Parallel monotone algorithm for a nonlinear convection-diffusion problem

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We are interested in the solution of singularly perturbed convection-diffusion parabolic problems described by the nonlinear differential equation

$$-\varepsilon\,(u_{xx}+u_{yy})+b_1(x,y,t)u_x+b_2(x,y,t)u_y+u_t=f(x,y,t,u),$$

where ε is a small positive parameter. For $\varepsilon \ll 1$, the problem is singularly perturbed and characterized by boundary layers near the boundary of the computational domain.

In the study of numerical solutions of nonlinear singularly perturbed problems by the finite difference method, the corresponding discrete problem is usually formulated as a system of nonlinear algebraic equations. A major point about this system is to obtain reliable and efficient computational algorithms for computing the solution.

We consider a parallel monotone algorithm which combines the monotone method (known as the method of lower and upper solutions) and the domain decomposition method based on the Schwarz alternating procedure.

The monotone method leads to iterative algorithms which converge globally and solve only linear discrete systems at each iterative step which is of great importance in practice. The initial iteration in the monotone iterative method is either upper or lower solutions, which can be constructed directly from the difference equation without any knowledge of the exact solution. This method eliminates the search for the initial iteration as is often needed in Newton's method.

Iterative domain decomposition algorithms based on Schwarz-type alternating procedures have received much attention for their potential as efficient algorithms for parallel computing. We split the computational domain in the space variables into many nonoverlapping subdomains with interface Γ and introduce small interfacial subdomains near Γ. We apply a variant of a block Gauss-Seidel iteration (or in the parallel context as a multicoloured algorithm) with a Dirichlet-Dirichlet coupling through the interface variables.

For small values of ε, the convergence factor $\tilde{\rho}$ of the monotone domain decomposition algorithm is estimated by $\tilde{\rho}=\rho+O(\tau)$, where ρ is the convergence factor of the monotone undecomposed method and τ is the step size in the t-direction. Uniform (in the perturbation parameter) convergence of the monotone domain decomposition algorithm based on layer-adapted meshes is investigated. Numerical experiments are presented.