Frequency Response Review

- Frequency response defined by poles and zeros
  - Equal number of poles and zeros, but some may be at origin or at infinity.
- Zero: increase magnitude slope by 20 dB/decade
  - 90 deg phase lag/lead for RHP/LHP zero
    \[ H(s) = \frac{1 + \frac{s}{\omega_z}}{H_p(s)} = \frac{1 + \frac{j\omega}{\omega_z}}{H_p(s)} \]
- Pole: decrease
  - 90 deg phase lead/lag for RHP/LHP pole
- 20dB/dec = 10x/10x
  \[ H(s) = \frac{H_z(s)}{1 + \frac{s}{\omega_p}} = \frac{H_z(s)}{1 + \frac{j\omega}{\omega_p}} \]
Frequency Response Review

- Single RC pole
  - pole at freq where $Z(C) = R$
  - 3 dB ($\sqrt{2}$) attenuation
  - 20 dB/decade after pole
  - 90° total phase lag, 45° at pole

$$H(s) = \frac{Z_C}{Z_C + R}$$
$$= \frac{1}{sC} \frac{1}{R}$$
$$= \frac{1}{1 + sCR}$$
Frequency Response Review

- RC time constant: single pole in left half plane
  - real part is negative.
- Exists over entire s plane, but evaluated over imaginary axis

\[ H(s) = \frac{1}{1 + \frac{s}{\omega_p}} \]

\[ \omega_p = \frac{1}{RC} \]

\[ s = \sigma + j\omega \]
Frequency Response Review

- Single RC pole
  - 90° total phase lag, 45° at pole
  - Phase lag = $\tan(\omega/\omega_p)$
- Why does phase matter?
  - Feedback stability
  - Waveform fidelity

$$H(s) = \frac{Z_C}{Z_C + R}$$

$$= \frac{1}{sC} \frac{1}{sC + R}$$

$$= \frac{1}{1 + sCR}$$
Single-Pole High Frequency Response

Can split into the low-frequency term and the pole. Pole associated with a node at \( \omega = 1/(R_T C_T) \)

\[
\frac{v_x}{v_i} = \frac{R_2 \left| \frac{1}{sC_1} \right|}{R_1 + R_2 \left| \frac{1}{sC_1} \right|} = \frac{\frac{R_2}{1 + sR_2C_1}}{R_1 + \frac{R_2}{1 + sR_2C_1}}
\]

\[
= \frac{R_2}{R_1 + R_2} \left( 1 + s \frac{R_1 R_2}{R_1 + R_2} C_1 \right)
\]

\[
= \frac{R_2}{R_1 + R_2} \left( 1 + s \left[ R_1 \parallel R_2 \right] C_1 \right)
\]
Substituting $s=j2\pi f$ and using $f_p=1/(2\pi [R_1||R_2]C_1)$

\[
\frac{v_x}{v_i} = \frac{R_2}{R_1 + R_2} \left(1 + \frac{jf}{f_p}\right)
\]

This expression has two parts, the midband gain, $R_2/(R_2+R_1)$, and the high frequency characteristic, $1/(1+jf/f_p)$. 
Frequency Response Review

- High-Pass RC
  - Zero at DC, Pole at 1/RC

\[
H(s) = \frac{R}{Z_C + R} = \frac{1}{sC + R} = RC \frac{s}{1 + sRC}
\]
Frequency Response Review
Transfer Function Analysis

\[ F_L = \frac{(s + \omega_{Z1}^L)}{(s + \omega_{P1}^L)} \frac{(s + \omega_{Z2}^L)}{(s + \omega_{P2}^L)} \cdots \frac{(s + \omega_{Zk}^L)}{(s + \omega_{Pk}^L)} \]

\[ F_H = \frac{(1 + s/\omega_{Z1}^H)}{(1 + s/\omega_{P1}^H)} \frac{(1 + s/\omega_{Z2}^H)}{(1 + s/\omega_{P2}^H)} \cdots \frac{(1 + s/\omega_{Zk}^H)}{(1 + s/\omega_{Pk}^H)} \]

\[ F_H \approx 1 \text{ for } \omega << \omega_{Zi}^H, \omega_{Pi}^H, i = 1 \ldots 1 \]

\[ \therefore A_{L}(s) = A_{mid} F_L(s) \]

\[ F_L \approx 1 \text{ for } \omega >> \omega_{Zj}^L, \omega_{Pj}^L, j = 1 \ldots k \]

\[ \therefore A_{H}(s) = A_{mid} F_H(s) \]

\[ A_{mid} \text{ is midband gain between upper and lower cutoff frequencies.} \]
High-Frequency Response

\[ F_H \approx \frac{1}{(1 + s/\omega_{P3})} \]
\[ \omega_H \approx \omega_{P3} \]
\[ \omega_{P3} < \text{all other poles} \]

With no dominant pole, poles and zeros interact to determine \( \omega_H \).

For \( s=j\omega \), at \( \omega_H \), \[ |A(j\omega_H)| = \frac{A_{mid}}{\sqrt{2}} \]

\[ \frac{1}{\sqrt{2}} = \sqrt{\frac{1+(\omega_H^2/\omega_{Z1}^2)}{1+(\omega_H^2/\omega_{Z2}^2)}} \frac{1+(\omega_H^2/\omega_{P1}^2)}{1+(\omega_H^2/\omega_{P2}^2)} \]

Pole \( \omega_H < \) all other pole and zero frequencies

In general, \( \omega_H \approx \frac{1}{\sqrt{\sum_{n} \frac{1}{\omega_{Pn}^2} - 2\sum_{n} \frac{1}{\omega_{Zn}^2}}} \)
High-Frequency Response

\[ |A(j\omega)| \text{ dB} \]

Dominant-pole approximation (dashed)

Actual

\[ \omega_H \]

34 dB

\[ 10^6 \quad 10^8 \quad 10^9 \quad \omega (\text{log scale}) \]
Low-Frequency Response

![Graph showing the frequency response of a system with a dominant pole approximation, including dB scale and frequency axis (log scale).]
Low-Frequency Response

Estimating low cut-off frequency without a dominant pole:

\[ A_L(s) = \frac{A_{mid} F_L(s)}{s + \omega Z_1} \left( s + \omega Z_2 \right) \]

For \( s = j\omega \), at \( \omega_L \),

\[ A(j\omega_L) = \frac{A_{mid}}{\sqrt{2}} \]

\[ \frac{1}{\sqrt{2}} = \sqrt{\frac{\omega_L^2 + \omega Z_1^2}{\omega_L^2 + \omega P_1^2}} \sqrt{\frac{\omega_L^2 + \omega Z_2^2}{\omega_L^2 + \omega P_2^2}} \]

\[ \Rightarrow \frac{1}{2} = \frac{1 + \frac{\omega^2}{\omega L^2} + \frac{\omega^2}{\omega L^4} + \frac{\omega^2}{\omega P_1^2} + \frac{\omega^2}{\omega P_2^2}}{1 + \frac{\omega^2}{\omega P_1^2} + \frac{\omega^2}{\omega P_2^2}} \]

Pole \( \omega_L > \) all other pole and zero frequencies

\[ \omega_L \equiv \sqrt{\omega^2 + \omega P_1^2 - 2\omega Z_1^2 - 2\omega Z_2^2} \]

In general, for \( n \) poles and \( n \) zeros,

\[ \omega_L \equiv \sqrt{\sum_{n} \omega^2 P_n^2 - 2\sum_{n} \omega Z_n^2} \]
Transfer Function Analysis and Dominant Pole Approximation Example

- **Problem:** Find midband gain, $F_L(s)$ and $f_L$ for
  \[ A_L(s) = 2000 \frac{s(s + 100)}{(0.1s + 1)(s + 1000)} \]

- **Analysis:** Rearranging the given transfer function to get it in standard form,
  \[ A_L(s) = 200 \frac{s(s + 100)}{(s + 10)(s + 1000)} \]

  Now,
  \[ A_L(s) = A_{\text{mid}} F_L(s) \]

  \[ A_{\text{mid}} = 200 \quad \text{and} \quad F_L(s) = \frac{s(s + 100)}{(s + 10)(s + 1000)} \]

  Zeros are at $s=0$ and $s=-100$. Poles are at $s=-10$, $s=-1000$

  \[ f_L = \frac{1}{2\pi} \sqrt{10^2 + 1000^2 - 2(0^2 + 100^2)} = 158 \text{Hz} \]

  All pole and zero frequencies are low and separated by at least a decade. Dominant pole is at $\omega=1000$ and $f_L = 1000/2\pi = 159 \text{ Hz}$. For frequencies $> \text{a few rad/s}$: \[ A_L(s) = 200 \frac{s}{(s + 1000)} \]
Transfer Function Analysis

\[ A_v(s) = \frac{N(s)}{D(s)} = \frac{a_0 + a_1 s + a_2 s^2 + \ldots + a_m s^m}{b_0 + b_1 s + b_2 s^2 + \ldots + b_n s^n} \]

\[ = A_{mid} F_L(s) F_H(s) \]

\[ A_{mid} \] is midband gain between upper and lower cutoff frequencies.

\[ F_L(s) = \left( \frac{s + \omega L}{Z_1} \right) \left( \frac{s + \omega L}{Z_2} \right) \ldots \left( \frac{s + \omega L}{Z_k} \right) \]

\[ F_H(s) = \left( \frac{1 + \frac{s}{H}}{\omega Z_1} \right) \left( \frac{1 + \frac{s}{H}}{\omega Z_2} \right) \ldots \left( \frac{1 + \frac{s}{H}}{\omega Z_l} \right) \]

\[ |F_H(j\omega)| \geq 1 \text{ for } \omega << \omega_{Z_i} \omega_{H}, i = 1 \ldots l \]

\[ : A_L(s) \equiv A_{mid} F_L(s) \]

\[ |F_L(j\omega)| \geq 1 \text{ for } \omega >> \omega_{Z_j} \omega_{P_j}, j = 1 \ldots k \]

\[ : A_H(s) \equiv A_{mid} F_H(s) \]
Direct Determination of Low-Frequency Poles and Zeros: C-S Amplifier

\[ V_o(s) = I_o(s)R_3 = -g_m V_{gs}(s) \frac{R_D}{R_D + (1/sC_3) + R_3} \]

\[ = -g_m (R_3 || R_D) \frac{s}{1/C_3(R_D + R_3)} V_{gs}(s) \]

\[ V_g(s) = \frac{s + C_1 R_G}{s + C_1 (R_I + R_G) + 1} V_i(s) \]

\[ V_{gs}(s) = V_g - V_S = \frac{s + (1/C_2 R_S)}{s + [1/(1/g_m) R_S]} V_g(s) \]

\[ A_v(s) = \frac{V_o(s)}{V_i(s)} = A_{mid} F_L(s) \]

\[ A_{mid} = -g_m (R_3 || R_D) \frac{R_G}{R_G + R_I} \]
Direct Determination of Low-Frequency Poles and Zeros: C-S Amplifier (contd.)

\[ F_L(s) = \frac{s^2 \left( s + \frac{1}{C_2 R_S} \right)}{\left( \frac{1}{C_1 (R_I + R_G)} \right) \left( \frac{1}{C_2 (1/g_m) R_S} \right) \left( \frac{1}{C_3 (R_D + R_3)} \right)} \]

- The three zero locations are: \( s = 0, 0, -1/(R_S C_2) \).
- The three pole locations are: \( s = -\frac{1}{C_1 (R_I + R_G)}, -\frac{1}{C_2 (1/g_m) R_S}, -\frac{1}{C_2 (R_D + R_3)} \)

- Each capacitor contributes one pole and one zero.
- Series capacitors \( C_1 \) and \( C_3 \) contribute the two zeros at \( s=0 \) (DC)
- Parallel combination of \( C_2 \) and \( R_S \) creates 3rd zero
  - Parallel paths generally create zeros: same polarity: LHP; opposite polarity: RHP
Direct Determination of Low-Frequency Poles and Zeros: C-S Amplifier (contd.)

\[ v_o(s) = g_m v_{gs}(s) \frac{R_D}{R_D + (1/sC_3) + R_3} \]

\[ = g_m v_{gs}(s) (R_D \parallel R_3) X \]

\[ \frac{R_D R_3}{R_D + R_3} X = \frac{R_D R_3}{R_D + (1/sC_3) + R_3} \]

\[ X = \frac{R_D + R_3}{R_D + (1/sC_3) + R_3} \]

Want poles, zeros in form \( s + \omega z/p \)

\[ X = \left[ \frac{s/(R_D + R_3)}{s/(R_D + R_3)} \right] \left[ \frac{R_D + R_3}{R_D + (1/sC_3) + R_3} \right] \]

\[ X = \frac{s}{s + 1/C_3(R_D + R_3)} \]
Short-Circuit Time Constant Method to Determine $\omega_L$

- Lower cutoff frequency for a network with $n$ coupling and bypass capacitors is given by:

$$\omega_L \approx \frac{1}{\sum_{i=1}^{n} R_{iS} C_i}$$

- $R_{iS}$ is resistance at terminals of $i$th capacitor $C_i$

- All other capacitors replaced by short circuits.

- Product $R_{iS} C_i$ is short-circuit time constant associated with $C_i$.

Direct analysis is precise, but tedious

Midband gain, $f_L, f_H$ are of more interest than complete transfer function.
Estimate of $\omega_L$ for C-E Amplifier

Using SCTC method, for $C_1$,

$$R_{1S} = R_I + (R_B \| R_{CE}) = R_2 + (R_B \| r_\pi)$$

For $C_2$,

$$R_{2S} = R_4 \| R_iE = R_4 \left\| \frac{r_\pi + R}{\beta_o + 1} \right\|$$

$$= R_4 \left\| \frac{r_\pi + (R_L \| R_B)}{\beta_o + 1} \right\|$$

For $C_3$,

$$R_{3S} = R_3 + (R_C \| R_{iC}) = R_3 + (R_C \| r_o)$$

$$= R_3 + R_C$$

$$\omega_L \equiv \sum_{i=1}^{3} \frac{1}{R_i S C_i} \quad f_L = 735 Hz$$
High-Frequency Response

- HF response may be dominated by $R_{\text{OUT}}$ and $C_{\text{Load}}$
- Often more complex
- Sometimes need to consider parasitic capacitances
  - Wiring
  - Devices
Frequency-dependent Hybrid-Pi Model for BJT

Collector-base capacitance is voltage dependent:

\[ C_\mu = \frac{C_{\mu o}}{\sqrt{1 + \left(\frac{V_{CB}}{\Phi_{jc}}\right)}} \]

where:
- \( C_{\mu o} \) is total collector-base junction capacitance at zero bias,
- \( \Phi_{jc} \) is its built-in potential.

Base-emitter capacitance, parallel with \( r_\pi \):

\[ C_\pi = g_m \tau_F \]

- \( \tau_F \) is forward transit-time of the BJT.
- At higher frequency, \( C_\pi \) reduces \( v_{be} \) for given input signal current; reduces current gain.
Unity-gain Frequency of BJT

\[ \omega_\beta = \frac{1}{r_\pi (C_\mu + C_\pi)} \] is the beta-cutoff frequency

where

\[ \beta(s) = \frac{\beta_o}{1 + \frac{s(C_\pi + C_\mu)}{r_\pi}} \]

\[ f_T = \frac{\omega_T}{2\pi} \] is the unity gain current bandwidth

\[ \omega_T = \frac{\beta_o \omega_\beta}{(C_\pi + C_\mu) r_\pi} = \frac{g_m}{C_\pi + C_\mu} \]

High-frequency right-half plane zero

\[ \omega_Z = \frac{g_m}{C_\mu} \] can be neglected.
High-frequency Model of MOSFET

\[ i_D(s) = (g_m - sC_{GD})v_{GS} \]
\[ = i_G(s)\frac{(g_m - sC_{GD})v_{GS}}{s(C_{GS} + C_{GD})} \]
\[ \therefore \beta(s) = \frac{i_D(s)}{i_G(s)} \]
\[ = \frac{\omega_T(s)}{s} \left(1 - \frac{s}{\omega_T(1 + (C_{GS}/C_{GD}))}\right) \]

\[ \omega_T = \frac{g_m}{C_{GS} + C_{GD}} \]
Limitations of High-frequency Models

- Above $0.3 f_T$, behavior of simple pi-models begins to deviate significantly from the actual device.
- $\omega_T$ depends on operating current as shown and is not constant as assumed.
- For given BJT, a collector current $I_{CM}$ exists that yield $f_{T_{\text{max}}}$.
- For FET in saturation, $C_{GS}$ and $C_{GD}$ are independent of Q-point current, so
  \[ \omega_T \propto g_m \propto \sqrt{I_D} \]
  - Efficiency/speed tradeoff
Effect of Base Resistance on Midband Amplifiers

Base material is resistive.

$r_x$ models voltage drop between base contact and active area of the BJT.

To account for base resistance $r_x$ is absorbed into equivalent pi model and can be used to transform expressions for C-E, C-C and C-B amplifiers.

\[ i = g_m v = g_m \frac{r_\pi}{r_\pi + r_x} v_{be} = g_m' v_{be} \]

\[ g_m' = g_m \frac{r_\pi}{r_\pi + r_x} = \frac{\beta_o}{r_\pi + r_x} \]

\[ r_\pi' = r_\pi + r_x \]

\[ \beta_o' = \beta_o \]
Miller Effect

We desire to replace $C_{xy}$ with $C_{eq}$ to ground. Starting with the definition of small-signal capacitance:

$$C = \frac{\Delta Q}{\Delta V}$$

Now write an expression for the change in charge for $C_{xy}$:

$$\Delta Q = C_{phy} (\Delta V_x - \Delta V_y) = C_{xy} (\Delta V_x - A_{xy} \Delta V_x) = C_{xy} \Delta V_x (1 - A_{xy})$$

We can now find and equivalent capacitance, $C_{eq}$:

$$C_{eq} = \frac{C_{xy} \Delta V_x (1 - A_{xy})}{\Delta V_x} = C_{xy} (1 - A_{xy})$$
Miller Effect - Alternate View

- Break cap into two series caps
  - size determined by Miller multiplier
- $v_x = v_i \frac{C_1}{(C_1+C_2)} + v_o \frac{C_2}{(C_1+C_2)}$
- $v_x = 0$ if
  - $C_1 = C_{GD}(1-A_V)$
  - $C_2 = C_{GD}(1-1/A_V)$
C-E Amplifier High Frequency Response using Miller Effect

First, find the simplified small-signal model of the C-E amp.

$C_1$, $C_2$, $C_3$ shorted. Focusing on mid-band and high frequency
C-E Amplifier High Frequency Response using Miller Effect (cont.)

Input gain is found as

\[ A_i = \frac{v_b}{v_i} = \frac{R_{in}}{R_i + R_{in}} \cdot \frac{r_{\pi}}{r_x + r_{\pi}} \]

\[ = \frac{R_1 \parallel R_2 \parallel (r_x + r_{\pi})}{R_i + R_1 \parallel R_2 \parallel (r_x + r_{\pi})} \cdot \frac{r_{\pi}}{r_x + r_{\pi}} \]

Terminal gain is

\[ A_{bc} = \frac{v_c}{v_b} = -g_m r_o \parallel R_C \parallel R_3 \equiv -g_m R_C \parallel R_3 \equiv -g_m R_L \]

Using the Miller effect, we find the equivalent capacitance at the base as:

\[ C_{eqB} = C_\mu (1 - A_{bc}) + C_\pi (1 - A_{be}) \]

\[ = C_\mu (1 - [ -g_m R_L ]) + C_\pi (1 - 0) \]

\[ = C_\mu (1 + g_m R_L ) + C_\pi \]
C-E Amplifier High Frequency Response using Miller Effect (cont.)

The total equivalent resistance at the base is

\[ R_{eqB} = (R_{th} + r_x) \parallel R_{inB} \]
\[ = (R_i \parallel R_1 \parallel R_2 + r_x) \parallel r_\pi = r_\pi 0 \]

The total capacitance and resistance at the collector is

\[ C_{eqC} = C_\mu + C_L \]
\[ R_{eqC} = r_o \parallel R_C \parallel R_3 \equiv R_C \parallel R_3 \equiv R_L \]

Because of interaction through \( C_\mu \), the two RC time constants interact, giving rise to a dominant pole

\[ \omega_{p1} = \frac{1}{r_\pi 0 [C_\mu (1 + g_m R_L) + C_\pi] + R_L [C_\mu + C_L]} \]
\[ C_T = [C_\mu (1 + g_m R_L) + C_\pi] + \frac{R_L}{r_\pi 0} [C_\mu + C_L] \]
\[ \omega_{p1} = \frac{1}{r_\pi 0 C_T} = \frac{1}{r_\pi 0 ([C_\mu (1 + g_m R_L) + C_\pi] + \frac{R_L}{r_\pi 0} [C_\mu + C_L])} \]
The small-signal model can be simplified by using Norton source transformation.

\[ R_L = R_3 \parallel R_C = 100 \, \text{k}\Omega \parallel 4.3 \, \text{k}\Omega \quad R_B = R_1 \parallel R_2 = 30 \, \text{k}\Omega \parallel 10 \, \text{k}\Omega \]

\[ v_{th} = v_i \frac{R_B}{R_I + R_B} \quad R_{th} = \frac{R_I R_B}{R_I + R_B} \]

\[ i_s = \frac{v_{th}}{R_{th} + r_x} \quad r_{\pi o} = r_\pi (R_{th} + r_x) \]
Direct High-Frequency Analysis: C-E Amplifier (Pole Determination)

From nodal equations and Cramer’s rule

\[ V_C(s) = I_s(s) \frac{(sC_\mu - g_m)}{\Delta} \]

\[ \Delta = s^2 \left( C_\pi \left( C_\mu + C_L \right) + C_\pi C_L \right) \]

\[ + s \left( C_\pi g_L + C_\mu g_L + g_\mu + g_\pi \right) + g_L g_\pi o \]

\[ f_H(s) \] given by 2 poles, one finite zero and one zero at infinity.

For a polynomial \( s^2 + sA_1 + A_0 \) with roots \( a \) and \( b \), \( a = A_1 \) and \( b = A_0 / A_1 \).

\[ C_T = C_\pi + C_\mu \left( 1 + g_m R_L + \frac{R_L}{r_{\pi o}} \right) + C_L \frac{R_L}{r_{\pi o}} \]

\[ \omega_{P1} \equiv \frac{A_0}{A_1} \frac{1}{r_{\pi o} C_T} \]

\[ \omega_{P2} \equiv \frac{g_m}{C_\pi \left( 1 + (C_L / C_\mu) \right) + C_L} \equiv \frac{g_m}{C_\pi + C_L} \]

First pole (smallest root) limits frequency response and determines \( \omega_H \).

Second pole is important for phase response in feedback amplifiers.
Direct High-Frequency Analysis: C-E Amplifier (Pole Determination)

From nodal equations and Cramer’s rule

\[ V_C(s) = I_S(s) \frac{(sC_\mu - g_m)}{\Delta} \]
\[ \Delta = s^2 \left( C_\pi \left( C_\mu + C_L \right) + C_\pi C_L \right) \]
\[ + s \left( C_\pi g_L + C_\mu \left( g_L + g_\mu + g_\pi \right) + C_L g_\pi_0 \right) + g_L g_\pi_0 \]

\( f_H(s) \) given by 2 poles, one finite zero and one zero at infinity.

For a polynomial \( s^2 + sA_1 + A_0 \) with roots \( a \) and \( b \), \( a = A_1 \) and \( b = A_0/A_1 \).

\[ C_T = C_\pi + C_\mu \left( 1 + g_m R_L + \frac{R_L}{r_\pi_0} \right) + C_L \frac{R_L}{r_\pi_0} \]

\[ \omega_{P1} = \frac{A_0}{A_1} \frac{1}{r_\pi_0 C_T} \]

\[ \omega_{P2} = \frac{g_m}{C_\pi \left( 1 + (C_L / C_\mu) \right) + C_L} \frac{C_L}{C_\pi + C_L} \]

First pole (smallest root) limits frequency response and determines \( \omega_H \).
Second pole is important for phase response in feedback amplifiers.
Direct High-Frequency Analysis: C-E Amplifier (Overall Transfer Function)

\[ V_o(s) = \frac{V_{th}(s)}{R_{th} + r_x \left( g_L g_{\pi \omega} \left[ 1 + \frac{s}{\omega P_1} \right] \right) \left[ 1 + \frac{s}{\omega P_2} \right]} \]

\[ V_o(s) = \frac{V_{th}(s)}{R_{th} + r_x \left( -g_m R_L r_{\pi \omega} \right)} \left( \frac{1}{1 + \frac{s}{\omega Z}} \right) \]

\[ \therefore V_o(s) \approx \frac{V_{th}(s) \left( g_m R_L r_{\pi \omega} \right)}{R_{th} + r_x \left( 1 + \frac{s}{\omega P_1} \right)} \]

\[ A_{vth}(s) = \frac{V_o(s)}{V_{th}(s)} \approx \frac{A_{mid}}{1 + \frac{s}{\omega P_1}} \]

\[ A_{mid} = \frac{\beta o R_L}{R_{th} + r_x + r_{\pi}} \quad \omega_{P_1} = \frac{1}{r_{\pi \omega} C_T} \]

Dominant pole model at high frequencies for C-E amplifier is as shown.
Direct High-Frequency Analysis: C-E Amplifier (Example)

- **Problem:** Find midband gain, poles, zeros and $f_L$.
- **Given data:** Q-point $= (1.60 \text{ mA}, 3.00 \text{V})$, $f_T = 500 \text{ MHz}$, $\beta_o = 100$, $C_\mu = 0.5 \text{ pF}$, $r_x = 250 \Omega$, $C_L = 0$
- **Analysis:**
  \[
g_m = 40I_c = 40(0.0016) = 64 \text{ mS}, \quad r_\pi = \frac{\beta_o}{g_m} = 1.56 \text{ k}\Omega.
  \]
  \[
f_{p1} = \frac{1}{2\pi r_\pi C_T} = 1.56 \text{MHz}
  \]
  \[
f_{p2} = \frac{g_m}{2\pi(C_\pi + C_L)} = 512 \text{ MHz}
  \]
  \[
f_Z = \frac{g_m}{2\pi C_\mu} = 20.4 \text{GHz}
  \]
  \[
A_{vth} = A_i A_{bc} = 0.512(-264) = -135
  \]

\[
R_L = R_3 \parallel R_C = 100\text{k}\Omega \parallel 4.3\text{k}\Omega = 4.12\text{k}\Omega
\]

\[
R_{th} = R_B \parallel R_I = 7.5\text{k}\Omega \parallel 1\text{k}\Omega = 882\Omega
\]

\[
r_\pi o = r_\pi (R_{th} + r_x) = 656\Omega
\]

\[
C_T = C_\pi + C_\mu \left(1 + g_m R_L\right) + \frac{R_L}{r_\pi o} (C_\mu + C_L) = 156 \text{pF}
\]
High Frequency Poles for the C-B Amplifier

\[
A_i \equiv \frac{1}{1 + g_m R_i}
\]

\[
A_{ec} = \frac{v_c}{v_e} = g_m R_{iC} \parallel R_L \equiv g_m R_L
\]

\[
R_{iC} = r_o (1 + g_m r_\pi \parallel R_L)
\]

Since \(C_\mu\) does not couple input and output, input and output poles can be found directly.

\[
C_{eqE} = C_\pi
\]

\[
R_{eqE} = \left(\frac{1}{g_m} \parallel R_E \parallel R_i\right) \approx \frac{g_m}{C_\pi}
\]

\[
\omega_{p1} = \frac{1}{(\frac{1}{g_m} \parallel R_E \parallel R_i)C_\pi} \approx \frac{g_m}{C_\pi}
\]

\[
C_{eqC} = C_\mu + C_L
\]

\[
R_{eqC} = R_{iC} \parallel R_L \approx R_L
\]

\[
\omega_{p2} = \frac{1}{(R_{iC} \parallel R_L)(C_\mu + C_L)} \approx \frac{1}{R_L (C_\mu + C_L)}
\]
Spice Simulation of Example C-E Amplifier
Estimation of $\omega_H$ using the Open-Circuit Time Constant Method

At high frequencies, impedances of coupling and bypass capacitors are small enough to be considered short circuits. Open-circuit time constants associated with impedances of device capacitances are considered instead.

$$\omega_H \equiv \frac{1}{m \sum_{i=1}^{m} R_{io} C_i}$$

where $R_{io}$ is resistance at terminals of $i$th capacitor $C_i$ with all other capacitors open-circuited.

For a C-E amplifier, assuming $C_L = 0$

$$R_{\pi o} = r_{\pi o}$$

$$\omega_H \equiv \frac{1}{R_{\pi o} C_\pi + R_{\mu o} C_\mu} = \frac{1}{r_{\pi o} C_T}$$
High-Frequency Analysis: C-S Amplifier

Analysis similar to the C-E case yields the following equations:

\[ R_{th} = R_I \parallel R_G \]

\[ R_L = R_D \parallel R_3 \]

\[ v_{th} = \frac{v_i R_G}{R_I + R_G} \]

\[ C_T = C_{GS} + C_{GD} \left(1 + g_m R_L\right) + \frac{R_L}{R_{th}} (C_{GD} + C_L) \]

\[ \omega_{P1} = \frac{1}{R_{th} C_T} \]

\[ \omega_{P2} = \frac{g_m}{C_{GS} + C_L} \]

\[ \omega_2 = \frac{g_m}{C_{GD}} \]
C-S Amplifier High Frequency Response with Source Degeneration Resistance

First, find the simplified small-signal model of the C-A amp.

Recall that we can define an effective $g_m$ to account for the unbypassed source resistance.

$$g_m' = \frac{g_m}{1 + g_m R_S}$$
Input gain is found as

\[ A_i = \frac{v_g}{v_i} = \frac{R_G}{R_i + R_G} = \frac{R_1 \parallel R_2}{R_i + R_1 \parallel R_2} \]

Terminal gain is

\[ A_{gd} = \frac{v_d}{v_g} = -g_m' (R_{ID} \parallel R_D \parallel R_3) \approx \frac{-g_m R_D \parallel R_3}{1 + g_m R_S} \]

Again, we use the Miller effect to find the equivalent capacitance at the gate as:

\[ C_{eqG} = C_{GD} (1 - A_{gd}) + C_{GS} (1 - A_{gs}) \]

\[ = C_{GD} (1 - \frac{[-g_m R_L]}{1 + g_m R_S}) + C_{GS} (1 - \frac{g_m R_S}{1 + g_m R_S}) \]

\[ = C_{GD} (1 + \frac{g_m R_D \parallel R_L}{1 + g_m R_S}) + \frac{C_{GS}}{1 + g_m R_3} \]
C-S Amplifier High Frequency Response with Source Degeneration Resistance (cont.)

The total equivalent resistance at the gate is

\[ R_{eqG} = R_G \parallel R_I = R_{th} \]

The total capacitance and resistance at the collector is

\[ C_{eqD} = C_{GD} + C_L \]
\[ R_{eqD} = R_{iD} \parallel R_D \parallel R_3 \equiv R_D \parallel R_3 = R_L \]

Because of interaction through \( C_{GD} \), the two RC time constants interact, giving rise to the dominant pole:

\[ \omega_p = \frac{1}{R_{th} \left[ C_{GD} \left( 1 + \frac{g_m R_L}{1 + g_m R_S} \right) + \frac{C_{GS}}{1 + g_m R_S} + \frac{R_L}{R_{th}} \left( C_{GD} + C_L \right) \right]} \]

And from previous analysis:

\[ \omega_{p2} = \frac{g_m'}{(C_{GS} + C_L)} = \frac{g_m}{(1 + g_m R_S)(C_{GS} + C_L)} \]
\[ \omega_z = \frac{+g_m'}{C_{GD}} = \frac{+g_m}{(1 + g_m R_s)(C_{GD})} \]
C-E Amplifier with Emitter Degeneration Resistance

Analysis similar to the C-S case yields the following equations:

\[ r_{\pi 0} = R_{eqB} = (R_{th} + r_x) \parallel [r_\pi + (\beta + 1)R_E] \]

\[ R_L = R_C \parallel R_3 \]

\[ \omega_{p1} = \frac{1}{r_{\pi 0}C_T} \]

\[ = \frac{1}{r_{\pi 0}([C_\mu (1 + \frac{g_mR_L}{1 + g_mR_E}) + \frac{C_\pi}{1 + g_mR_E}] + \frac{R_L}{r_{\pi 0}}[C_\mu + C_L])} \]

\[ \omega_{p2} \approx \frac{g_m}{2\pi(1 + g_mR_E)(C_\pi + C_L)} \]

\[ \omega_z = \frac{+g_m}{2\pi[1 + g_mR_E][C_\mu]} \]
Gain-Bandwidth Trade-offs Using Source/Emitter Degeneration Resistor

Adding source resistance to the CS amp caused gain to decrease and dominant pole frequency to increase.

\[ A_{gd} = \frac{v_d}{v_g} = \frac{-g_m R_D \parallel R_3}{1 + g_m R_S} \]

\[ \omega_{p1} = \frac{1}{R_{th}[C_{GD}(1 + \frac{g_m R_L}{1 + g_m R_S}) + \frac{C_{GS}}{1 + g_m R_S} + \frac{R_L}{R_{th}}(C_{GD} + C_L)]} \]

\[ \omega_{p2} = \frac{g_m}{(1 + g_m R_S)(C_{GS} + C_L)} \]

However, decreasing the gain also decreased the frequency of the second pole.

Increasing the gain of the C-E/C-S stage causes pole-splitting, or increase of the difference in frequency between the first and second poles.
High Frequency Poles for the C-G Amplifier

Similar to the C-B, since $C_{GD}$ does not couple the input and output, input and output poles can be found directly.

\[
C_{eqS} = C_{GS}
\]
\[
R_{eqS} = \frac{1}{g_m} \parallel R_4 \parallel R_I
\]
\[
\omega_{p1} = \frac{1}{\left(\frac{1}{g_m} \parallel R_4 \parallel R_I\right)C_{GD}} \approx \frac{g_m}{C_{GD}}
\]

\[
C_{eqD} = C_{GD} + C_L
\]
\[
R_{eqD} = R_{iD} \parallel R_L \approx R_L
\]
\[
\omega_{p2} = \frac{1}{(R_{iD} \parallel R_L)(C_{GD} + C_L)} \approx \frac{1}{R_L(C_{GD} + C_L)}
\]
High Frequency Poles for the C-C Amplifier

\[ A_i = \frac{v_b}{v_i} = \frac{R_{in}}{R_i + R_{in}} \]

\[ A_{be} = \frac{v_e}{v_b} = \frac{g_m R_L}{1 + g_m R_L} \]

\[ C_{eqB} = C_\mu (1 - A_{bc}) + C_\pi (1 - A_{be}) = C_\mu (1 - 0) + C_\pi (1 - \frac{g_m R_L}{1 + g_m R_L}) \]

\[ = C_\mu + \frac{C_\pi}{1 + g_m R_L} \]

\[ R_{eqB} = R_i \parallel R_{in} = [(R_i \parallel R_B) + r_x] \parallel [r_\pi + (\beta + 1)R_L] = (R_{th} + r_x) \parallel [r_\pi + (\beta + 1)R_L] \]

\[ C_{eqE} = C_\pi + C_L \]

\[ R_{eqE} = R_{iE} \parallel R_L \approx \left[ \frac{1}{g_m} + \frac{(R_{th} + r_x)}{\beta + 1} \right] \parallel R_L \]
The low impedance at the output makes the input and output time constants relatively well decoupled, leading to two poles.

\[ \omega_{p1} = \frac{1}{\left([R_{th} + r_x] \parallel [r_\pi + (\beta + 1)R_L]\right)(C_\mu + \frac{C_\pi}{1 + g_m R_L})} \]

\[ \omega_{p2} = \frac{1}{\left[R_{iE} \parallel R_L\right][C_\pi + C_L]} \approx \frac{1}{\left[\frac{1}{g_m} + \frac{R_{th} + r_x}{\beta + 1}\right] \parallel R_L\left[C_\pi + C_L\right]} \]

The feed-forward high-frequency path through \( C_p \) leads to a zero in the C-C response. Both the zero and the second pole are quite high frequency and are often neglected, although their effect can be significant with large load capacitances.

\[ \omega_z \approx \frac{g_m}{C_\pi} \]
Similar to the C-C amplifier, the high frequency response is dominated by the first pole due to the low impedance at the output of the C-C amplifier.

\[
\omega_{p1} = \frac{1}{R_{th} \left(C_{GD} + \frac{C_{GS}}{1 + g_m R_L}\right)}
\]

\[
\omega_{p2} = \frac{1}{\left[R_{is} \parallel R_L\right] \left[C_{GS} + C_L\right]} \approx \frac{1}{\left[1/g_m \parallel R_L\right] \left[C_{GS} + C_L\right]}
\]

\[
\omega_z \approx \frac{g_m}{C_{GS}}
\]
<table>
<thead>
<tr>
<th>Amplifier Type</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common-emitter</td>
<td>[ \frac{1}{\omega_0 r_{\pi 0} C_T} = \frac{1}{r_{\pi 0} \left[ C_\pi + C_\mu (1 + g_m R_L) + \frac{R_L}{r_{\pi 0}} \right]} ]</td>
</tr>
<tr>
<td>Common-source</td>
<td>[ \frac{1}{R_m C_T} = \frac{1}{R_h \left[ C_{GS} + C_{GD} \left( 1 + g_m R_L \right) \left( 1 + \frac{R_L}{R_h} \right) \right]} ]</td>
</tr>
<tr>
<td>Common-emitter with emitter resistor</td>
<td>[ r_{\pi 0} = r_\pi \left( 1 + g_m R_L \right) ]</td>
</tr>
<tr>
<td>Common-source with source resistor</td>
<td>[ r_{\pi 0} = r_\pi \left( 1 + g_m R_L \right) ]</td>
</tr>
<tr>
<td>Common-base</td>
<td>[ \frac{1}{R_L \left( C_\mu + C_L \right)} ]</td>
</tr>
<tr>
<td>Common-gate</td>
<td>[ \frac{1}{R_L \left( C_{GD} + C_L \right)} ]</td>
</tr>
<tr>
<td>Common-collector</td>
<td>[ \frac{1}{\left( R_I \parallel R_B \right) \left( \frac{C_\pi}{1 + g_m R_L} + C_\mu \right)} ]</td>
</tr>
<tr>
<td>Common-drain</td>
<td>[ \frac{1}{\left( R_I \parallel R_G \right) \left( \frac{C_{GS}}{1 + g_m R_L} + C_{GD} \right)} ]</td>
</tr>
</tbody>
</table>
Frequency Response: Differential Amplifier

$C_{EE}$ is total capacitance at emitter node of the differential pair.

Differential mode half-circuit is similar to a C-E stage. Bandwidth is determined by the $r_{\pi o} C_T$ product. As emitter is a virtual ground, $C_{EE}$ has no effect on differential-mode signals.

For common-mode signals, at very low frequencies,

$$|A_{cc}(0)| \approx \frac{RC}{2R_{EE}} \ll 1$$

Transmission zero due to $C_{EE}$ is

$$s = -\omega Z = -\frac{1}{C_{EE} R_{EE}}$$
Common-mode half-circuit is similar to a C-E stage with emitter resistor $2R_{EE}$.
OCTC for $C_\pi$ and $C_\mu$ is similar to the C-E stage. OCTC for $C_{EE}/2$ is:

$$\omega_P \approx \left( \frac{C_\pi + C_{EE}}{2g_m} + C_\mu (R_C + r_x) \right) \approx \frac{1}{C_\mu (R_C + r_x)}$$

$$R_{EEO} = 2R_{EE} \left\{ \frac{r_\pi + r_x}{\beta_o + 1} - \frac{1}{g_m} \right\}$$

As $R_{EE}$ is usually designed to be large,
Frequency Response: Common-Collector/ Common-Base Cascade

$R_{EE}$ is assumed to be large and neglected.

$$R_{CC1}^{out} = \frac{r_{\pi 1} + r_{x1}}{\beta_{o1} + 1} \approx \frac{1}{g_{m1}}$$

$$R_{CB2}^{in} = \frac{r_{\pi 2} + r_{x2}}{\beta_{o2} + 1} \approx \frac{1}{g_{m2}}$$

The intermediate node pole is neglected since the impedance is quite low. We are left with the input pole for a C-D and the output pole of a C-B stage.

$$\omega_{pB1} = \frac{1}{([R_{th} + r_{x1}] \parallel r_{\pi 1} + (\beta + 1)R_{L1}))(C_{\mu 1} + \frac{C_{\pi 1}}{1 + g_{m}R_{L}})}$$

$$= \frac{1}{([R_{th} + r_{x1}] \parallel 2r_{\pi 1}))(C_{\mu 1} + \frac{C_{\pi 2}}{2})}$$

$$\omega_{pC2} \approx \frac{1}{R_{C}(C_{\mu} + C_{L})}$$
Frequency Response: Cascode Amplifier

There are two important poles, the input pole for the C-E and the output pole for the C-B stage. The intermediate node pole can usually be neglected because of the low impedance at the input of the C-B stage. \( R_{L1} \) is small, so the second term in the first pole can be neglected. Also note the \( R_{L1} \) is equal to \( 1/g_{m2} \).

\[
\omega_{pB1} = \frac{1}{r_{\pi 0} C_T} = \frac{1}{r_{\pi 01} \left( [C_{\mu_1} (1 + \frac{g_{m1} R_{L1}}{1 + g_{m1} R_{E1}} + C_{\pi 1}) + \frac{R_{L1}}{r_{\pi 0}} [C_{\mu_1} + C_{L1}] \right)}
\]

\[
\approx \frac{1}{r_{\pi 01} \left( 2C_{\mu_1} + C_{\pi 1} \right)}
\]

\[
\omega_{pC2} \approx \frac{1}{R_L (C_{\mu_2} + C_L)}
\]
Frequency Response: MOS Current Mirror

This is very similar to the C-S stage simplified model, so we will apply the C-S equations with relevant changes.

\[ \omega P_1 = \frac{1}{(1/g_{m1})C_T} \]

\[ = \frac{1/g_{m1}(C_{GS1} + C_{GS2} + C_{GD2}(1 + g_{m2}r_{o2}) + \frac{r_{o2}}{1/g_{m1}}C_{GD2})}{2C_{GS1} + 2C_{GD2}r_{o2}} \]

Assumes matched transistors.
Frequency Response: Multistage Amplifier

**Problem:** Use open-circuit and short-circuit time constant methods to estimate upper and lower cutoff frequencies and bandwidth.

**Approach:** Coupling and bypass capacitors determine low-frequency response, device capacitances affect high-frequency response.

At high frequencies, ac model for multi-stage amplifier is as shown.
Frequency Response: Multistage Amplifier Parameters

Parameters and operation point information for the example multistage amplifier.

<table>
<thead>
<tr>
<th>Transistor Parameters</th>
<th>$g_m$</th>
<th>$r_\pi$</th>
<th>$r_o$</th>
<th>$\beta_o$</th>
<th>$C_{GS}/C_{\pi}$</th>
<th>$C_{GD}/C_{\mu}$</th>
<th>$r_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>10 mS</td>
<td>$\infty$</td>
<td>12.2 kΩ</td>
<td>$\infty$</td>
<td>5 pF</td>
<td>1 pF</td>
<td>0 Ω</td>
</tr>
<tr>
<td>$Q_2$</td>
<td>67.8 mS</td>
<td>2.39 kΩ</td>
<td>54.2 kΩ</td>
<td>150</td>
<td>39 pF</td>
<td>1 pF</td>
<td>250 Ω</td>
</tr>
<tr>
<td>$Q_3$</td>
<td>79.6 mS</td>
<td>1.00 kΩ</td>
<td>34.4 kΩ</td>
<td>80</td>
<td>50 pF</td>
<td>1 pF</td>
<td>250 Ω</td>
</tr>
</tbody>
</table>
Frequency Response: Multistage Amplifier (SCTC Estimate of $\omega_L$)

SCTC for each of the six independent coupling and bypass capacitors are calculated as follows:

- $R_{1S} = R_I + (R_G \| R_{in1}) = 10k\Omega + 1M\Omega \| \infty = 1.01M\Omega$

- $R_{2S} = R_{S1} \left| \frac{1}{g_{m1}} \right| \frac{1}{0.01S} = 66.7 \Omega$

- $R_{3S} = (R_{D1} \| R_{O1}) + (R_{B2} \| R_{in2})$
  $= (R_{D1} \| r_{o1}) + (R_{B2} \| r_{\pi2}) = 2.69k\Omega$

- $R_{th2} = R_{B2} \| R_{D1} \| r_{o1} = 571 \Omega$

- $R_{4S} = R_{E2} \left| \frac{R_{th2} + r_{\pi2}}{\beta_{o2} + 1} \right| = 19.4 \Omega$

- $R_{5S} = (R_{C2} \| R_{O2}) + (R_{B3} \| R_{in3})$
  $= (R_{C2} \| r_{o2}) + (R_{B3} \| r_{\pi3} + (\beta_{o3} + 1)(R_{E3} \| R_L))$
  $= 18.4k\Omega$

- $R_{th3} = R_{B3} \| R_{C2} \| r_{o2} = 3.99k\Omega$

- $R_{6S} = R_L + R_{E3} \left| \frac{R_{th3} + r_{\pi3}}{\beta_{o3} + 1} \right| = 311 \Omega$

- $\omega_L \equiv \frac{1}{\sum_{i=1}^{n} \frac{1}{R_{iS} C_i}} = 3330 \text{rad/s}$

- $f_L = \frac{\omega_L}{2\pi} = 530\text{Hz}$
Frequency Response: Multistage Amplifier (High-Frequency Poles)

High-frequency pole at the gate of M1: Using our equation for the C-S input pole:

\[
f_{p1} = \frac{1}{2\pi R_{th1}[C_{GD1}(1 + g_{m1} R_{L1}) + C_{GS1} + \frac{R_{L1}}{R_{th1}}(C_{GD1} + C_{L1})} \]

\[R_{L1} = R_{I12} \parallel r_{\pi2} \parallel r_{o1} = 598\Omega \parallel (2.39k\Omega + 250\Omega) \parallel 12.2k\Omega = 469\ \Omega\]

\[C_{L1} = C_{\pi2} + C_{\mu2}(1 + g_{m2} R_{L2})\]

\[R_{L2} = R_{I23} \parallel R_{in3} \parallel r_{o2} = R_{I23} \parallel [r_{x3} + r_{\pi3} + (\beta_{03} + 1)(R_{E3} \parallel R_{L})] \parallel r_{o2} = 3.33k\Omega\]

\[C_{L1} = 39pF + 1pF[1 + 67.8mS(3.33k\Omega)] = 266pF\]

\[
f_{p1} = \frac{1}{2\pi 9.9k\Omega[1pF(1 + 0.01S(469\Omega)] + 5pF + \frac{469\Omega}{9.9k\Omega}(1pF + 266pF)]} = 689\ \text{KHz}\]
High-frequency pole at the base of Q2: From the detailed analysis of the C-S amp, we find the following expression for the pole at the output of the M1 C-S stage:

\[ f_{p2} = \frac{C_{GS1}g_{L1} + C_{GD1}(g_{m1} + g_{th1} + g_{L1}) + C_{L1}g_{th1}}{2\pi[C_{GS1}(C_{GD1} + C_{L1}) + C_{GD1}C_{L1}]} \]

For this particular case, \( C_{L1} \) (Q2 input capacitance) is much larger than the other capacitances, so \( f_{p2} \) simplifies to:

\[ f_{p2} \approx \frac{C_{L1}g_{th1}}{2\pi[C_{GS1}C_{L1} + C_{GD1}C_{L1}]} \approx \frac{1}{2\pi R_{th1}(C_{GS1} + C_{GD1})} \]

\[ f_{p2} = \frac{1}{2\pi 9.9k\Omega(5pF+1pF)} = 2.68 \text{ MHz} \]
High-frequency pole at the base of Q3: Again, due to the pole-splitting behavior of the C-E second stage, we expect that the pole at the base of Q3 will be set by equation 16.95:

\[
f_{p3} \cong \frac{g_{m2}}{2\pi [C_{\pi 2} (1 + \frac{C_{L2}}{C_{\mu 2}}) + C_{L2} ]}
\]

The load capacitance of Q2 is the input capacitance of the C-C stage.

\[
C_{L2} = C_{\mu 3} + \frac{C_{\pi 3}}{1 + g_{m3} R_{E3} || R_L} = 1\text{pF} + \frac{50\text{pF}}{1 + 79.6\text{mS}(3.3k\Omega || 250\Omega)} = 3.55\text{ pF}
\]

\[
f_{p3} \cong \frac{67.8\text{mS}[1k\Omega/(1k\Omega + 250\Omega)]}{2\pi[39\text{pF}(1 + \frac{3.55\text{pF}}{1\text{pF}}) + 3.55\text{pF}]} = 47.7\text{ MHz}
\]
Frequency Response: Multistage Amplifier \( (f_H \text{ estimate}) \)

There is an additional pole at the output of Q3, but it is expected to be at a very high frequency due to the low output impedance of the C-C stage. We can estimate \( f_H \) from eq. 16.23 using the calculated pole frequencies.

\[
f_H = \frac{1}{\sqrt{\frac{1}{f_{p1}^2} + \frac{1}{f_{p2}^2} + \frac{1}{f_{p3}^2}}} = 667 \text{ kHz}
\]

The SPICE simulation of the circuit on the next slide shows an \( f_H \) of 667 KHz and an \( f_L \) of 530 Hz. The phase and gain characteristics of our calculated high frequency response is quite close to that of the SPICE simulation. It was quite important to take into account the pole-splitting behavior of the C-S and C-E stages. Not doing so would have resulted in a calculated \( f_H \) of less than 550 KHz.
Frequency Response: Multistage Amplifier (SPICE Simulation)
Intro to RF Amplifiers

- Amplifiers with narrow bandwidth are often required in radio frequency (RF) applications to be able to select one signal from a large number of signals.
- Frequencies of interest > unity gain frequency of op amps, so active RC filters can’t be used.
- These amplifiers have high Q ($f_H$ and $f_L$ close together relative to center frequency)
- These applications use resonant RLC circuits to form frequency selective tuned amplifiers.
The Shunt-Peaked Amplifier

- As the frequency goes up, the gain is enhanced by the increasing impedance of the inductor.

\[
A_v(s) = \frac{(-gmR)(1+sL/R)}{1+sRC + s^2LC}
\]

where \( C = C_L + C_{GD} \)

- The gain improvement can be plotted as a function of parameter, \( m \), defined below:

\[
A_{vn}(s) = \frac{1+ms}{1+s+ms^2}
\]

where \( L = mR^2C \)
The Shunt-Peaked Amplifier

![Graph showing normalized voltage gain vs. normalized frequency for different peaking factors.]

- No peaking ($m = 0$)
- Maximally flat ($m = 0.41$)
- Maximum bandwidth ($m = 0.71$)
- BW reference ($A_{vn} = 0.707$)
Single-Tuned Amplifiers

- RLC network selects the frequency, parallel combination of $R_D$, $R_3$ and $r_o$ set the Q and bandwidth.

$$A_v(s) = \frac{V_o(s)}{V_i(s)} = \frac{sC_{GD} - g_m}{G_P + s(C + C_{GD}) + (1/sL)}$$

$$G_P = g_o + G_D + G_3$$

- Neglecting right-half plane zero,

$$A_v(s) = A_{mid} \frac{s^{\omega_0}}{Q} \left( \frac{1}{s^2 + s^{\omega_0} + \omega_0^2} \right)$$

$$\omega_0 = \frac{1}{\sqrt{L(C + C_{GD})}} \quad Q = \omega_0 R_P (C + C_{GD}) = \frac{R_P}{\omega_0 L}$$
Single-Tuned Amplifiers (contd.)

- At center frequency, \( s = j\omega_o \),
  \[ A_v = A_{mid}. \]
  \[ A_{mid} = -g_m R_P = -g_m \left( r_o \left| R_D \right| R_3 \right) \]
  \[ BW = \frac{\omega_o}{Q} = \frac{1}{R_P \left( C + C_{GD} \right)} = \frac{\omega_o^2 L}{R_P} \]
Use of tapped Inductor- Auto Transformer

$C_{GD}$ and $r_o$ can often be small enough to degrade characteristics of the tuned amplifier. Inductor can be made to work as an auto transformer to solve this problem.

$$\frac{V_o(s)}{I_2(s)} = \frac{nV_1(s)}{I_s(s)/n} = n^2 \frac{V_1(s)}{I_s(s)} \quad Z_s(s) = n^2 Z_p(s)$$

These results can be used to transform the resonant circuit and higher $Q$ can be obtained and center frequency doesn’t shift significantly due to changes in $C_{GD}$.

Similar solution can be used if tuned circuit is placed at amplifier input instead of output.
Multiple Tuned Circuits

\[ BW_n = BW_1 \sqrt{2^{1/n} - 1} \]
CS Amp with Inductive Degeneration

- Typically need to match input resistance to antenna impedance at center frequency, usually 50 ohms.
- Using our follower analyses, the input impedance is found as:

\[
Z_{in}(s) = Z_{gs} + Z_s + (g_m Z_{gs}) Z_s
\]

\[
Z_{in}(s) = \frac{1}{s C_{GS}} + s L_s + R_{eq}
\]

where \( R_{eq} = +g_m L_s / C_{GS} \)

- The following slide shows a complete low noise CS amp where a series inductor resonates with the input capacitance to leave only the resistance at the center frequency.
Complete Cascode LNA
Mixer Introduction

- A mixer is a circuit that multiplies two signals to produce sum and difference frequencies:

\[ S_0 = S_2 \cdot S_1 = \sin\omega_2 t \cdot \sin\omega_1 t \]

\[ = \frac{\cos(\omega_2 - \omega_1)t - \cos(\omega_2 - \omega_1)t}{2} \]

- A filter is usually used to reject either the sum or difference frequency to implement up-conversion or down-conversion.
Single-Balanced Mixer

- This basic mixer form is essentially a switched circuit that 'chops' the sine wave input with a square wave function

\[ v_1(t) = A \sin \omega_1 t \]

\[ s_s(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n \text{ odd}} \frac{1}{n \omega_2 t} \]
\[ v_o(t) = \frac{A}{2} \sin \omega_1 t + \frac{A}{\pi} \sum_{n \text{ odd}} \cos(n\omega_2 - \omega_1) t - \cos(n\omega_2 + \omega_1) t \]
Differential Pair as Single-Balanced Mixer

\[ v_o(t) = \sum_{n \text{ odd}} \frac{4}{n\pi} \left[ I_{EE} R C \sin n\omega_2 t + I_1 R C \frac{\cos(n\omega_2 - \omega_1)t - \cos(n\omega_2 + \omega_1)t}{2} \right] \]
Gilbert Multiplier as a Double-Balanced Mixer

- The Gilbert Multiplier is an extension of the differential single-balanced mixer.
- The input polarity is reversed on the second diff pair and the signal v1 selects between the two diff pairs.
- The currents are summed in the load resistors and the DC component is zero.
- Only sum and difference frequencies are present at the output.
$\nu(t) = V_m \frac{R}{R_1} C \sum_{n \text{ odd}} \frac{2}{n\pi} \left[ \cos(n\omega_c - \omega_m)t - \cos(n\omega_c + \omega_m)t \right] \quad \rho$