Heterogeneous Maximal-Throughput Bursty Traffic Model with Application to Packet Switches

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Abstract— **This paper presents an analysis of a discrete-time Markov-modulated process for generating heterogeneous bursty traffic in input-queued switches. The goals of the model described are two fold. First, it constitutes an expansion of the classic ON/OFF process to address scenarios in which multiple queues receive bursts from a shared input stream. Second, in concert with the first goal, it enables 100% link utilization (hence maximal traffic generation throughput) by avoiding the mandatory OFF slots introduced by classic ON/OFF models. This work is especially valuable in evaluating the performance of packet switching fabrics under bursty conditions.**

Keywords— **Bursty Traffic Model, Input-Queued Switch**

I. Introduction

SCALABLE packet scheduling algorithms that offer high-performance for input buffered switches and routers have been the focus of many academic and industrial research activities. Switch fabrics, residing at the core of routers, can be implemented in a variety of ways ranging from shared-memory architecture to fully distributed nearly-stateless architecture, and anywhere in between. One of the most challenging aspects of evaluating switching fabrics pertains to the definition of the traffic models applied. It is widely acknowledged that pragmatic packet arrival patterns are bursty in nature. This is due to processes occurring both at the data sources as well as at the network. To that end, there have been many attempts at modeling and validating such traffic [1][2][3], including, recently, a related industrial effort [4].

The embraced scalable switch architectures deployed are input-queued. The latter is particularly true for large-scale switches and routers, where by output queueing would proof infeasible. In input-queued switches, each arriving packet is stored in a designated queue which corresponds to its destination, as shown in figure 1. Markov modulated ON/OFF models have been repeatedly incorporated as a building block for constructing more complex, pragmatic traffic scenarios. Multimedia traffic, which by nature tends to be correlated on several levels, is commonly modeled by a superposition of several ON/OFF sources [1]. However, the basic ON/OFF model is limited in its ability to support 100% throughput due to a mandatory OFF slot separating two consecutive bursts. In this paper we present analysis for a modified Markov-modulated arrival process, which support non-uniform distribution of load between the destination ports as well as heterogeneous mean burst sizes, while retaining the ability to fully maximize the generate traffic load.

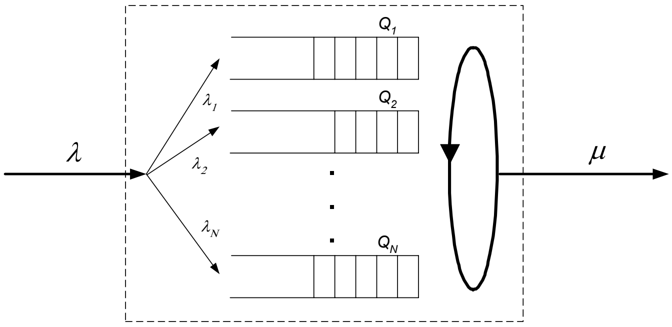

Fig. 1. Virtual output queued (VOQ) switch architecture in which bursty traffic is non-uniformly distributed between the destination.

II. Multi-Queue On/Off Arrival Process

Consider a discrete-time, two-state Markov chain generating arrivals modeled by an ON/OFF source which alternates between the ON and OFF states. An arrival is generated for each time slot that the Markov chain spends in the ON state. Let the parameters α and β denote the probabilities that the Markov chain remains in states ON and OFF, respectively. Using α and β the load and mean burst sizes may be directly obtained. Moreover, since at least a single OFF state separates two consecutive bursts, the maximal arrival rate is bounded by $B/(1+B)$ where B denotes the mean burst size. This bound is a significant limiting factor in the evaluation of high-speed packet switching fabrics. Moreover, in such evaluations there is a clear need for a heterogeneous model in which both burst sizes and average rates can be flexibly determined.

III. Maximal-Throughput Model

To overcome the maximal traffic load constraint, we introduce the $ON/OFF/\Omega$ Markov-modulated arrival process, whereby in a transition from the ON state the process visits the Ω state while generating an additional packet to the same destination. It is only from the Ω state that the process can transition back to the OFF state. Consider a general case where an arriving packet can be destined to each of the N different destination. As shown in figure 2, an arrival destined to the i^{th} output is generated for each time slot that the Markov chain spends in the ON_i state, $i = 1, 2, \ldots, N$, while no arrivals are noted for each when in the OFF state. Instead of direct transition from ON_i to OFF, a transition from ON_i to Ω_i is required. An

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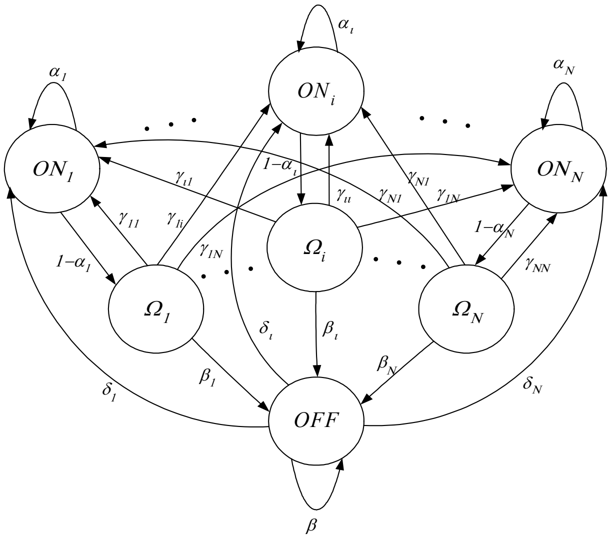

Fig. 2. The proposed maximal throughput Markov modulated arrival process.

arrival destined to the ith output is still generated while the Markov chain is in the Ω_i state. To that end, there are a total of $2N+1$ states in the proposed model, the transition probabilities of which are given by

$$
\begin{array}{c|ccccccccccc}
 & OFF & ON_1 & \cdots & ON_i & \cdots & ON_N & \Omega_1 & \cdots & \Omega_i & \cdots & \Omega_N \\
\hline
OFF & \beta & \delta_1 & \cdots & \delta_i & \cdots & \delta_N & 0 & \cdots & 0 & \cdots & 0 \\
ON_1 & 0 & \alpha_1 & \cdots & 0 & \cdots & 0 & 1-\alpha_1 & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
ON_i & 0 & 0 & \cdots & \alpha_i & \cdots & 0 & 0 & \cdots & 1-\alpha_i & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
P=ON_N & 0 & 0 & \cdots & 0 & \cdots & \alpha_N & 0 & \cdots & 0 & \cdots & 1-\alpha_N \\
\Omega_1 & \beta_1 & \gamma_{11} & \cdots & \gamma_{1i} & \cdots & \gamma_{1N} & 0 & \vdots & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\Omega_i & \beta_i & \gamma_{i1} & \cdots & \gamma_{ii} & \cdots & \gamma_{iN} & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\Omega_N & \beta_N & \gamma_{N1} & \cdots & \gamma_{Ni} & \cdots & \gamma_{NN} & 0 & \cdots & 0 & \cdots & 0
\end{array}
$$

The limiting distribution of the model can be derived from the following set of equations:

$$\pi_{OFF} = \beta\pi_{OFF} + \sum_{i=1}^{N} \beta_i \pi_{\Omega_I} \tag{1}$$

$$\pi_{ON_i} = \delta_i \pi_{OFF} + \alpha_i \pi_{ON_i} + \sum_{k=1}^{N} \gamma_{ki} \pi_{\Omega_i} \tag{2}$$

$$\pi_{\Omega_i} = (1-\alpha_i)\,\pi_{ON_i} \tag{3}$$

$$\pi_{OFF} + \sum_{k=1}^{N} \pi_{ON_i} + \sum_{k=1}^{N} \pi_{\Omega_i} = 1 \tag{4}$$

The mean arrival rate per output can be expressed as $\lambda_i = \pi_{ON_i} + \pi_{ON_i}$, while the mean burst size for output i is given by $MBS_i = 1 + \frac{1}{1-\alpha_i}$. For the case of uniform distribution and identical mean burst sizes ($\beta_i \equiv \beta$, $\lambda_i \equiv \lambda/N$), we have $\gamma_{ij} = \frac{1-\beta}{N-1}$, $\forall i \neq j$, and $\alpha_i = \alpha$. Solving under these assumptions yields

$$\lambda = \frac{(1-\beta)(2-\alpha)}{2-\alpha-\beta} \tag{5}$$

$$\lambda_i = \frac{(1-\beta)(2-\alpha)}{N(2-\alpha-\beta)} \tag{6}$$

$$MBS_i = 1 + \frac{1}{1-\alpha} \tag{7}$$

The key attribute of this model is that given any set of mean burst sizes, $MBS_i > 1$, and any traffic load distribution, λ_i, the Markov chain in figure 2 can be constructed so as to yield the desired traffic generation engine. More importantly, the latter can achieve 100% traffic load. Without loss of generality, we can prove this statement by showing it holds for the uniform case. We observe that

$$\alpha = \frac{MBS-2}{MBS-1}, \tag{8}$$

which means that given $MBS \geq 2$, we have $0 \leq \alpha \leq 1$. Moreover, observing that

$$\beta = \frac{(1-\lambda)(2-\alpha)}{2-\alpha-\lambda}, \tag{9}$$

we differentiate both sides with respect to λ, to obtain

$$\frac{d\beta}{d\lambda} = \frac{(\alpha-1)(2-\alpha)}{(2-\alpha-\lambda)^2} < 0. \tag{10}$$

The latter suggests that β is a decreasing function of λ, therefore, clearly, given $MBS \geq 2$ and $0 \leq \lambda \leq 1$, directly implies that $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$.

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