j\omega})$ is a continuous periodic function and would have a Fourier series representation
- $H(e^{j\omega})$ has a Fourier-series expansion in terms of complex exponentials
- The Fourier series coefficients then become the values of the unit sample response

$$H(e^{j\omega}) = \sum_{n} h[n]e^{-j\omega n}$$

$$h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega$$

Fourier Transform

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Properties

■ Frequency-domain representation of arbitrary sequence x[n]

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

The convolution property

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Properties

$$x[n] * y[n] \leftrightarrow X(e^{j\omega})Y(e^{j\omega})$$

$$e^{j\omega_0 n} \to H(e^{j\omega_0})e^{j\omega_0 n}$$

$$\sum_k A_k e^{j\omega_k n} \to \sum_k A_k H(e^{j\omega_k})e^{j\omega_k n}$$

$$x[n] = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(e^{j\omega})e^{j\omega n} d\omega$$

A decomposition of x[n] as a linear combination of complex exponentials

$$y[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) H(e^{j\omega}) e^{j\omega n} d\omega$$

$$Y(e^{j\omega}) = X(e^{j\omega}) H(e^{j\omega})$$

$$y[n] = x[n] * h[n]$$

The symmetry property

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Properties

For real sequences x[n], the symmetry property indicates that the FT of x[n] is a conjugate symmetric function of ω

$$X(e^{j\omega}) = X^*(e^{-j\omega})$$

Other useful properties:

■ The real part of the FT is an even function:

$$X_R(e^{j\omega}) = X_R(e^{-j\omega})$$

- The imaginary part of the FT is an odd function: $X_i(e^{j\omega}) = -X_i(e^{-j\omega})$
- The amplitude of the FT is an even function: $|X(e^{j\omega})|$
- The phase of the FT is an odd function $X(e^{j\omega})$