Follower Circuits: Output Resistance

- Follower $R_{Out} = (R_{iE} \parallel R_L) \sim 1/g_m$
- But, does driving resistance matter?
  - Consider extreme case: Base driven by current source.
Follower Circuits: Output Resistance

Current is injected into emitter of BJT.

\[ v_e = \frac{\alpha_o i}{g_m} + \frac{R_{th} i}{\beta_o + 1} \]

Current \( \alpha_o i \) coming out of collector must be supported by \( v_{eb} = \frac{\alpha_o i}{g_m} \), given by the first term. \( i_b = -\frac{i}{\beta_o} + 1 \) creates a voltage drop in \( R_{th} \) given by second term

In case of FET, \( R_{IS} = \frac{1}{g_m} \)

Thus equivalent resistance looking into emitter or source of a transistor is approximately \( \frac{1}{g_m} \).
Emitter (Source) Degeneration

- Emitter (source) resistor often used in CE (CS) amps.
- What is its effect?
- Examine transconductor by itself.
- Assume $V_C$ is small-signal ground.
Emitter (Source) Degeneration

- We will view NPN+RE as transconductor
- From CD analysis: gain from \( v_x \) to \( v_e \)?
  - \( g_m R_E/(1+g_m R_E) \)
- For \( G_m = i_c/v_x \), find \( v_{be}/v_x \)
- \( v_{be}/v_x = 1 - g_m R_E/(1+g_m R_E) \)
  = \( 1/(1+g_m R_E) \)
- \( i_c = g_m v_{be} \), so \( G_m = g_m/(1+g_m R_E) \)
- For \( g_m R_E \gg 1 \), \( G_m \to 1/R_E \)
- Why reduce \( G_m \)? Sensitivity, linearity
Emitter (Source) Degeneration

- $R_{\text{In}}$?
- $v_{be} = v_x/(1 + g_m R_E)$
- $i_b = v_{be}/r_\pi$
  
  \[= v_x/[r_\pi(1 + g_m R_E)]\]
- $R_{\text{In}} = v_x/i_b = r_\pi(1 + g_m R_E)$
- $R_{\text{Out}}$?
Emitter (Source) Degeneration

- $R_{out}$? (Ignore $i_b$)
- Find $i_c$
  \[ i_c = -g_m v_e + \frac{(v_c - v_e)}{r_o} \]
  \[ = \frac{v_c}{r_o} - v_e \left( g_m + \frac{1}{r_o} \right) \]
- Get rid of $v_e$? KCL @ $v_e$
  \[ v_e \left( \frac{1}{R_E} + \frac{1}{r_o} + g_m \right) = \frac{v_c}{r_o} \]
  \[ v_e = \frac{v_c}{r_o \left( \frac{1}{R_E} + \frac{1}{r_o} + g_m \right)} \]
Emitter (Source) Degeneration

- $R_{Out}$? (Ignore $i_b$)
- Combine

$$i_c = v_c \left[ \frac{1}{r_o} - \frac{g_m + 1/r_o}{1 + g_m r_o + r_o/R_E} \right]$$

$$= \frac{1/r_o + g_m + 1/R_E - g_m - 1/r_o}{1 + g_m r_o + r_o/R_E}$$

$$\frac{i_c}{v_c} = \frac{1}{R_E + g_m r_o R_E + r_o}$$

$R_{Out} \sim r_o (1 + g_m R_E)$ If $r_o >> R_E$

- Error significant if $R_E >> r_\pi$
Terminal Impedances - Collector

- Current across $R_E$ decreases $i_E$, increasing $R_{iC}$.
- Some “escapes” through base, increases $v_{BE}$ & limiting $R_{iC}$

\[ i_c = g_m v_{be} + (v_c - v_e)/r_o \]
\[ v_e = i_c R_E || (R_B + r_\pi) \]
\[ = i_c \frac{R_E (R_B + r_\pi)}{R_E + R_B + r_\pi} \]
\[ v_b = i_c \frac{R_E}{R_E + R_B + r_\pi} R_B \]
\[ v_{be} = i_c \frac{R_E R_B - R_E R_B - R_E r_\pi}{R_E + R_B + r_\pi} \]
\[ i_c = g_m i_c \frac{-R_E r_\pi}{R_E + R_B + r_\pi} + v_c/r_o - \frac{1}{r_o} \left( i_c \frac{R_E (R_B + r_\pi)}{R_E + R_B + r_\pi} \right) \]
\[ v_c/r_o = i_c \left( 1 + \frac{g_m R_E r_\pi + 1/r_o (R_E (R_B + r_\pi))}{R_E + R_B + r_\pi} \right) \]
\[ R_{iC} = \frac{v_c}{i_c} = r_o \left( 1 + R_E \frac{\beta + (R_B + r_\pi)/r_o}{R_E + R_B + r_\pi} \right) \]
\[ R_{iC} = \frac{v_c}{i_c} = r_o \left( 1 + R_E \frac{\beta + R_B/r_o + \beta/(g_m r_o)}{R_E + R_B + r_\pi} \right) \]
\[ R_{iC} = \frac{v_c}{i_c} = r_o \left( 1 + R_E \frac{\beta + R_B/r_o}{R_E + R_B + r_\pi} \right) \]
Terminal Impedances - Collector

- Expression is cumbersome and unhelpful. Make some approximations.
- Important when trying to maximize gain

\[ R_{iC} = r_o \left( 1 + R_E \frac{\beta + R_B/r_o}{R_E + R_B + r_\pi} \right) \]

\[ R_{iC} \approx r_o \left( 1 + \frac{R_E \beta}{R_E + R_B + r_\pi} \right) \]

If \( \beta \to \infty \) (MOS); recall \( r_\pi = \beta/g_m \)

\[ R_{iC} \approx r_o \left( 1 + R_E \frac{\beta}{\beta/g_m} \right) = r_o (1 + g_m R_E) \]

If \( \beta \) finite and \( R_E \gg r_\pi \)

\[ R_{iC} \approx r_o (1 + \beta) \]