A Distributed Computational Approach
For Frequency Diverse RF Systems

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Abstract

Conventional array processing has an assumption of band limited signal processing techniques for beamforming. Unfortunately, such approaches do not account for different types of waveforms, particularly when signals of interest may vary in instantaneous bandwidth and center frequency. We propose a signal processing computational approach that can accomodate signals of varying bandwidth and center frequency by linking the structure of the computational method to the type of waveform used in the application.

1. Introduction

We will first develop the methodology for a composite chirp signal and show how this method produces a narrow high resolution pulse for multiple narrow bandwidth signals. We will then show how this composite signal can be integrated into a beam forming technique. Finally we will show the formulation of the beamforming technique in a method that can be scaled based on instananeous bandwidth and number of array elements being used in the waveform.

2. Chirp Signal

The basis optimization algorithm begins by defining a linear FM or ‘chirp’ signal with sufficient bandwidth to provide the desired resolution on the target being observed. The chirp waveform with carrier frequency ω_0 can be described as

$$f_n(t) = \tilde{u}(t/T)e^{jbt^2}e^{jt\omega_0} \tag{1}$$

where $\tilde{u}(t)$ denotes a rectangular pulse of unit height and width, T, centered at the origin and b is the chirp slope describing the rate over the frequency interval between $\omega_i \le \omega \le \omega_{i+n}$. Because the chirp is over the time interval $-T/2 \le t \le T/2$, the instantaneous frequency varies over a bandwidth given by

$$B_c = bT \tag{2}$$

To achieve wider bandwidth response we have multiple chirp increments of n = 1,.....,N where all chirp increments are combined in post processing. If we have an impulse response *I(t)* then our received signal for each chirp increment, is equal to

$$h_n(t) = f_n(t) \otimes I(t) \tag{3}$$

where $\otimes$ denotes the convolution operation. We also define the vector of outputs from the correlation over a finite window of time. If we now Fourier transform $h_n(t)$ you have $H_n(\omega)$. Combining each chirp increment we have

$$\tilde{H}(\omega) = \sum_{n=1}^{N} H_n(\omega) \tag{4}$$

We inverse Fourier transform (4) to obtain $\tilde{r}(t)$ the impulse response of the combined chirp function. In the case of a target response we have

$$s_n(t) = f_n(t) \otimes T(t) \tag{5}$$

If we now Fourier transform $s_n(t)$ you have $S_n(\omega)$. The combined target response function from our object is then

$$\tilde{S}(\omega) = \sum_{n=1}^{N} S_n(\omega) \tag{6}$$

The matched filter response of the target function in the Fourier domain is then

$$\tilde{K}(\omega) = F(\tilde{s}(t))F(\tilde{h}(-t)) \tag{7}$$

Inverse Fourier transforming $k(t)$ is the combined target response. It is important to note that even though we have a high resolution pulse, this technique does not require high sampling rates due to the ability to sample the chirp pulses over slow time intervals thus reducing the real time computational burden on the algorithm.

3. Time Delay Array Structure

We now develop a measurement process for signal returns from an object radiated by a composite chirp signal. To locate a signal return in three dimensions we need to measure the time delay between the signal at 4 points in space . The distance between the source of the backscatter and any of the points is.

$$r_n = \sqrt{(x_n - x_0)^2 + (y_n - y_0)^2 + (z_n - z_0)^2} \tag{8}$$

The corresponding time delay to each point is given by.

$$t_n = \frac{r_n}{c} \tag{9}$$

The angle to the target in 4 dimensions is given by the following two equations

$$\tilde{\theta}_x(n) = \sin^{-1}\frac{x_n}{r_n} \tag{10}$$

$$\tilde{\theta}_y(n) = \cos^{-1}\frac{y_n}{r_n} \tag{11}$$

If we now assume our signal is a single frequency measured from the radiation center of two dimensional rectangular isotropic phased array with spacing d at wavelength λ with radiation pattern as

$$G(\theta_x, \theta_y) = \frac{\sin[N\pi(d/\lambda)\sin\theta_y]^2}{N^2\sin[\pi(d/\lambda)\sin\theta_y]^2}\frac{\sin[M\pi(d/\lambda)\sin\theta_x]^2}{M^2\sin[\pi(d/\lambda)\sin\theta_x]^2} \tag{12}$$

where N = the number of vertical columns of the array that give rise to the vertical angle θ_y and M = the number of horizontal rows that generate the angle θ_x The above relationship assumes that the spacing between elements in the two directions is the same. Unfortunately such a pattern for element spacings that are not equal to $\frac{\lambda}{4}$ results in repeated grating lobes as is shown in Figure 1.

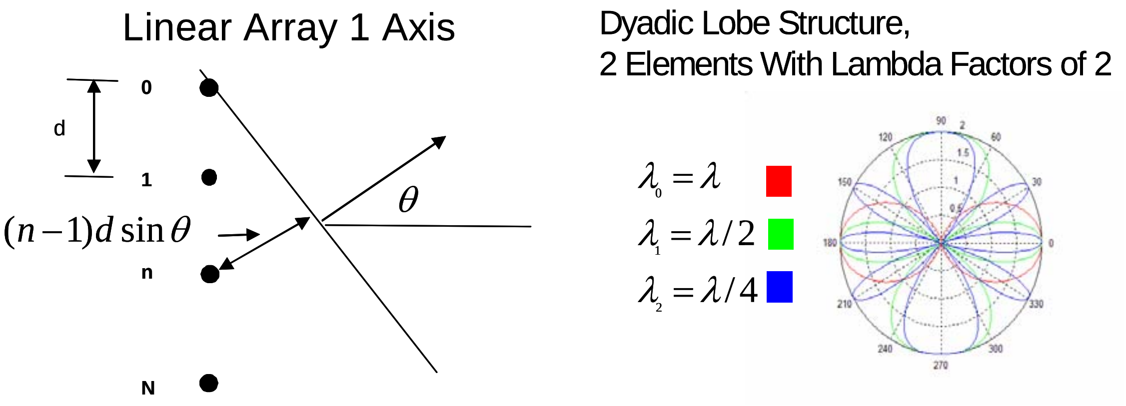

Figure 1 Grating lobe structure for uniform linear array

If we now introduce the composite chirp signal from the previous section as is shown in the following equation as the combination of phased array signals. The impulse response of this signal delayed by the amount that causes all of the return pulses to intersect at the time according to their distances from the reflector is.

$$h(\tau) = h(\tau - t_1) + h(\tau - t_2) + \ldots h(\tau - t_N) \tag{13}$$

The corresponding correlated target response from a point reflector in space is:

$$k(\tau) = k(t_1 - \tau) + k(t_2 - \tau) + \ldots k(t_N - \tau) \tag{14}$$

4. Computational Approach

If you have an individual point scatterer sampled with the chirp waveform in equation 1 $f_n(m)$ with bandwith B_c , this waveform must be critically sampled with bandwidth $2B_c$. The time series of this sampled waveform correlated with with the original wavefom is then $k_n(m)$. For varying time delays l we have $k_n(m-l)$. Thus the discrete equation corresponding to 14 is.

$$\tilde{k}(l) = \sum_{i=1}^{N} k_i(m-l_1) + \sum_{i=1}^{N} k_i(m-l_2) + \ldots \sum_{i=1}^{N} k_i(m-l_M) \tag{15}$$

or

$$\tilde{k} = \sum_{i=1}^{N} \sum_{j=1}^{M} k_i(m - l_j) \tag{16}$$

Thus in equation 16 we see that we can integrate resolution from the number of spatial samples equivalent to the number of spatial elements N or over instananeous bandwidth M. In either case, whether more spatial samples are used or more instantaneous bandwidth, this approach conserves computational overhead for equivalent resolution by integrating in spatial samples or samples in time. Thus applications requiring a given amount of resolution can be scaled according to the waveform being used with the same computational architecture. For the same amount of compuational overhead, computational load can be distributed over numbers of elements N or numbers of time samples M.

5. Conclusion

We have shown a computational formulation for array processing at varying center frequencies and amounts of instantaneous bandwidth. This approach allows scalability in beamforming for varying bandwidths and center frequencies in an aperture.

6. References

1. Brown, R.D., Lynch E.D., Mokry, D.W., VanDamme J.M., Schneible R.A., Wicks M.C., “Near field focusing algorithm for ground penetration imaging radar”, Radar Conference 1999.

2. Skolnik, M., *Introduction to Radar Systems 3rd Ed.*, McGraw Hill, Boston, 2001.

3. Soumekh, M., *Synthetic Aperture Radar Signal Processing with Matlab Algorithms,* Wiley, New York, 1999.

4. Steinburg, B., *Microwave Imaging With Large Antenna Arrays Radio Camera Principles and Techniques,* Wiley, New York, 1984.