Test Instructions: On your test provide the last four digits of your UT student identification number in the space provided at the top right of each page. Do not include your name on the test, just the last four digits your UT ID#. Carefully read the question before solving the problem. Show all your work in the space provided. Note the following suggestions:

- If necessary, write on the back of the problem page, but indicate where your work is continuing for that problem.
- If you are unable to obtain an intermediate value that is needed for subsequent steps in a problem, make an assumption, state it, and use your assumption for the subsequent steps.
- If you realize that your final answer is wrong, but you run out of time to fix it or are unable to find the mistake, indicate that you believe your answer to be wrong and why.
- Clearly mark your final answer with a box or circle.

Calculators are allowed, but they may not have any communication capability. Additional scratch paper is available on request.

1. (35 points) Shown at right is an amplifier with two differential stages with a single supply of 12V. Use these assumptions, parameter values, and definitions:
   - $K_N = 1 \text{mA/V}^2$
   - $I_D = 250 \mu A$ for all four transistors
   - $V_{TN} = 0.5 V$
   - $R_S = 10k\Omega$; $R_D = 24k\Omega$
   - Ignore channel-length modulation, i.e. assume that $r_0=\infty$.
   - The DC common-mode input level is $V_{DD}/2=6V$.
   - Differential input voltage $v_{id} = v_{i+} - v_{i-}$.
   - Common-mode input voltage $v_{ic} = (v_{i+} + v_{i-})/2$
   - First stage common-mode output voltage $v_{o1} = (v_{o1+} + v_{o1-})/2$
   - First stage differential-mode output voltage $v_{od1} = v_{o1+} - v_{o1-}$.

   ![Amplifier Circuit Diagram]

   a) Calculate the common-mode gain ($A_{CM} = A_{CC} = v_{o1}/v_{ic}$) of the first stage.

   The common-mode gain equation for a cascode amplifier is:

   $A_{CM} = \frac{-2g_m R_D}{1+2g_m R_S}$

   Therefore, based on the gain equations for a cascode amplifier:

   $g_m = \sqrt{2 \left( \frac{200}{10} \right)} \approx 3 \text{mS}$

   $A_{CM} = \frac{-2 \times 3 \text{mS} \times 2 \text{k}\Omega}{1+2 \times 3 \text{mS} \times 10 \text{k}\Omega}$

   $= \frac{-6 \times 2 \text{k}\Omega}{1+60}$

   $= -1.12$
b) Calculate the differential-mode gain ($A_{DM} = A_{DD} = V_{od}/V_{id}$) of the first stage.

\[ A_{DM} = \frac{V_{od}}{R_D} = (0.707 m S)(24 k\Omega) \]

\[ A_{DM1} = 16.97 \]

\[ A_{DM, Total} = 14.4 \]

d) Calculate the common-mode gain ($A_{CM} = V_{o}/V_{IC}$) of the entire amplifier.

\[ A_{CM, Total} = A_{CM1} \cdot A_{CM2} = (-1.12)(-1.12) \]

\[ A_{CM, Total} = 1.25 \]

e) Calculate the common-mode rejection ratio (CMRR) of the entire amplifier.

\[ CMRR = \frac{A_{DM}}{A_{CM}} = \frac{14.4}{1.25} = 114.6 \]

\[ CMRR = 41.2 \text{ dB} \]
2. (40 points) Consider the amplifier at right, using the following parameters and assumptions.

- $\beta = 100; V_A = 75V$
- $I_1 = 100\mu A; I_2 = 200\mu A$
- $I_S = 6.91 \times 10^{-17} A$
- For the first stage, you may ignore the Early effect, i.e. assume that $r_o = \infty$, but in the second stage, you must account for it.

a) Find a value for $R_C$ that results in the DC output voltage $V_o = 7.0V$. You may neglect the base current of the PNP for this calculation.

$$I_c_3 = I_2 = 200\mu A \quad V_{CE_3} = 12V - 7V = 5.0V; \quad -V_{BC} = \frac{I_2}{\beta} R_C$$
$$I_c_3 = I_S e^{\frac{-V_{CE_3}}{A_T}} \quad V_{BC} = U_T \ln \left[ \frac{I_c_3}{I_S} \left( \frac{V_A}{V_A + V_{CE_3}} \right) \right]$$

$$V_{BE} = 0.716V = \frac{I_2}{\beta} R_C$$

$$R_C = \frac{0.716V}{200\mu A} = 14.3 k\Omega$$

b) Calculate the voltage gain of each stage and the overall gain for the amplifier

$(A_V = V_o / V_i)$.

Unloaded fully differential gain of 1st stage:

$$A_{V, Total_1} = A_{Op_1} \left( \frac{1}{2} \right) \left( \frac{R_{in_2}}{R_{in_2} + R_{out_2}} \right) A_{V_2}$$

Only taking single-ended output of 1st stage, attenuation due to resistive loading of stage 1 by stage 2.

$A_{Op_1} = 3m A = \frac{I_2}{2V_T} R_C = \left( \frac{50\mu A}{25\mu V} \right) 14.3 k\Omega = 28.6 A_{Op_1}$

$R_{in_2} = R_{in_3} = \frac{B, U_T}{I_c_3} = 12.5 k\Omega$

$R_{out_1} = R_C = 14.3 k\Omega$

$A_{V_2} = 3m A = \frac{I_{CE_3}}{V_A} = \frac{75V}{25\mu V} = 3000 = A_{V_2}$

$A_{V, Total_1} = 28.6 \left( \frac{1}{2} \right) \left( \frac{12.5}{12.5 + 14.3} \right) 3000$

$b = 20,000 \left( \frac{12.5}{12.5 + 14.3} \right) 3000$

$\Rightarrow 86 dB$
c) Find the output resistance of the amplifier.

\[ R_{out} = \frac{V_A}{I_2} = \frac{7.5V}{200mA} \]

\[ R_{out} = 37.5 \, k\Omega \]

d) Suppose that you wish to increase the second-stage current \( I_2 \) in order to reduce the output resistance. What is the maximum allowable value for \( I_2 \) without reducing the overall voltage gain by more than 6dB (1/2).

Increasing \( I_2 \) causes \( g_m \) to increase and \( r_{\pi 3} \) to decrease. This leads down the first stage, in the term \( \frac{r_{\pi 3}}{r_e + r_{\pi 3}} \) from (b). If we increase \( I_2 \) by a factor \( \kappa \), \( r_{\pi 3} \) decreases by a factor \( \kappa \).

\[ \frac{-r_{\pi 3}/\kappa}{r_e + r_{\pi 3}/\kappa} = \frac{1}{2} \left( \frac{r_{\pi 3}/\kappa}{r_e} \right) \rightarrow 2 (r_e + r_{\pi 1}) = \kappa r_e + r_{\pi 1} \]

\[ r_e (1 - \kappa) = r_{\pi 1} \rightarrow \kappa = \frac{r_{\pi 1}}{r_e} + 2 = \frac{12.5k\Omega}{14.3k\Omega} + 2 = 2.87 \approx 5.75 \, mA \]

\[ I_{2, max} = 5.75 \, mA \]

e) Explain why it is reasonable to neglect the Early effect for the first stage but not for the second stage.

In the 1st stage, \( r_o \) of \( Q_1 \) and \( Q_2 \) are insignificant compared to \( r_e \).

In the 2nd stage, \( r_{\pi 3} \) is the only impedance on the output node. It must be considered or the gain would be infinite.
(25 points) In this problem, you will choose values for the capacitors in the amplifier at right. Use these assumptions and parameter values:

- Beta = 100. Neglect Early effect.
- $R_S = 100 \Omega$; $R_L = 100 \, k\Omega$
- $R_C = 20 \, k\Omega$; $R_E = 5 \, k\Omega$
- $R_{B1} = 180 \, k\Omega$; $R_{B2} = 50 \, k\Omega$
- $I_C = 400 \, \mu A$

a) Find the small-signal resistance looking into the base $R_{ib}$.

$$R_{ib} = R_T + (\beta + 1) R_C$$

$$R_T = \frac{\beta}{\beta + 1} \frac{U_T}{I_C} = \frac{100 \times 25 \, mV}{400 \, mA} = 6.25 \, k\Omega$$

$$R_{ib} = 6.25 \, k\Omega + (10) 5 \, k\Omega$$

$$R_{ib} = 511.25 \, k\Omega$$

b) Find the value of $C_1$ such that it appears as a short for frequencies above 1 kHz.

$$\frac{1}{2\pi f C_1} \ll R_S + R_{ib}/R_{B1}/R_{B2} = 100\Omega + 511.25\, k\Omega/180\, k\Omega/50\, k\Omega$$

$$= 36.4\, k\Omega$$

$$C_1 = \frac{10}{2\pi (1000) 36.4\, k\Omega} = 43.7 \times 10^{-9}$$

$$C_1 = 43.7 \, nF \approx 4$$
c) Find the value of $C_2$ such that it can be considered a short for frequencies above 1 kHz.

\[
\frac{1}{2\pi f_c} \ll R_L + R_C = 120 \text{ k} \Omega
\]

\[
C_2 = \frac{\frac{10}{2\pi (1000)} \text{ k} \Omega}{120 \text{ k} \Omega} = 13.3 \times 10^{-9}
\]

$C_2 = 13.3 \text{ nF}$