Test Instructions: On your test provide the last four digits of your UT student identification number in the space provided at the top right of each page. Do not include your name on the test, just the last four digits your UT ID#. Carefully read the question before solving the problem. Show all your work in the space provided. Note the following:

- If necessary, write on the back of the problem page, but indicate where your work is continuing for that problem.
- If you are unable to obtain an intermediate value that is needed for subsequent steps in a problem, make an assumption, state it, and use your assumption for the subsequent steps.
- If you realize that your final answer is wrong, but you run out of time to fix it or are unable to find the mistake, indicate that you believe your answer to be wrong and why.
- Put your final answer in the space provided.

Calculators are allowed, but may not have any communication capability. Additional scratch paper is available on request.

1. (35 points) Consider the current mirror shown here. The reference current, provided by the ideal current source $I_{In}$, is duplicated ten times. Use the parameters below for your analysis.
   - $V_A = 100$ V; $\beta = 100$; $V_{BE} = 0.7$ V
   - $I_{In} = 100$ µA; $R_E = 1$ kΩ

   a) Find $I_{Out,N}$ (where $N$ is any number 1-10). For this part, neglect the Early effect and assume all transistors are in the forward-active region.

Because $V_E$ is the same for all transistors Q0-10 and emitter condition is the same, $I_C$ is equal for Q0-Q10, and therefore base current is also equal.

$$
I_{C0} = I_C(1-10) \\
I_{B0} = I_B(1-10) \\
I_{In} = I_{C0} + I_{B(0-10)} \\
= I_{C0} + 11I_{B0} \\
= I_{C0} \left(1 + \frac{11}{\beta}\right) \\
I_{C0} = \frac{I_{In} \beta}{\beta + 11} = 0.9I_{In} \\
I_{Out,N} = I_{C,N} = 0.9I_{In} = 90.1$ $\mu$A

$
I_{Out} = 90.1$ $\mu$A
b) Q0 and its emitter resistor provide the bias voltage for the other current sources. What is the equivalent resistance for this voltage source?

There are (at least) two ways to do this, but either way we’ll need to know \( gm \):

\[
g_m = I_C / U_T = 90 \mu A / 25 mV = 3.6 mS
\]

Method 1: A the diode connected resistor has an equivalent resistance of \( 1/gm \parallel ro \sim 1/gm \), and the current source has infinite resistance, so

\[
R_{Eq} = R_E + 1/gm = 1.27 \, k\Omega
\]

Method 2: For a source-degenerated transistor, as we’ve discussed before:

\[
G_m = i_d / v_b = g_m / (1 + g_m R_E)
\]

\[
R_{Eq} = 1/G_m = (1 + g_m R_E) / g_m = 1.27 \, k\Omega
\]

\[
R_{Eq} = \underline{1.27 \, k\Omega}
\]

c) Find the output resistance of the current sources. (You must include the Early effect for this part.)

\[
R_{Out} = r_o \left( 1 + \frac{\beta R_E}{R_{th} + R_E + r_\pi} \right)
\]

\[
r_o = \frac{V_A}{I_C} = 1.1 M\Omega
\]

\[
r_\pi = \frac{\beta}{g_m} = 27.8 k\Omega
\]

\[
R_{th} = R_X |\frac{R_{ib}}{10}
\]

\[
R_X = R_{Eq} \text{ from part (b)}
\]

\[
R_{ib} = r_\pi (1 + g_m R_E) = 6 k\Omega
\]

\[
R_{th} = 1.15 k\Omega
\]

\[
R_{Out} = 4.7 M\Omega
\]

\[
R_{Out} = \underline{4.7 \, M\Omega}
\]
2. (20 points) Consider the amplifier at right. Assume that the DC operating point is known and that the transistor operates in the forward active region. Use these parameters:

- $I_C = 2 \text{ mA}$; $\beta = 100$
- $R_1 = 5 \text{ k}\Omega$; $R_2 = 65 \text{ k}\Omega$
- $R_3 = 5 \text{ k}\Omega$; $R_L = 5 \text{ k}\Omega$
- $V_{CC} = V_{EE} = 12\text{ V}$
- Neglect the Early effect

Choose the correct size for $C_2$ such that it can be treated as a short circuit at 100 Hz.

\[
R_{C2} = R_3||R_{ie} + R_L
\]

\[
R_{ie} = 1/g_m + \frac{R_{th}}{\beta}
\]

\[
R_{th} = R_1||R_2 = 4.6k\Omega
\]

\[
1/g_m = \frac{25\text{ mV}}{2\text{ mA}} = 12.5\Omega
\]

\[
R_{ie} = 58.9\Omega
\]

\[
R_{C2} = 5.1k\Omega
\]

\[
C_2 = \frac{10}{2\pi R_{C2}f_L} = 3.15\mu F
\]

\[
C_2 = \underline{3.15\mu F}
\]
3. (35 points) In this problem you will examine a two-stage amplifier comprising a differential input stage and a PMOS common-drain (source follower) second stage. Use these assumptions and parameter values:

- \( V_{TN} = 0.6 \, \text{V} \); \( V_{TP} = -0.6 \, \text{V} \)
- \( K_N = 400 \, \mu\text{A/V}^2 \); \( K_P = 250 \, \mu\text{A/V}^2 \)
- \( I_1 = 200 \, \mu\text{A} \); \( R_1 = 100 \, \text{k}\Omega \) (i.e. \( I_1 \) has 100 k\Omega parallel resistance).
- \( R_2 = 24 \, \text{k}\Omega \)
- DC common-mode input voltage = 6 V
- Ignore channel-length modulation

a) Find \( R_D \) such that the output (\( V_o \)) is biased at 6V DC. Neglect the output resistance of \( R_1 \) for this part.

We'll call the output PFET M3.

\[
I_{D3} = \frac{V_{DD} - V_{out}}{R_2} = \frac{12V - 6V}{24k\Omega} = 250\mu\text{A}
\]

\[
V_{GS3} = -\sqrt{2I_{D3}/K_P} - V_{TP} = -2V
\]

\[
V_{O1-} = V_{out} - (-V_{GS3}) = 4V
\]

\[
I_{D1} = I_{D2} = I_1/2 = 100\mu\text{A}
\]

\[
R_D = \frac{V_{DD} - V_{O1-}}{I_{D1}} = \frac{8V}{100\mu\text{A}} = 80k\Omega
\]

\[ R_D = \boxed{80 \, \text{k}\Omega} \]

b) Find the overall small-signal differential gain.

\[
g_{m1} = \sqrt{2K_NI_{D1}} = 280\mu\text{S}
\]

\[
g_{m3} = \sqrt{2K_PI_{D3}} = 353\mu\text{S}
\]

\[
A_{DM1} = g_{m1}R_D/2 = 11.3
\]

\[
A_{V2} = \frac{g_{m3}R_2}{1 + g_{m3}R_2} = 0.89
\]

\[
A_{DM,Total} = A_{V1}A_{V2} = 10.1
\]

\[ A_{DM} = \boxed{10.1} \]

c) Find the overall small-signal common-mode gain.
4. (10 points) As shown at right, a source-degeneration resistor can be used to increase a MOS current source’s output resistance. Let $K$ represent the factor by which output resistance is increased, such that $R_{Out} = K r_o$. Find $K$ in terms of $V_S$, the voltage across the degeneration resistor, and the overdrive voltage $V_{OV} = V_{GS} - V_{TN}$.

$$A_{CM1} = \frac{g_{m1}R_D/2}{1 + 2g_{m1}R_1} = 0.39$$
$$A_{CM,Total} = A_{CM1} A_{V2} = 0.35$$

$$A_{CM} = \underline{0.35}$$

$$R_{Out} = r_o(1 + g_m R_S)$$

$$K = \frac{R_{Out}}{r_o}$$

$$= 1 + g_m R_S$$

$$= 1 + \frac{2I_D}{V_{OV}} R_S$$

$$= 1 + \frac{2V_S}{V_{OV}}$$