# Texture Mapping

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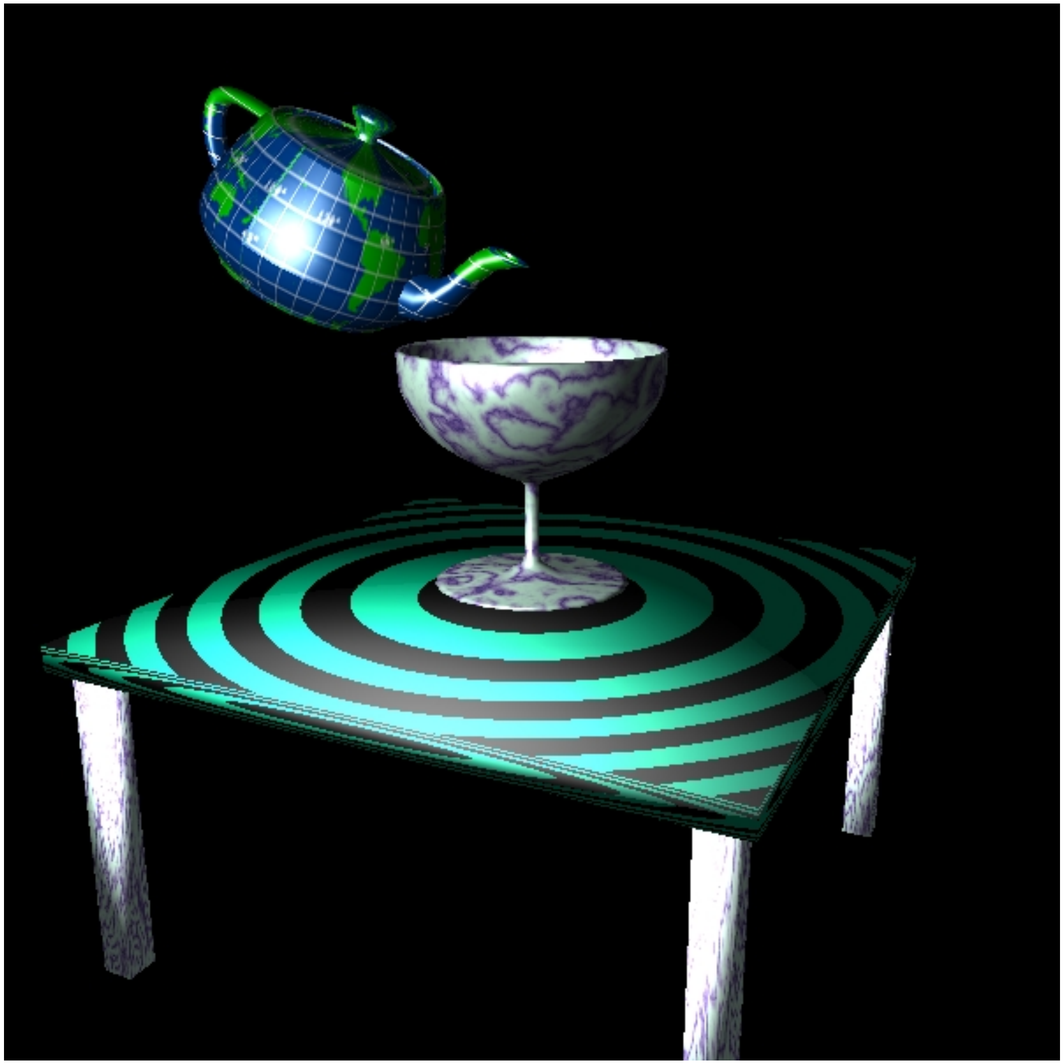
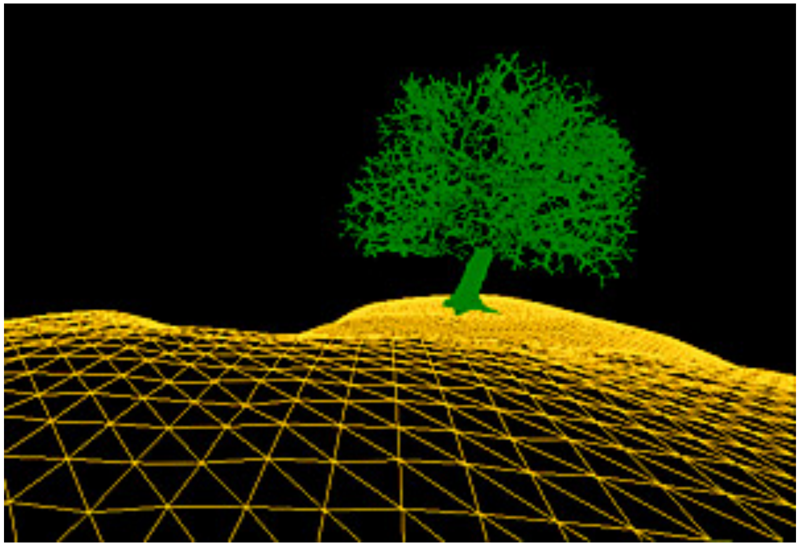
This set of slides references the ones used at Ohio State for instruction.

# Can you do this …



# What Dreams May Come



# Texture Mapping

- Of course, one can model the exact micro-geometry + material property to control the look and feel of a surface
- But, it may get extremely costly
- So, graphics use a more practical approach – texture mapping

# Texture Mapping

- Particles and fractals
  - gave us lots of detail information
  - not easy to model
  - mathematically and computationally challenging

# Texture Mapping

- (Sophisticated) Illumination models
  - gave us "photo"-realistic looking surfaces
  - not easy to model
  - mathematically and computationally challenging
- Phong illumination/shading
  - easy to model
  - relatively quick to compute
  - only gives us dull surfaces

# Texture Mapping

- Surfaces "in the wild" are very complex
- Cannot model all the fine variations
- We need to find ways to add **surface detail**
- How?

# Texture Mapping

- Solution - (its really a cheat!!)

MAP surface detail from a predefined multi-dimensional table ("texture") to a simple polygon

- How?

# Textures Make A Difference

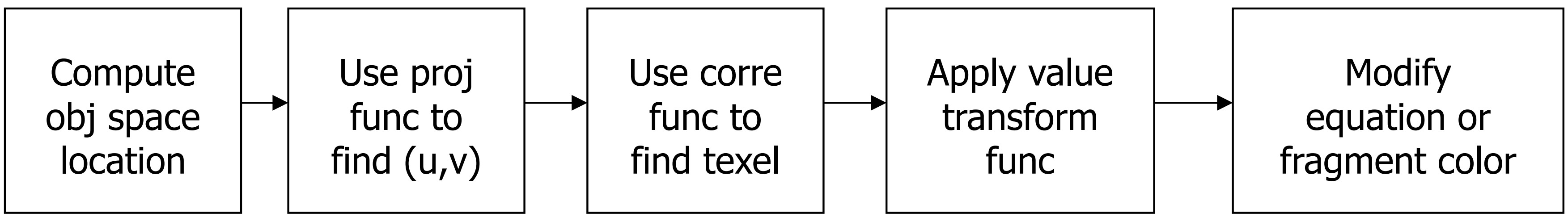
- Good textures, when applied correctly, make a world of difference!

# A Texture can be?

- F(u,v) ==> a continuous or discrete function of:
  - { R(u,v), G(u,v), B(u,v) }
  - { I(u,v) }
  - { index(u,v) }
  - { alpha(u,v) } (transparency)
  - { normals(u,v) } (bump map)
  - { surface_height(u,v) } (displacement map)
  - Specular color (environment map)
  - …

# The Generalized Pipeline

- The generalized pipeline of texture mapping



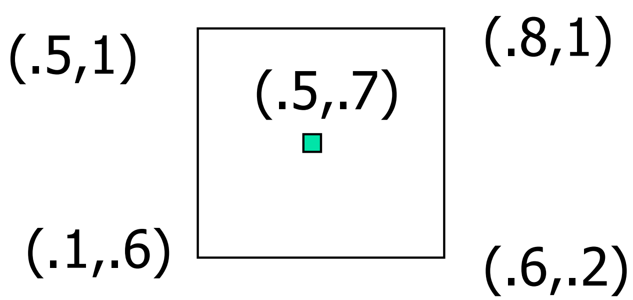

- Fragment: after rasterization, the data are not pixels yet, but are fragments. Each fragment has coordinate, color, depth, and undergo a series of tests and ops before showing up in the framebuffer

# Texture Mapping

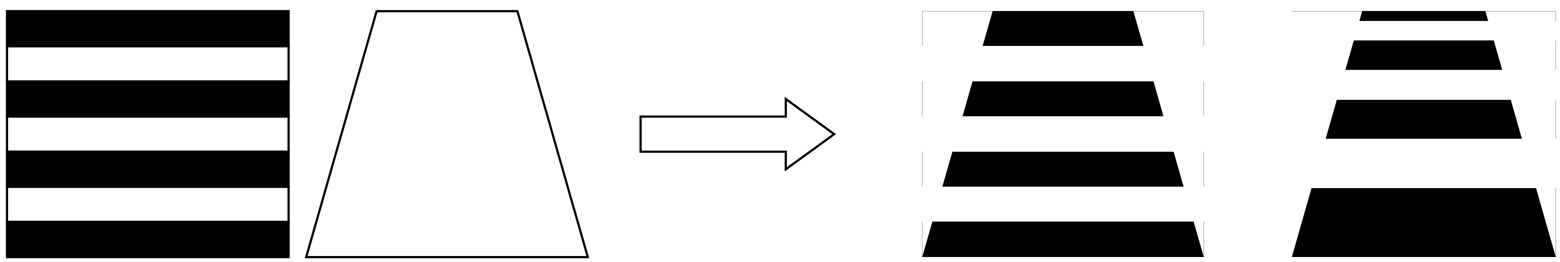
- Problem #1
  - Fitting a square peg in a round hole
  - We deal with non-linear transformations
  - Which parts map where?

# Inverse Mapping

- Need to transform back to obj/world space to do the interpolation
- Orientation in 3D image space




- Foreshortening



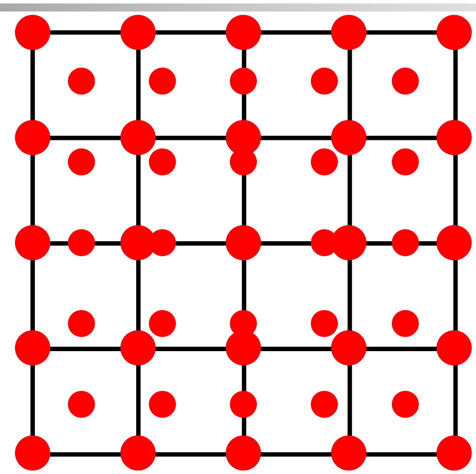
# Texture Mapping

- Problem #2
  - Mapping from a pixel to a "texel"
  - Aliasing is a huge problem!

# Mapping to A Texel ?

- Basically map to an image
- Need to interpolate
- Same as ....
  - How can I find an appropriate value for an arbitrary (not necessarily integer) index?
    - How would I rotate an image 45 degrees?
    - How would I translate it 0.5 pixels?

# Interpolation



Nearest neighbor

Linear Interpolation

# How do we get F(u,v)?

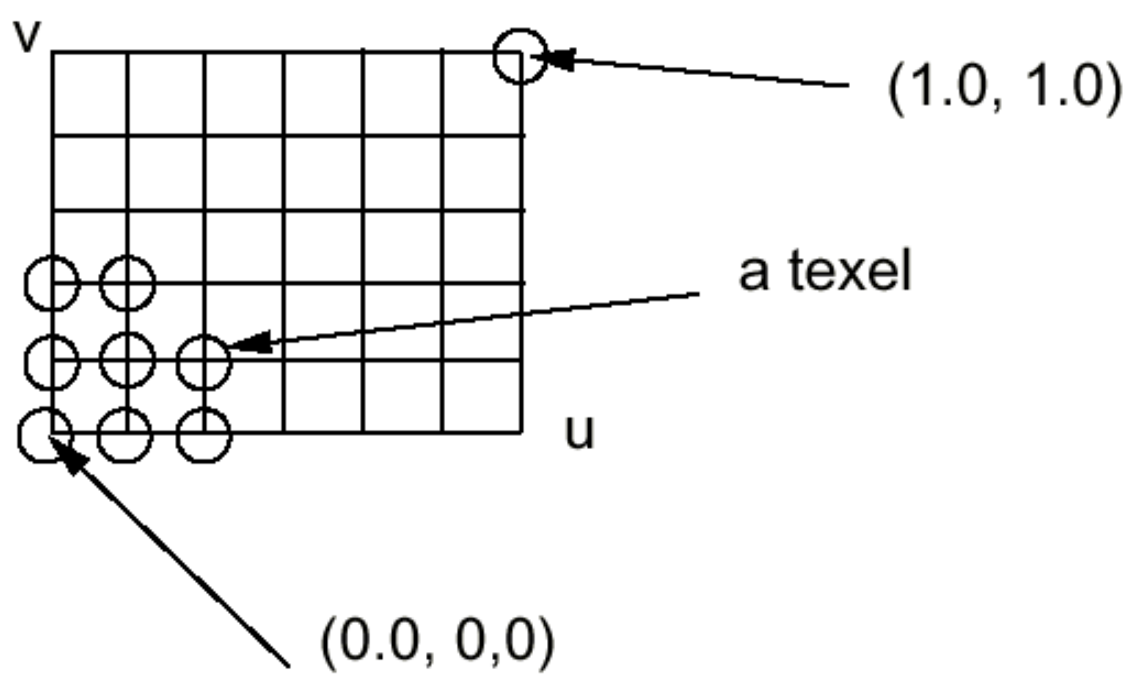
- We are given a discrete set of values:
  - F[i,j] for i=0,…,N, j=0,…,M
- Nearest neighbor:
  - F(u,v) = F[ round(N*u), round(M*v) ]
- Linear Interpolation:
  - i = floor(N*u), j = floor(M*v)
  - interpolate from F[i,j], F[i+1,j], F[i,j+1], F[i+1,j+1]
- Filtering in general !

# How do we get F(u,v)?

- Higher-order interpolation
  - $F(u,v) = \sum_i \sum_j F[i,j]\, h(u,v)$
  - *h(u,v) is called the reconstruction kernel*
    - *Gaussian*
    - *Sinc function*
    - *splines*
  - Like linear interpolation, need to find neighbors.
    - Usually four to sixteen

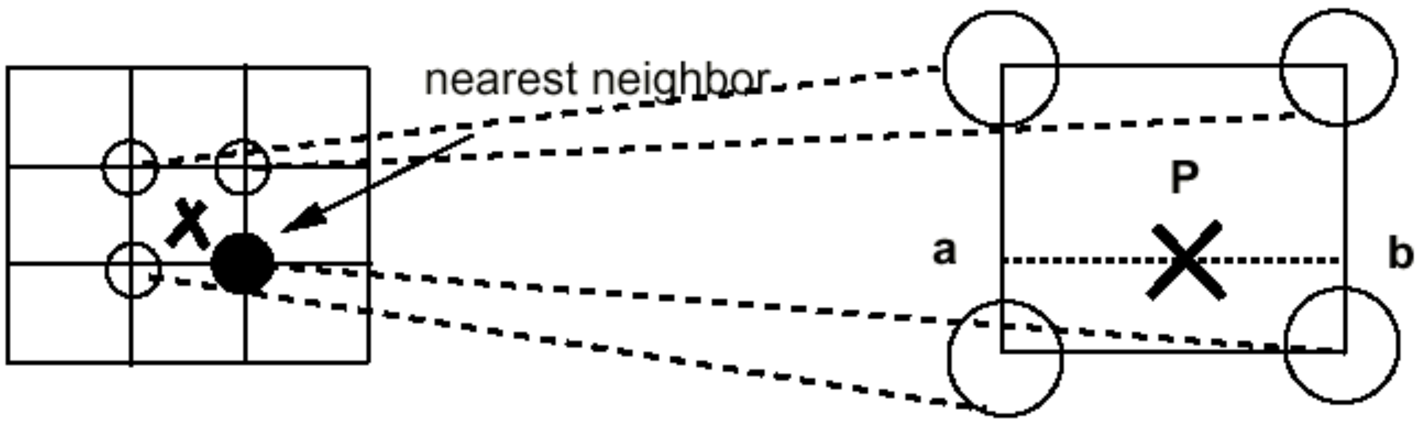
# Texture and Texel

- Each pixel in a texture map is called a Texel
- Each Texel is associated with a (u,v) 2D texture coordinate
- The range of u, v is [0.0,1.0]



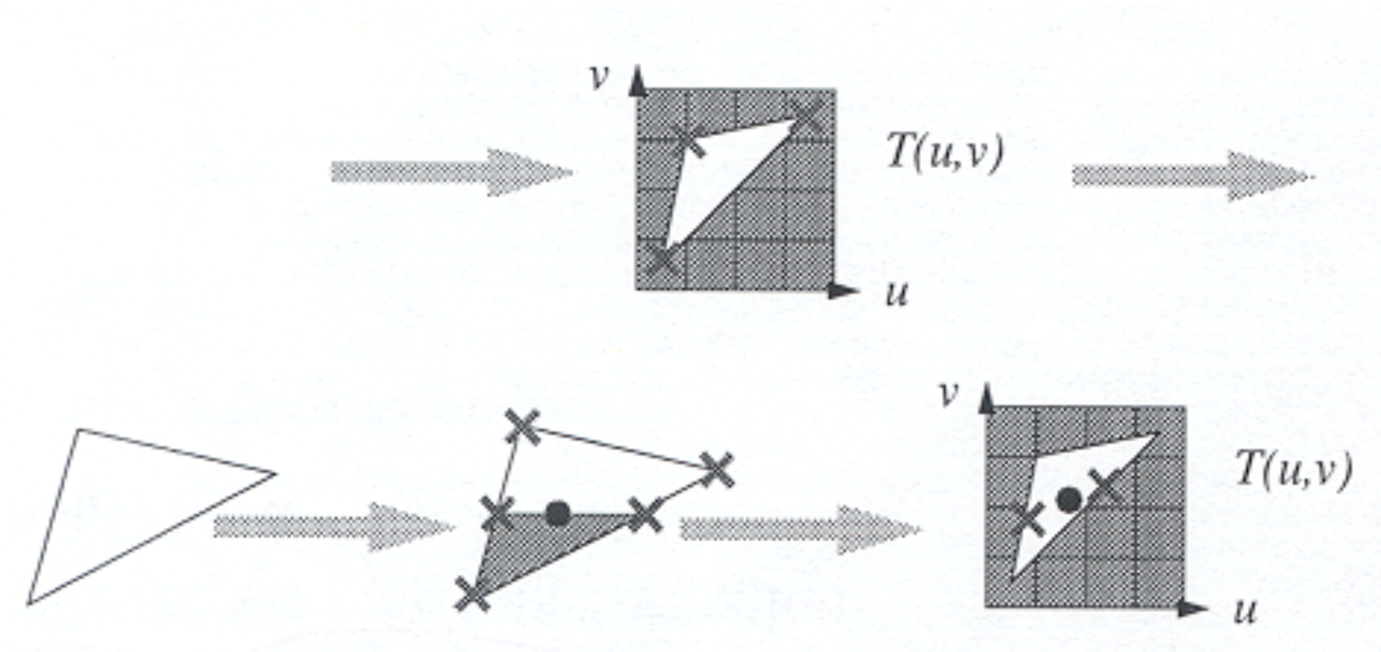
# (u,v) tuple

- For any (u,v) in the range of (0-1, 0-1), we can find the corresponding value in the texture using some interpolation



# The Projector Function

1. Model the mapping: (x,y,z) -> (u,v)
2. Do the mapping



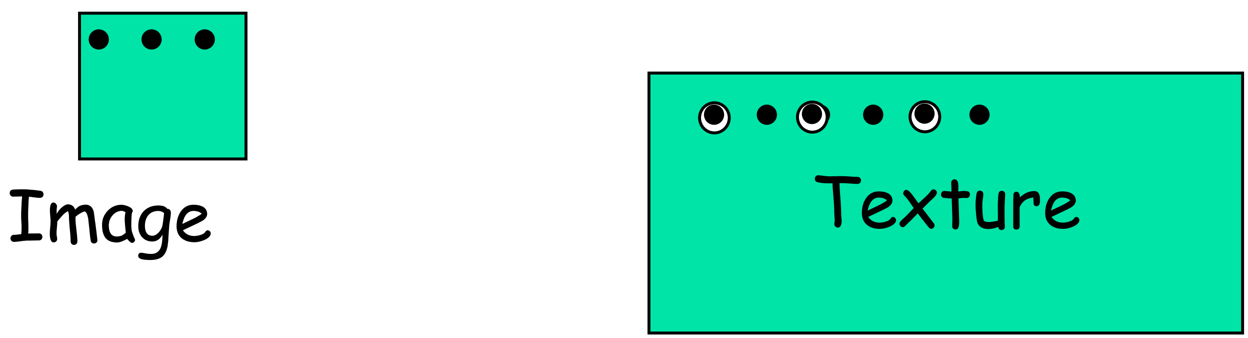
# Image space scan

```
For each y    /* scan-line */
    For each x /* pixel on scan-line */
        compute u(x,y) and v(x,y)
        copy texture(u,v) to image(x,y)
```

- Samples the warped texture at the appropriate image pixels.
- <u>inverse</u> mapping

# Image space scan

- Problems:
  - Finding the inverse mapping
    - Use one of the analytical mappings
    - Bi-linear or triangle inverse mapping
  - May miss parts of the texture map



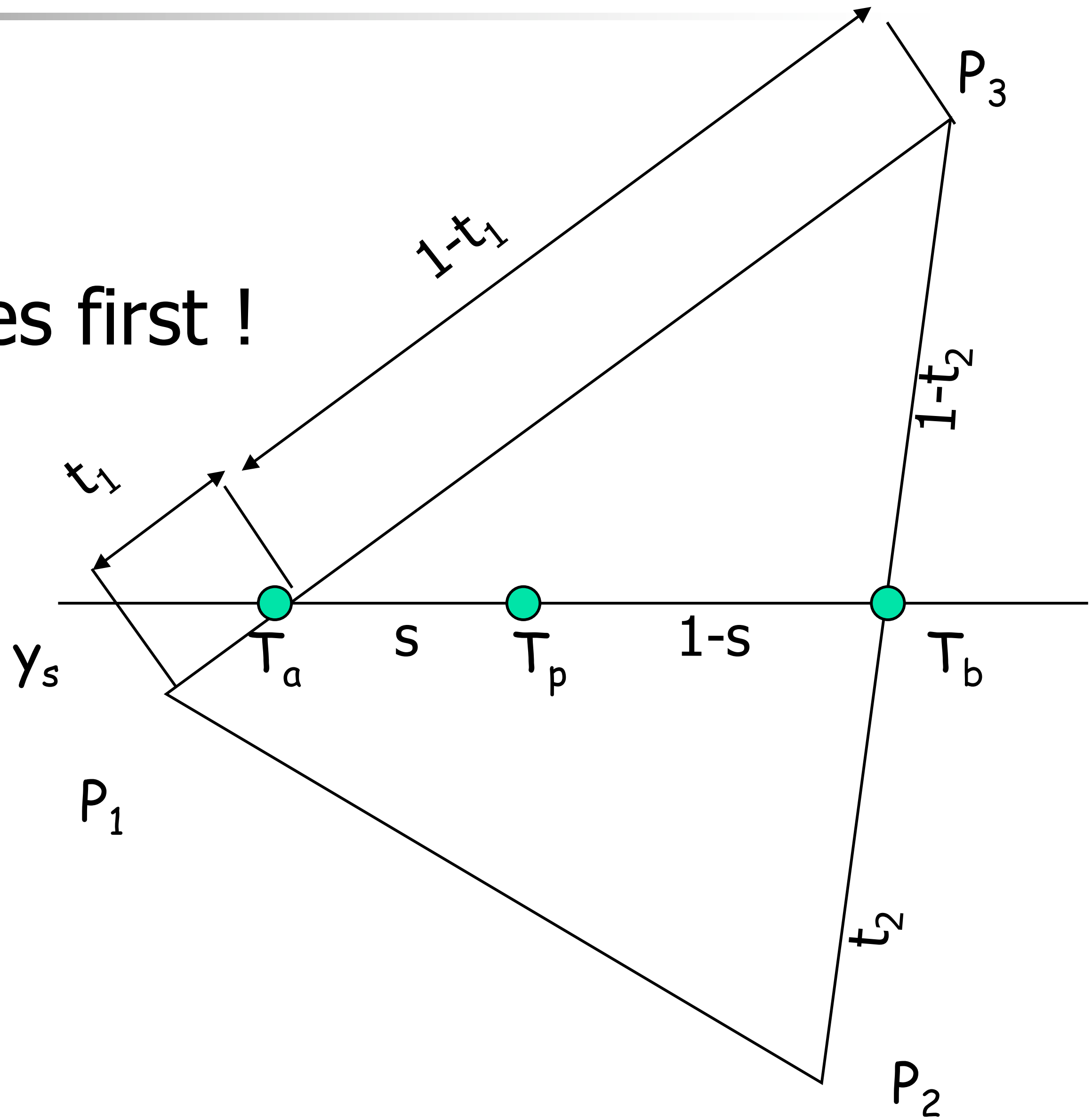
# Texture Parameterization

- Definition:
  - The process of assigning texture coordinates or a texture mapping to an object.
- The mapping can be applied:
  - Per-pixel
  - Per-vertex

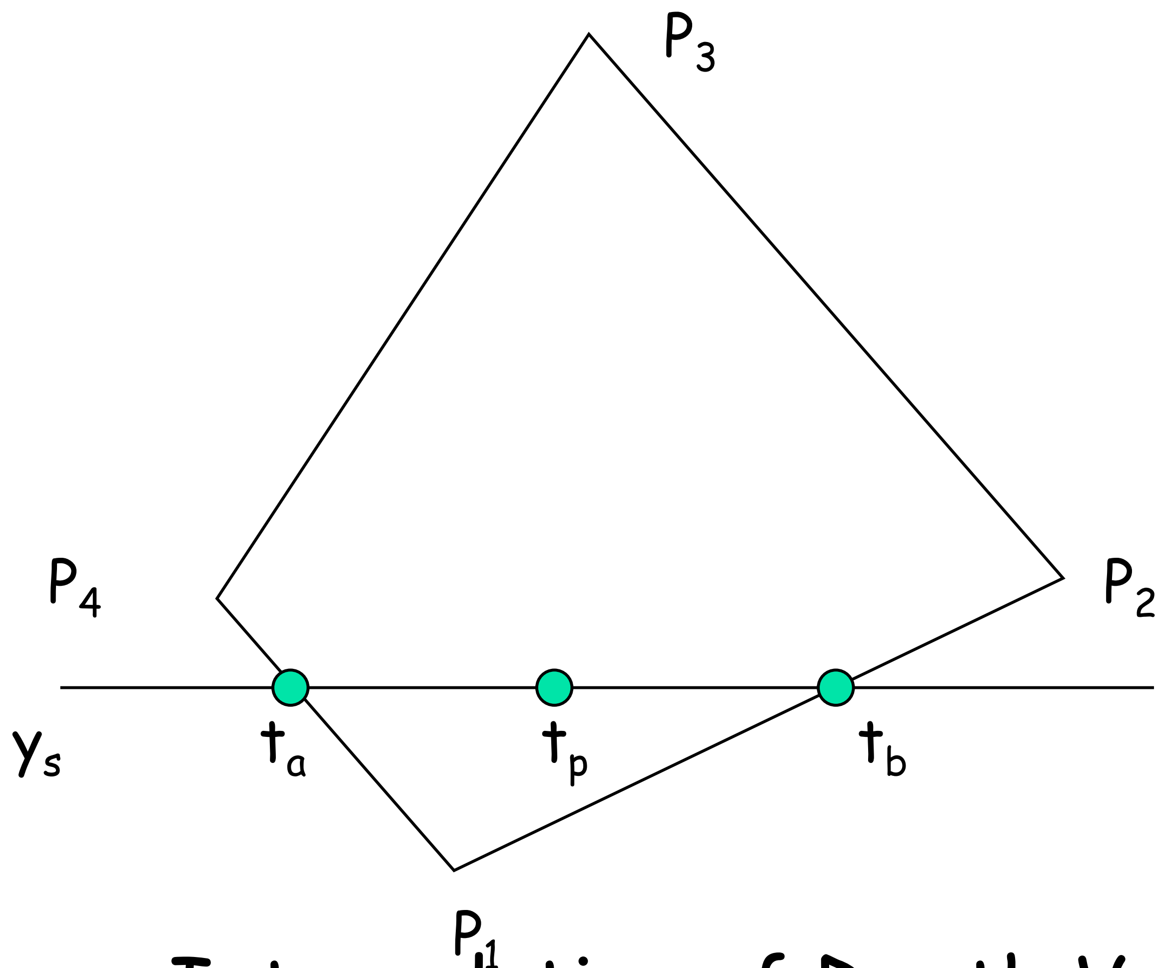
# Interpolation Concepts

T is texture

Find textures at vertices first !



# Quads ?




Bilinear Interpolation of Depth Values

# Texture space scan

For each v

    For each u

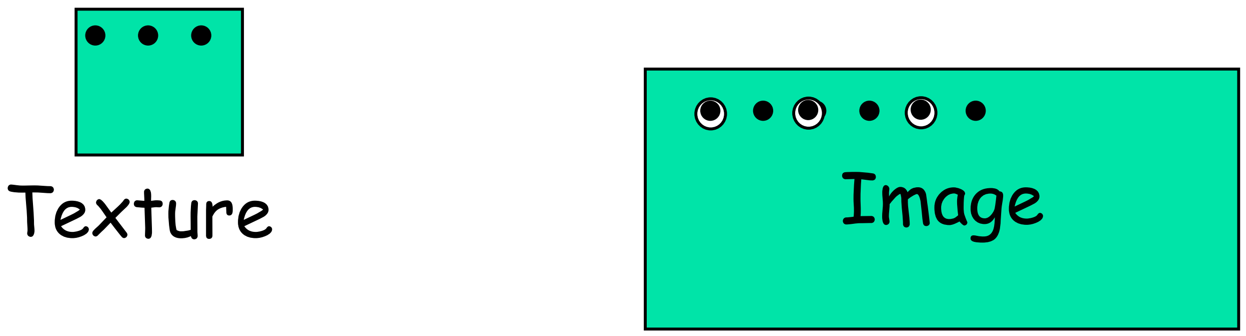
        compute x(u,v) and y(u,v)

        copy texture(u,v) to image(x,y)

- Places each texture sample to the mapped image pixel.
- **Forward** mapping

# Texture space scan

- Problems:
  - May not fill image
  - Forward mapping needed



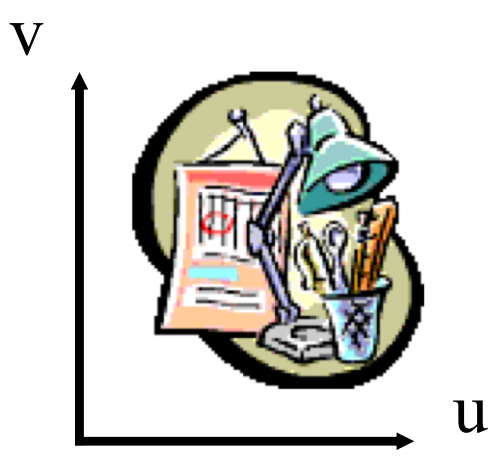
# Simple Projector Functions

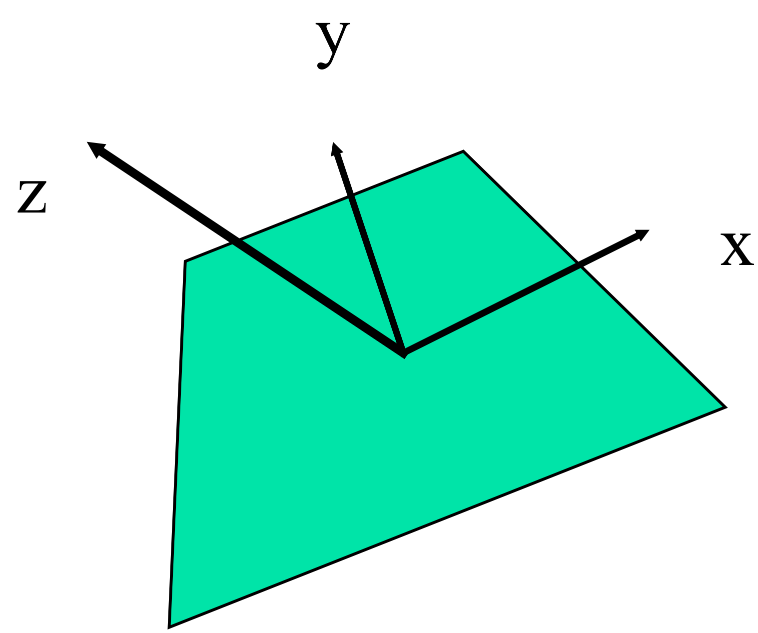
- Spherical
- Cylindrical
- Planar

- For some model, a single projector function suffices. But very often, an artist may choose to subdivide each object into parts that use different projector

# Planar

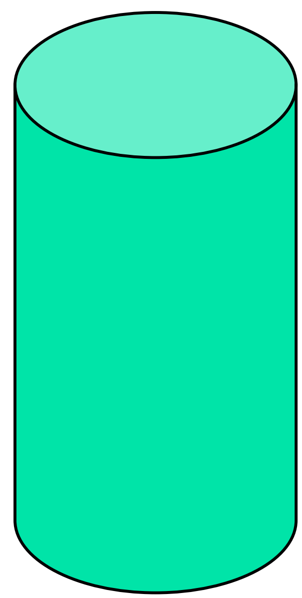
- Mapping to a 3D Plane
  - Simple Affine transformation
    - rotate
    - scale
    - translate



# Cylindrical

- Mapping to a Cylinder
  - Rotate, translate and scale in the uv-plane
  - u -> θ
  - v -> z
  - x = r cos(θ), y = r sin(θ)

# Spherical

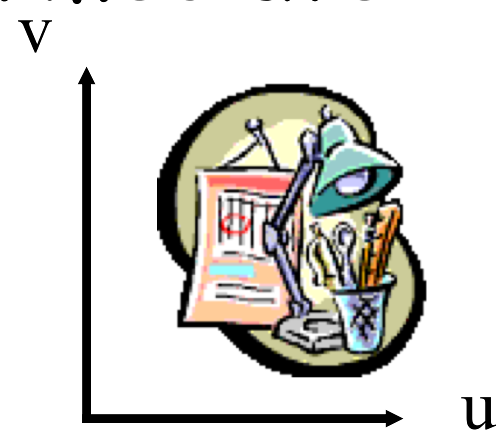
- Mapping to Sphere
  - Impossible!!!!
  - Severe distortion at the poles
  - u -> $\theta$
  - v -> $\phi$
  - $x = r \sin(\theta) \cos(\phi)$
  - $y = r \sin(\theta) \sin(\phi)$
  - $z = r \cos(\theta)$

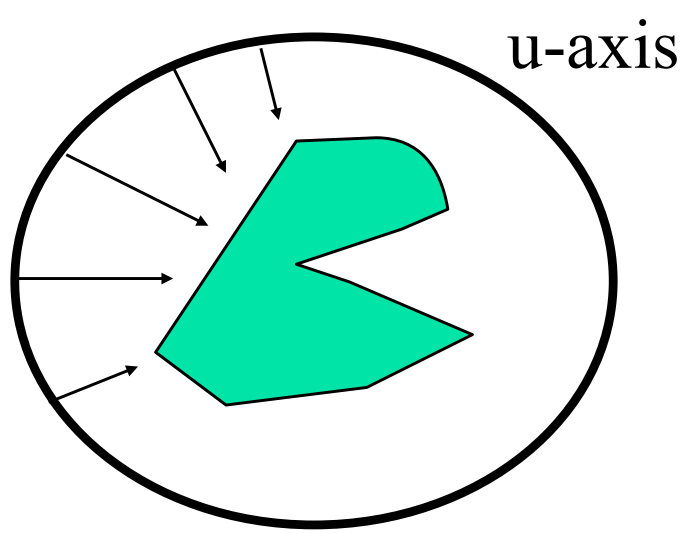
# Two-pass Mapping

- Idea by Bier and Sloan
- S: map from texture space to intermediate space
- O: map from intermediate space to object space

# Two-pass Mapping

- Map texture to intermediate:
  - Plane
  - Cylinder
  - Sphere
  - Box
- Map object to same.



# Texture Mapping

- O mapping:
  - reflected ray (environment map)
  - object normal
  - object centroid
  - intermediate surface normal (ISN)
- that makes 16 combinations
- only 5 were found useful

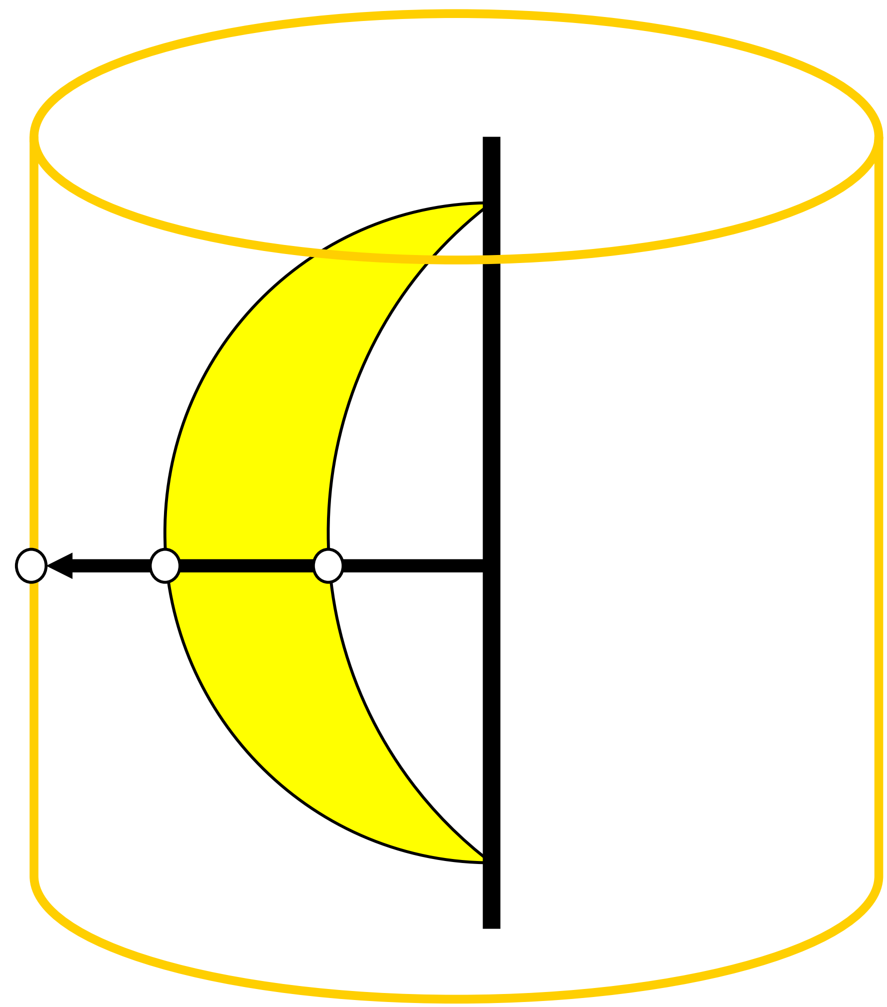
# Texture Mapping

- Cylinder/ISN (shrinkwrap)
  - Works well for solids of revolution
- Plane/ISN (projector)
  - Works well for planar objects
- Box/ISN
- Sphere/Centroid
- Box/Centroid

Works well for roughly spherical shapes

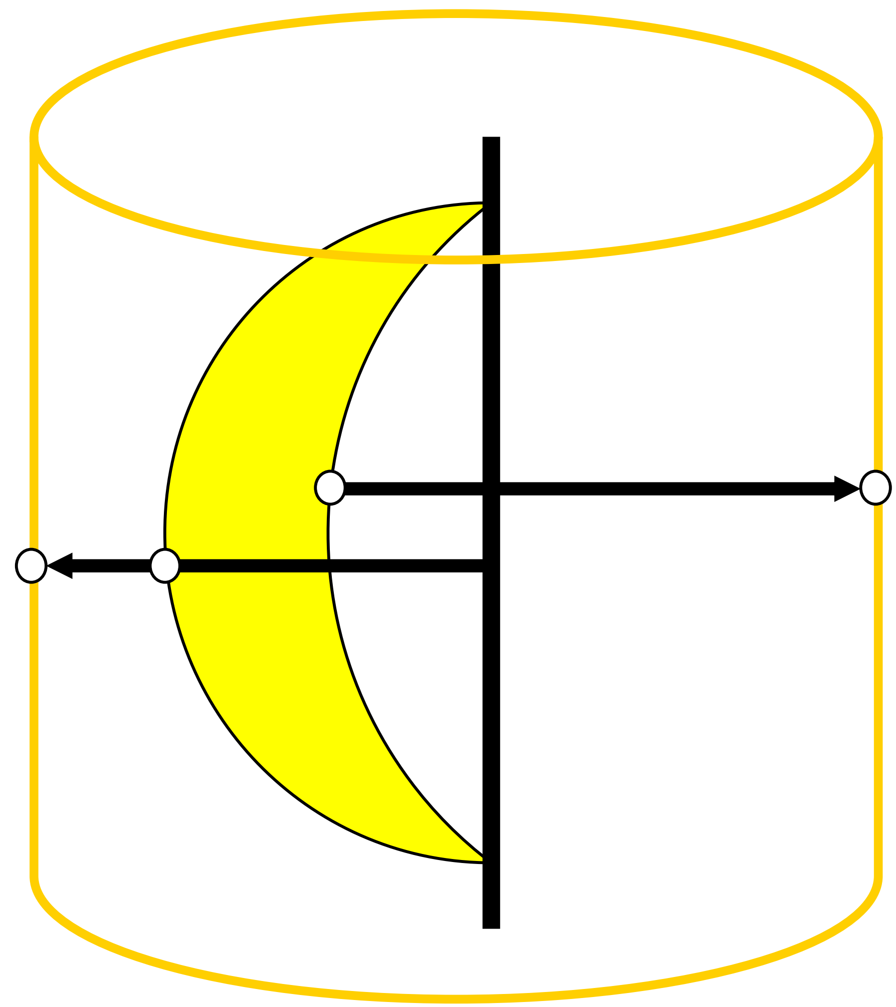
# Texture Parameterization

- What is this ISN?
  - Intermediate surface normal.
  - Needed to handle concave objects properly.
  - Sudden flip in texture coordinates when the object crosses the axis.



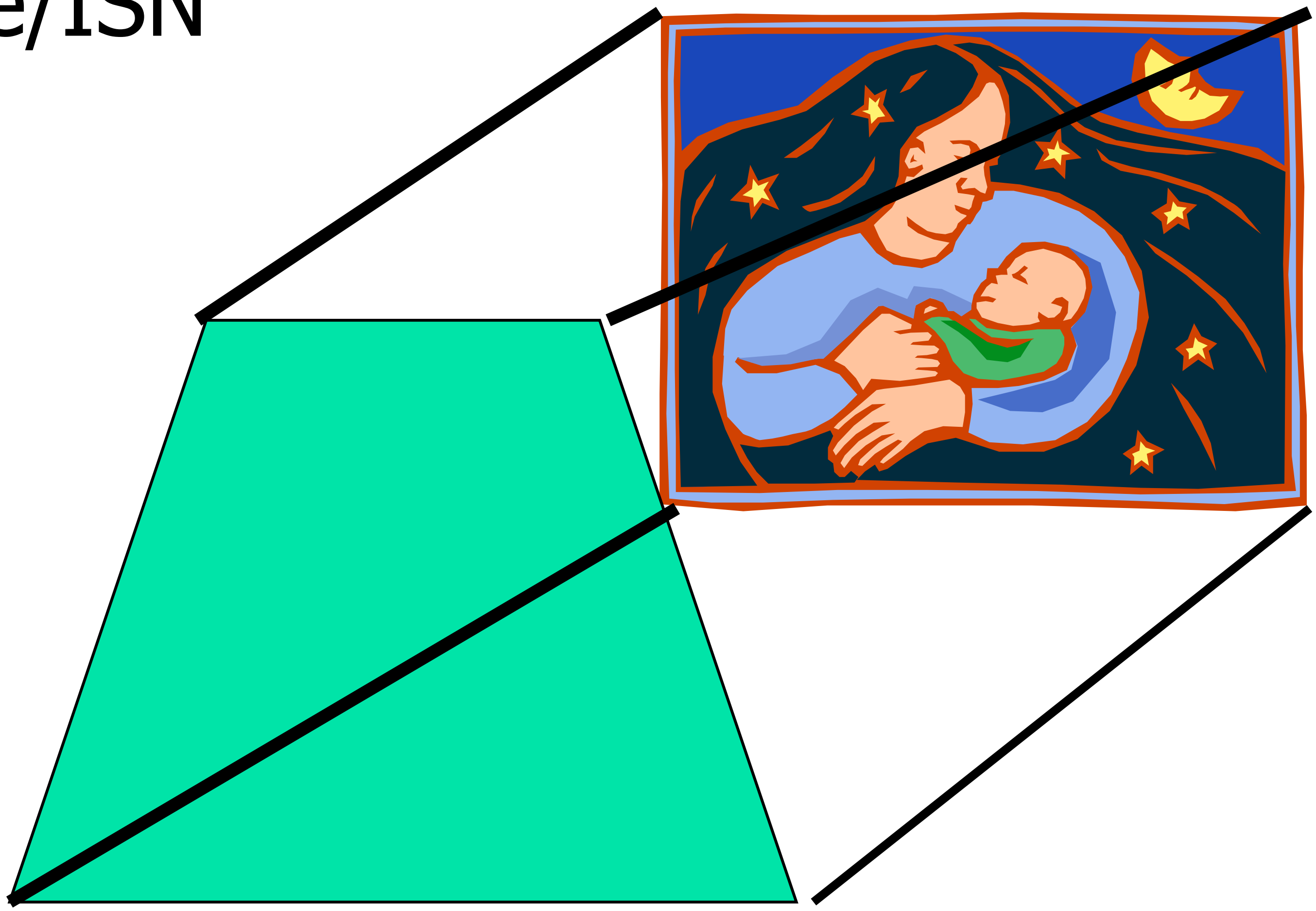
# Texture Parameterization

- Flip direction of vector such that it points in the same half-space as the outward surface normal.



# Texture Parameterization

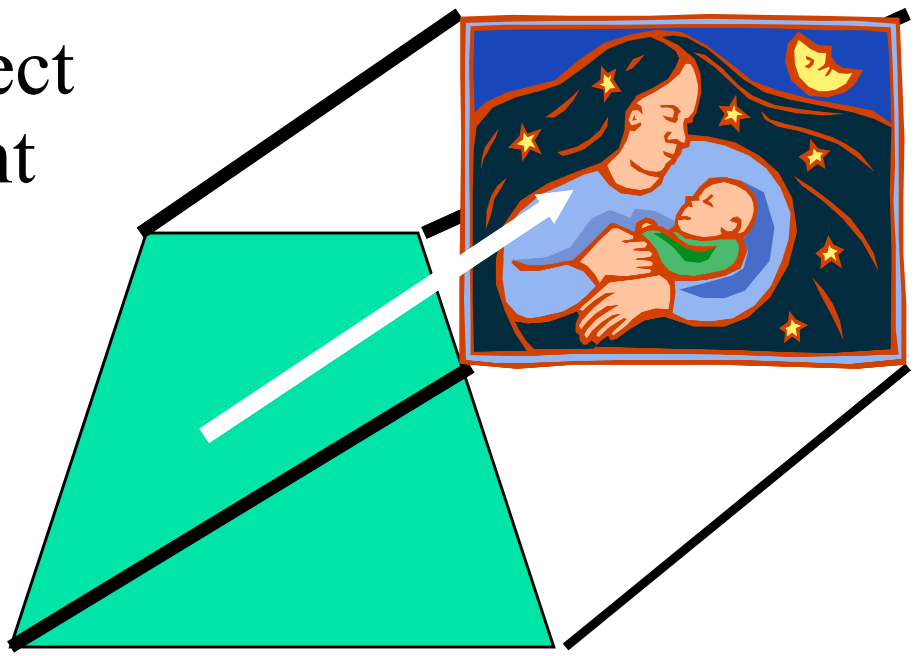
- Plane/ISN



# Texture Parameterization

- Plane/ISN
  - Draw vector from point (vertex or object space pixel point) in the direction of the texture plane.

– The vector will intersect the plane at some point depending on the coordinate system



# Texture Parameterization

- Plane/ISN
  - Resembles a slide projector
  - Distortions on surfaces perpendicular to the plane.

# Texture Parameterization

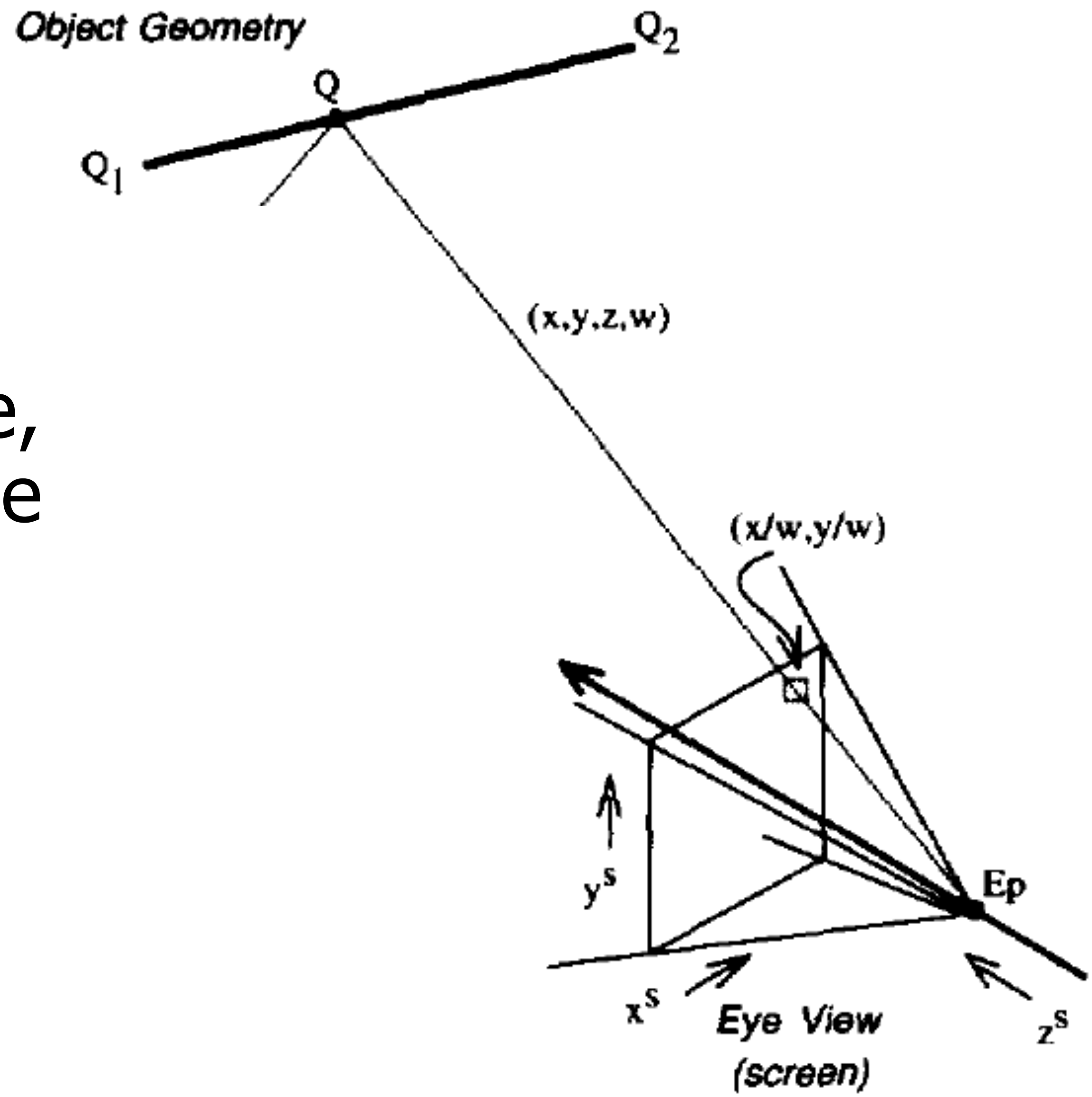
- Cylinder/ISN
  - Distortions on horizontal planes
  - Draw vector from point to cylinder
  - Vector connects point to cylinder axis

# Texture Parameterization

- Sphere/ISN
  - Small distortion everywhere.
  - Draw vector from sphere center through point on the the surface and intersect it with the sphere.

# Interpolating Without Explicit Inverse Transform

- Scan-conversion and color/z/normal interpolation take place in screen space, but really, what space should it be in?
- What about texture coordinates?
  - Do it in clip space, or homogenous coordinates



# In Clip space

- Two end points of a line segment (scan line)

$$\mathbf{Q}_1 = (x_1, y_1, z_1, w_1) \qquad \mathbf{Q}_2 = (x_2, y_2, z_2, w_2)$$

- Interpolate for a point Q in-between

$$\mathbf{Q} = (1 - t)\mathbf{Q}_1 + t\mathbf{Q}_2$$

# In Screen Space

- From the two end points of a line segment (scan line), interpolate for a point Q in-between:

$$\mathbf{Q}^s = (1 - t^s)\mathbf{Q}_1^s + t^s \mathbf{Q}_2^s$$

- Where: $\mathbf{Q}_1^s = \mathbf{Q}_1 / w_1$ and $\mathbf{Q}_2^s = \mathbf{Q}_2 / w_2$.
- Easy to show: in most occasions, t and $t^s$ are different

# From $t^s$ to t

- Change of variable: choose
  - a and *b* such that $1 - t^s = a/(a + b)$, $t^s = b/(a + b)$
  - A and B such that $(1 - t) = A/(A + B)$, $t = B/(A + B)$.
- Easy to get

$$\mathbf{Q}^s = \frac{a\mathbf{Q}_1/w_1 + b\mathbf{Q}_2/w_2}{(a+b)} = \frac{A\mathbf{Q}_1 + B\mathbf{Q}_2}{Aw_1 + Bw_2}$$

- Easy to verify: $A = aw_2$ and $B = bw_1$ is a solution

# Texture Coordinates

- All such interpolation happens in homogeneous space.
- Use A and B to linearly interpolate texture coordinates
- The homogeneous texture coordinate is: (u,v,1)

# Homogeneous Texture Coordinates

- $u^l = A/(A+B)\ u_1^l + B/(A+B) u_2^l$
- $w^l = A/(A+B)\ w_1^l + B/(A+B) w_2^l = 1$
- $u = u^l/w^l = u^l = (Au_1^l + Bu_2^l)/(A + B)$
- $u = (au_1^l + Bu_2^l)/(A + B)$
- $u = (au_1^l/w_1^l + bu_2^l/w_2^l\ )/(a\ ^1/w_1^l + b\ ^1/w_2^l)$

# Homogeneous Texture Coordinates

- The homogeneous texture coordinates suitable for linear interpolation in screen space is computed simply by
  - Dividing the texture coordinates by screen w
  - Linearly interpolating (u/w,v/w,1/w)
  - Dividing the quantities u/w and v/w by 1/w at each pixel to recover the texture coordinates