Test Instructions: On your test provide the \textbf{last four digits of your UT student identification number} (not your SSN) in the space provided at the top right of each page. Do not include your name on the test, just the last four digits your UT ID#. \textit{Carefully} read the question before solving the problem. Show all your work in the space provided. Note the following suggestions:

- If necessary, write on the back of the problem page, but indicate where your work is continuing for that problem.
- If you are unable to obtain an intermediate value that is needed for subsequent steps in a problem, make an assumption, state it, and use your assumption for the subsequent steps.
- If you realize that your final answer is wrong, but you run out of time to fix it or are unable to find the mistake, indicate that you believe your answer to be wrong and why.
- If you find yourself short of time, it is more important to set a problem up and demonstrate the approach you would take than it is to complete the computation.
- Clearly \textbf{mark your final answer} with a box or circle.

Calculators are allowed, but they may not have any communication capability. Additional scratch paper is available on request.

1. (20 points) Shown at right is a single-stage differential amplifier with a single supply of 12V. Use these assumptions, parameter values, and definitions:

- $I_s = 1 \text{ fA} = 10^{-15} \text{ A}$
- Neglect base current and Early effect. ($V_A = \beta = \infty$)
- $V_{cc} = 12 \text{ V}$, $I_{ee} = 100 \mu \text{A}$
- $R_C = 100 \text{ k}\Omega$
- $V_X = 2.3 \text{ V}$ and $V_Y = 2.35 \text{ V}$

\begin{align*}
I_{c1} &= I_s \frac{V_X - V_Y}{V_T} \\
I_{c2} &= I_s \frac{V_Y}{V_T} \\
I_{c1} + I_{c2} &= I_s \left(1 + \frac{1}{55}ight) = I_{ee} = 100 \mu \text{A} \\
I_{c1} &= \frac{100 \mu \text{A}}{1 + \frac{1}{55}} = 11.9 \mu \text{A} \\
I_{c2} &= 89.1 \mu \text{A} \\
V_e &= V_{cc} - I_{c1} R_c = 12 - (11.9 \mu \text{A})(100 \text{ k}\Omega) = 10.8 \text{ V} \\
V_3 &= V_{ee} - I_{c2} R_c = 3.2 \text{ V} \\
V_{ce} &= I_{c2} e^{V_{ee}/V_T} \rightarrow V_{be} = V_T \log \left(\frac{I_{c2}}{I_s}\right) = .58 \text{ V} \\
V_1 &= V_X - V_{be} = 1.72 \text{ V} \\
V_2 &= 10.8 \text{ V} \\
V_3 &= 3.2 \text{ V}
\end{align*}
2. (35 points) At right is shown a common-drain amplifier. In this problem, you will analyze signal propagation from the positive supply to the output. Assume that the input is an AC ground. Leave your answers in terms of component values and small-signal parameters (e.g. $g_m$, $r_o$, $R_L$, etc.).

a) (20 pts) Find the low-frequency small-signal gain from the positive supply to the output $A_{ps} = \frac{V_{dd}}{V_o}$. Be sure to account for channel-length modulation ($r_o$).

For (a) neglect capacitors

$V_{gs} = -V_o$

$V_o \left( \frac{1}{V_o} + \frac{1}{R_L} \right) = \frac{V_{dd}}{r_o}$

$\frac{V_o}{V_{dd}} = \frac{1}{r_o \left( \frac{1}{V_o} + \frac{1}{R_L} \right)}$

$A_{ps} = \frac{V_o}{V_{dd}} = \frac{1}{1 + g_m r_o + \frac{r_o}{R_L}}$

b) (15 pts) Using either the open-circuit time-constant technique or direct analysis, find the high-frequency cutoff $f_h$ of $A_{ps}(s)$.

Some model, but include capacitors.

Direct Analysis:

$\frac{V_{dd}}{r_o} = V_o \left( \frac{1}{V_o} + \frac{1}{R_L} + s C_{gs} + C_L \right)$

$\frac{V_o}{V_{dd}} = \frac{1}{1 + g_m r_o + \frac{r_o}{R_L} + s r_o \left( C_{gs} + C_L \right)}$

$A_{ps}(s) = \frac{V_o}{V_{dd}} = \left( \frac{1}{1 + g_m r_o + \frac{r_o}{R_L}} \right) \frac{1}{1 + s \left( \frac{r_o}{1 + g_m r_o + \frac{r_o}{R_L}} \right) \left( C_{gs} + C_L \right)}$

Only 1 pole, so high-frequency cutoff is

$f_h = \frac{1}{2 \pi \left( V_o \left| g_m \right| R_L \left( C_{gs} + C_L \right) \right)}$
3. (10 pts) In the circuit below $A_1$ is an ideal op-amp. Find values for $R_1$ and $R_2$ such that the input resistance (as seen from the signal source $v_i$) is 100 kΩ and the small-signal voltage gain ($v_{out}/v_i$) is -5.

\[
R_{in} = \frac{R_2}{R_1} = 100 \text{kΩ} \\
A_v = -\frac{R_2}{R_1} = -5, \quad R_2 = 5 \text{R}_1 = 500 \text{kΩ}
\]

4. (5 points) Consider the differential amplifier shown at right. Of $v_a$ and $v_b$, determine which is the positive input and which is the negative input.

There is one inversion from $v_b$ to $v_x$ and one from $v_x$ to $v_a$, so $v_b$ is the positive input.

$V_b$ positive

$V_a$ negative.
5. (30 points) Consider the common-base amplifier shown here. Assume the following parameter values:
- $\beta = 100$, $I_S = 10^{-15}$ A.
- Neglect Early effect.
- $C_L = 2 \text{ pF}$
- $C_C = 2 \text{ pF}$, $C_m = .5 \text{ pF}$
- $V_{CC} = V_{EE} = 12 \text{ V}$
- $R_I = 200 \Omega$

a) (15 pts) Assuming that $C_1$ and $C_2$ are very large, choose $R_E$ and $R_C$ such that $g_m = 1 \text{ mS}$ and $A_V = V_Y / V_X = 10$.

$$S_n = \frac{I_e}{V_T} = 1 \text{ mS} \Rightarrow I_e = 2.5 \text{ mA} \Rightarrow I_C = I_e / (\beta + 1)$$

$$V_{BE} = V_{BEU} = V_T \log \left( \frac{I_e}{I_S} \right) = 0.83 \text{ V}$$

$$R_C = \frac{12V - 0.83}{5.98 \text{ mA} (\frac{1}{100})} = 451.5 \Omega$$

$$A_V = \frac{g_m R_C}{R_E + \frac{R_C}{1 + \frac{1}{S_m}}} = 10 \Rightarrow R_C = \frac{10}{1.85 g_m} = \frac{12.5 \text{ k}\Omega}{R_C}$$

b) (15 pts) Using direct analysis, find the two most significant poles in the high-frequency response.

The small-signal model is:

$$N_x = \frac{R_E}{R_E} - \frac{N_e}{N_e}$$

$$V_Y \left( \frac{1}{R_E + s(C_m + C_L)} \right) = V_Y$$

$$\frac{V_Y}{V_X} = \frac{N_e}{N_X}$$

$$N_e \left( \frac{1}{R_E + s(C_m + C_L)} \right) = N_e$$

$$N_e = \frac{I_m}{R_C} \left( \frac{1}{\frac{1}{R_E} + \frac{1}{C_m} \left( \frac{1}{R_E} \right)} \right)$$

$$\frac{V_Y}{V_X} = \frac{V_X \left( \frac{R_E + R_C}{R_E + \frac{R_C}{1 + \frac{1}{S_m}}} \right)}{R_E + R_C / R_C} = \frac{R_E + R_C / R_C}{R_E}$$

The poles are:

$$f_{1} = \frac{1}{2 \pi R_C \left( R_E + \frac{1}{S_m} \right)} = 478 \text{ MHz}$$

$$f_{2} = \frac{1}{2 \pi \left( R_E \left( C_m + C_L \right) \right)} = 5.3 \text{ MHz}$$