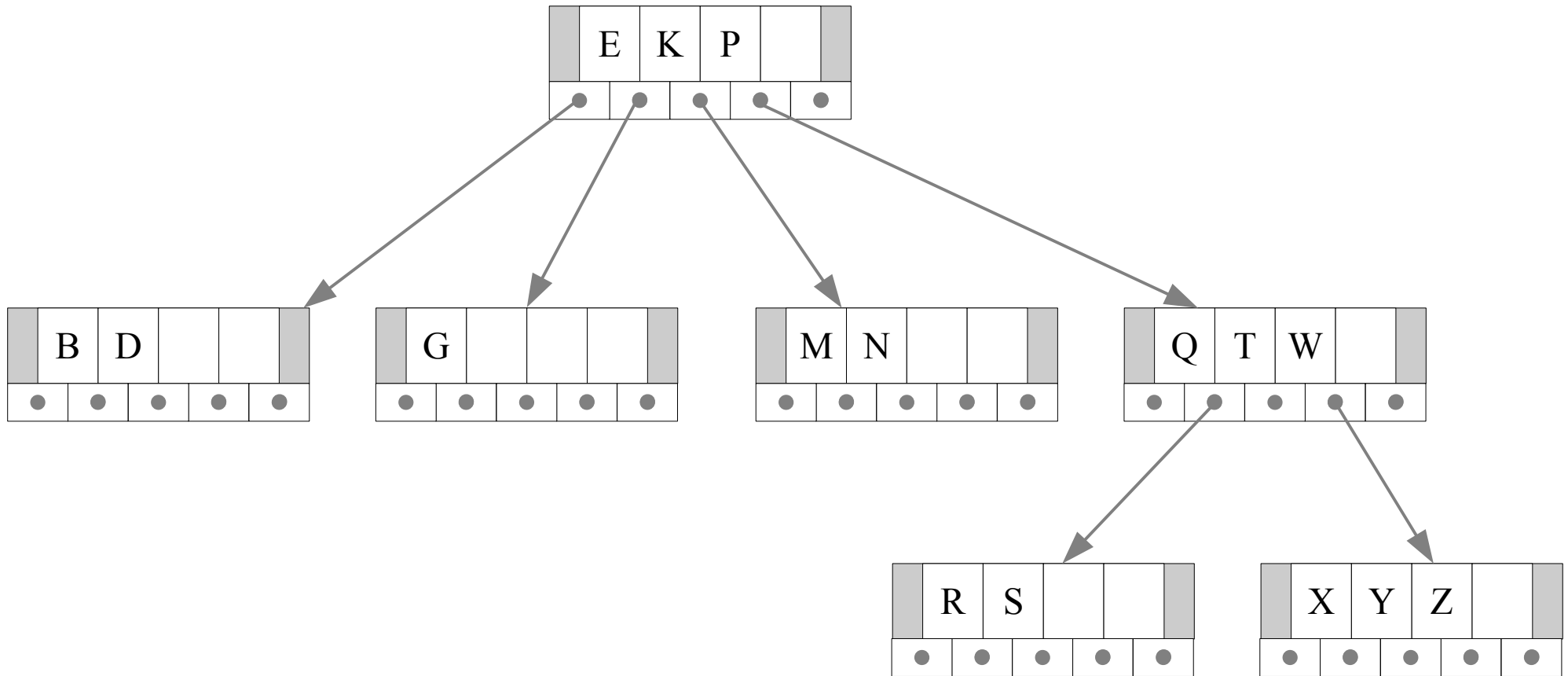


# Example multiway tree of order 5 (This is not a B-Tree, BTW)



# B-Tree Invariants

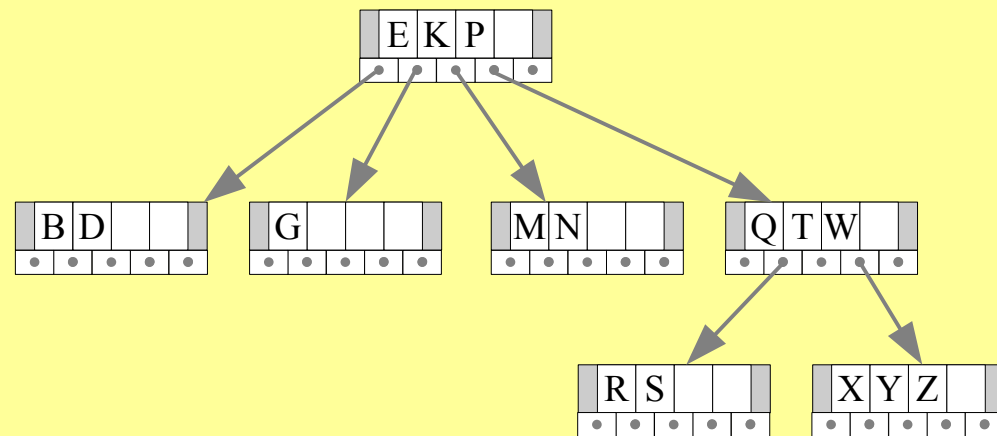
---

- Affix the order -  $m$
- All nodes, except the top, have  $\text{ceil}(m/2)-1$  keys.
- All external nodes are at the same level.

# B-Tree Invariants

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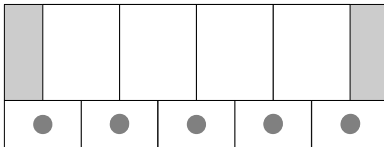
The tree from the first page is not a B-Tree.



# To Construct a B-Tree

---

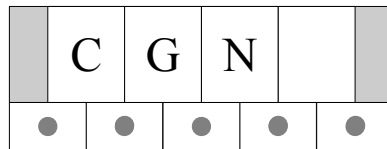
- Fill in the first node's keys, until it's full
- Example: Insert C, G and N into a tree of order 5:



# To Construct a B-Tree

---

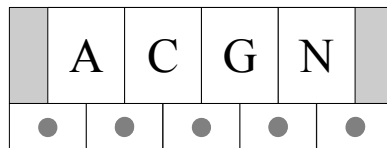
- Fill in the first node's keys, until it's full
- Example: Insert C, G and N into a tree of order 5:
- Next, insert A:



# To Construct a B-Tree

---

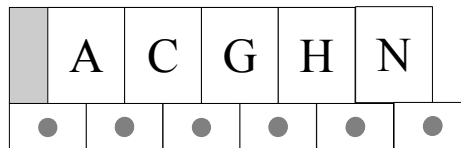
- Fill in the first node's keys, until it's full
- Example: Insert C, G and N into a tree of order 5:
- Next, insert A:
- Suppose we now insert H.



# To Construct a B-Tree

---

- Fill in the first node's keys, until it's full
- Example: Insert C, G and N into a tree of order 5:
- Next, insert A:
- Suppose we now insert H.



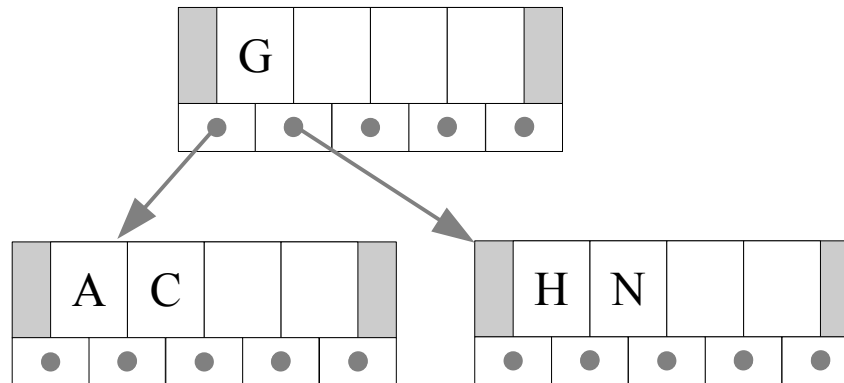
It's too big now,  
so we “split” it into  
three nodes:



# To Construct a B-Tree

---

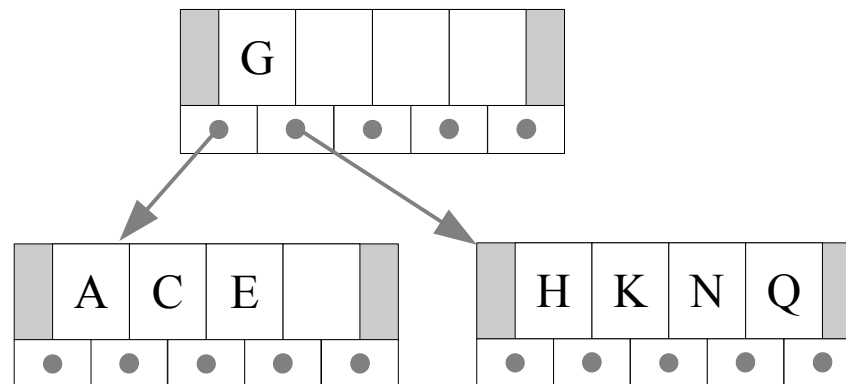
- Next, we'll insert E, K and Q – these all go into leaf nodes.





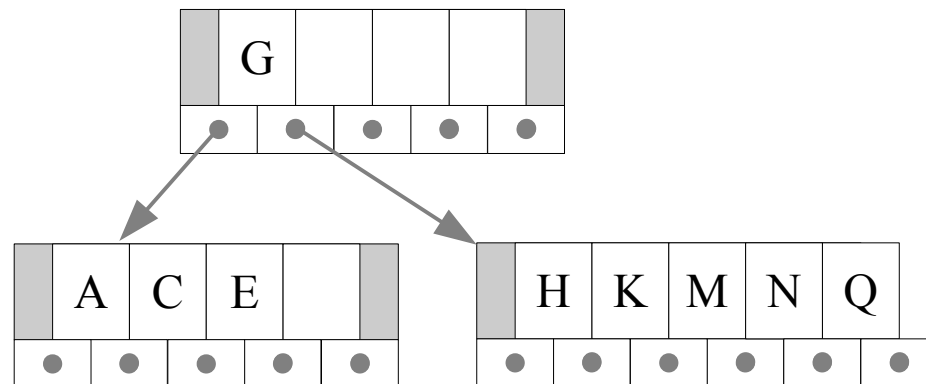
# To Construct a B-Tree

- Next, we'll insert E, K and Q – these all go into leaf nodes.
- Next, insert M – this requires a split:



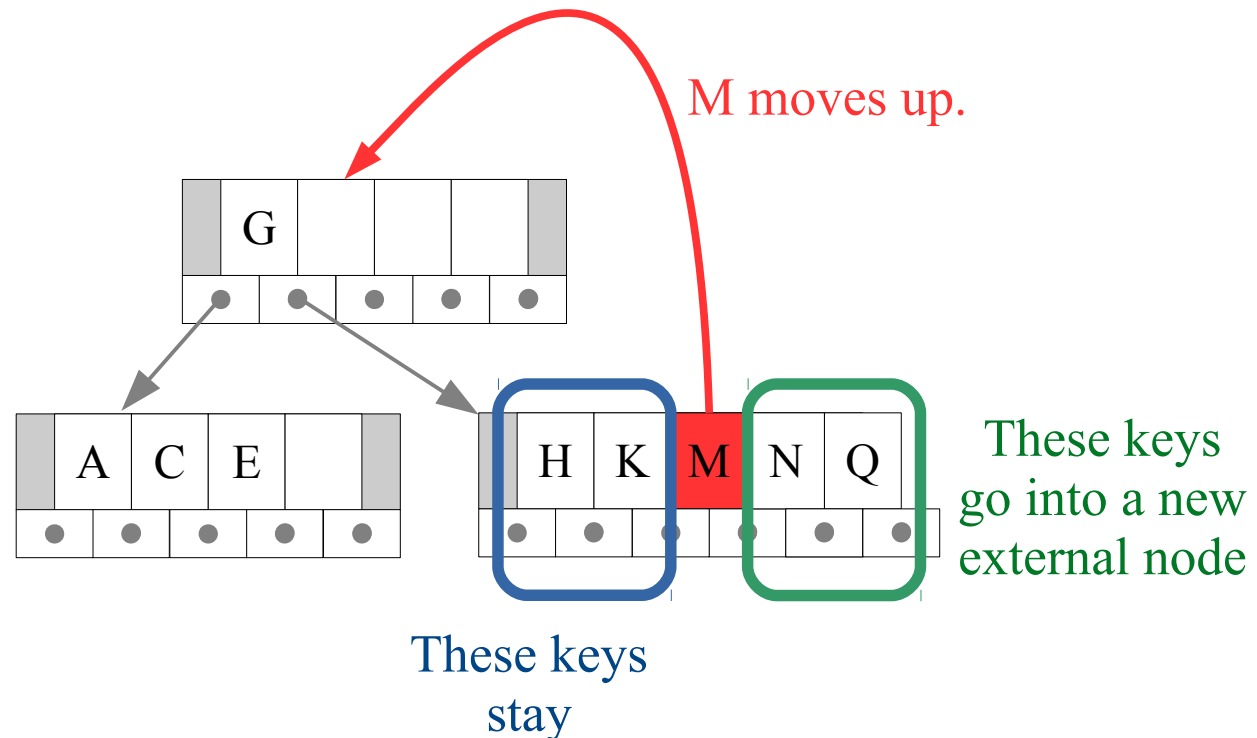
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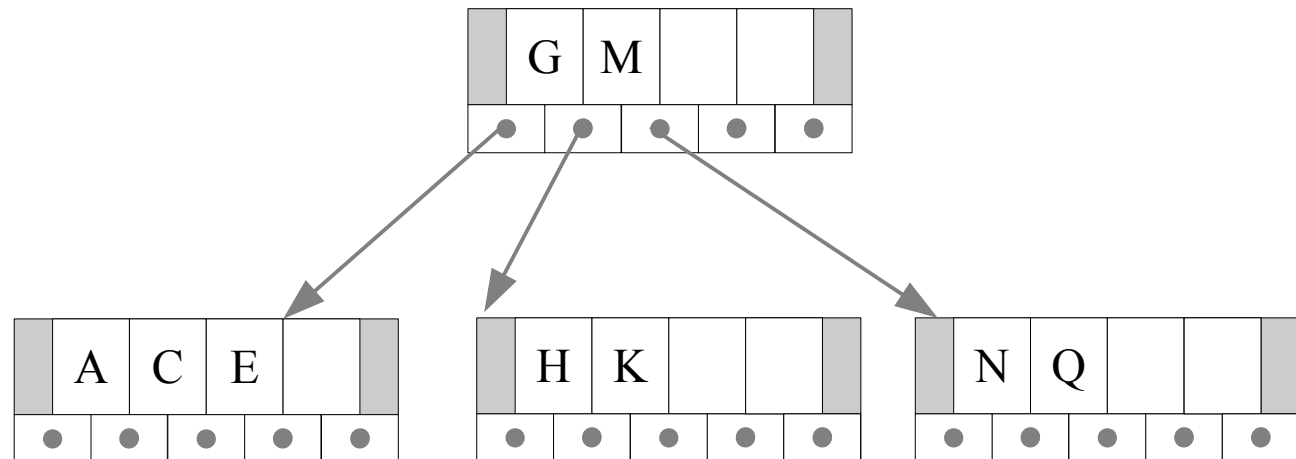
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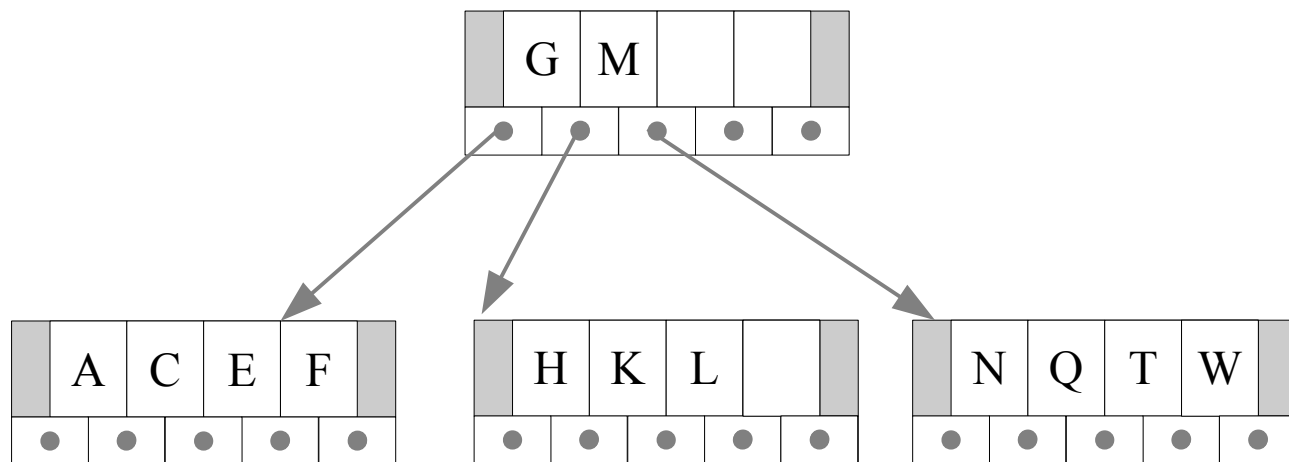
# To Construct a B-Tree

- Next, we'll insert E, K and Q – these all go into leaf nodes.
- Next, insert M – this requires a split.
- Insert F, W, L, T



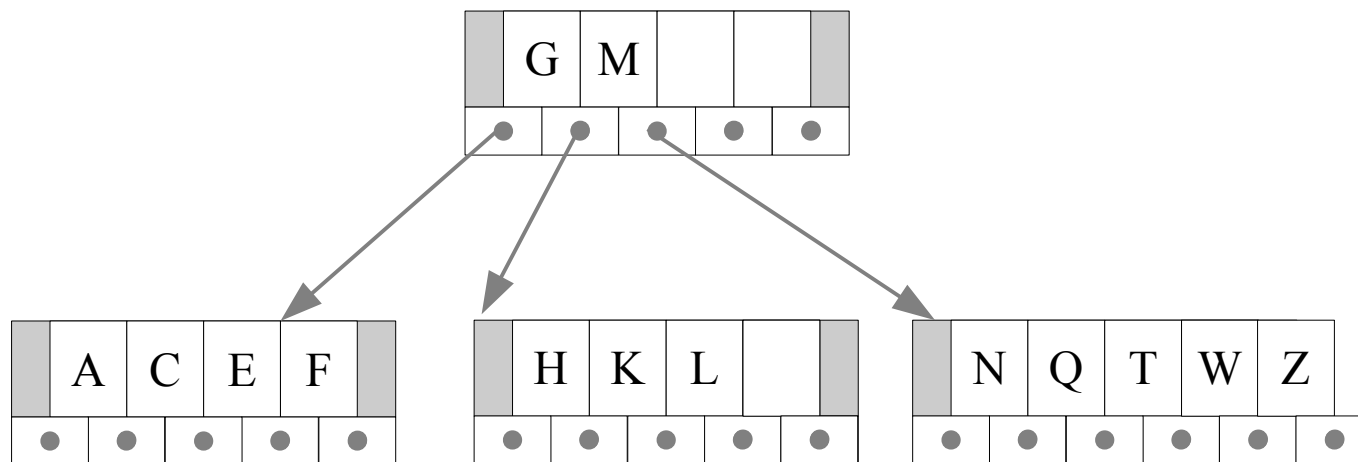
# To Construct a B-Tree

- Next, we'll insert E, K and Q – these all go into leaf nodes.
- Next, insert M – this requires a split.
- Insert F, W, L, T
- Insert Z: Another split



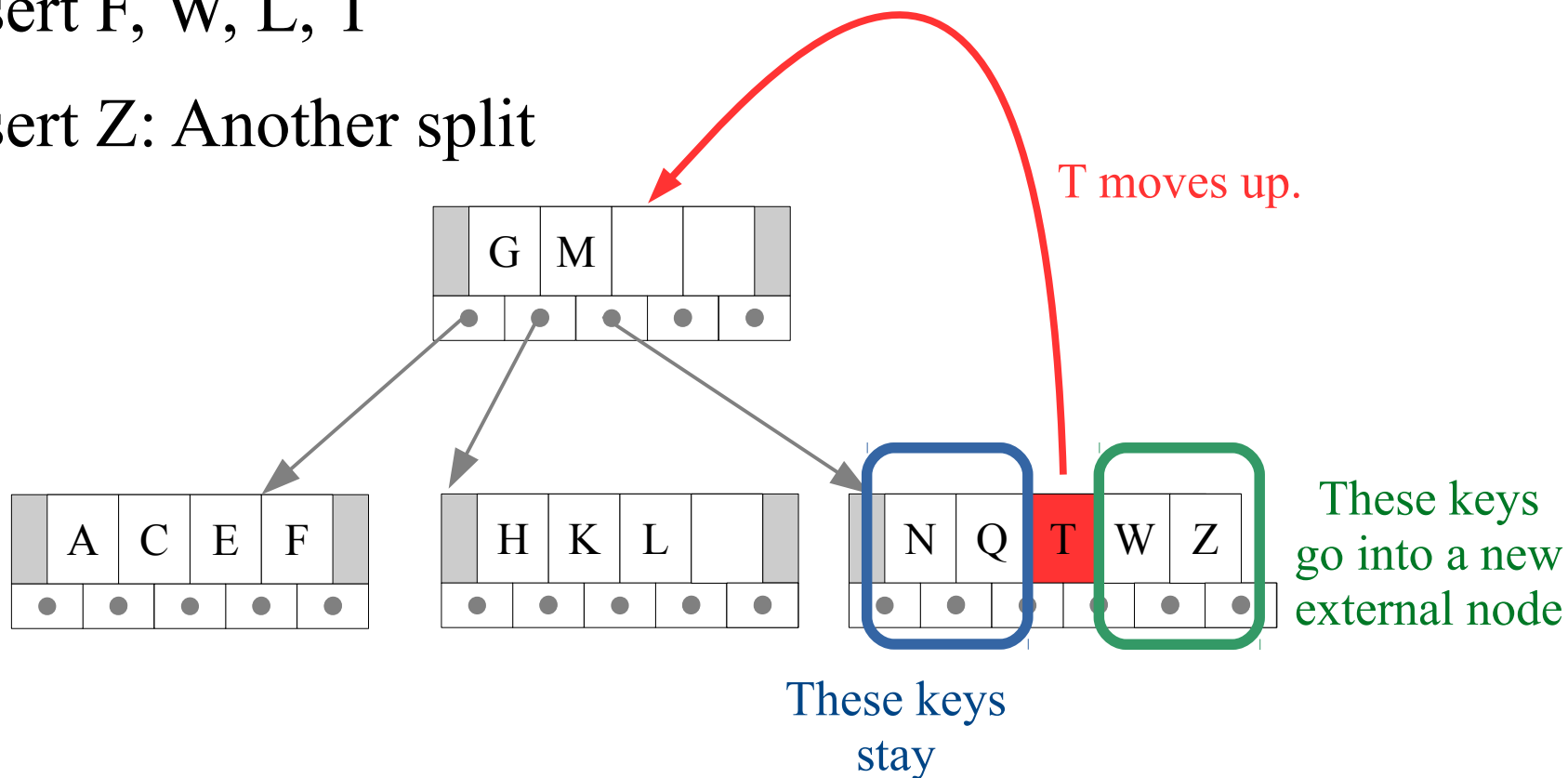
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- Next, we'll insert E, K and Q – these all go into leaf nodes.
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- Insert F, W, L, T
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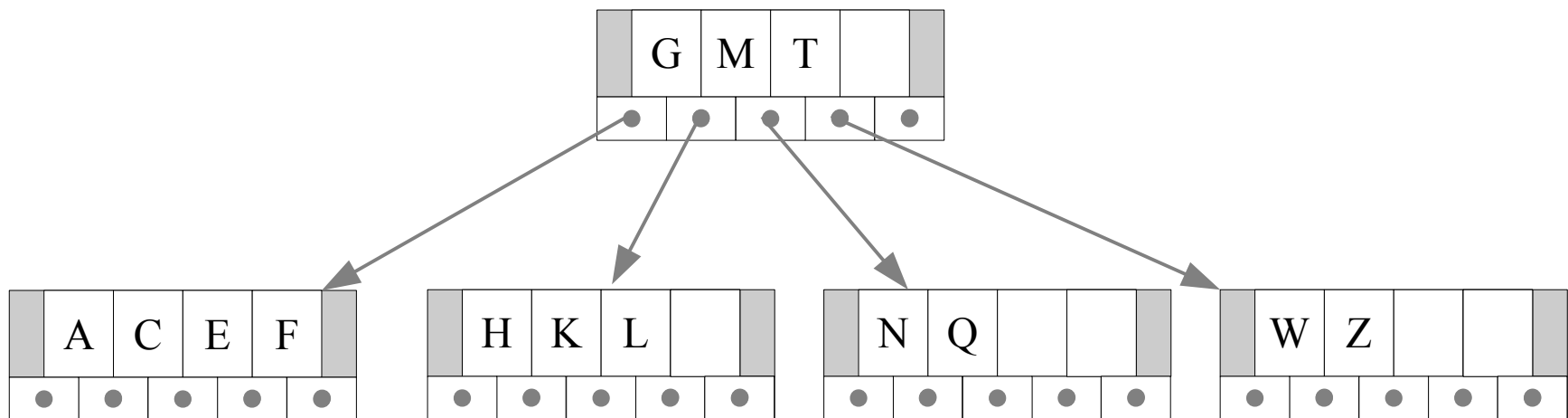
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- Next, we'll insert E, K and Q – these all go into leaf nodes.
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- Insert F, W, L, T
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# To Construct a B-Tree

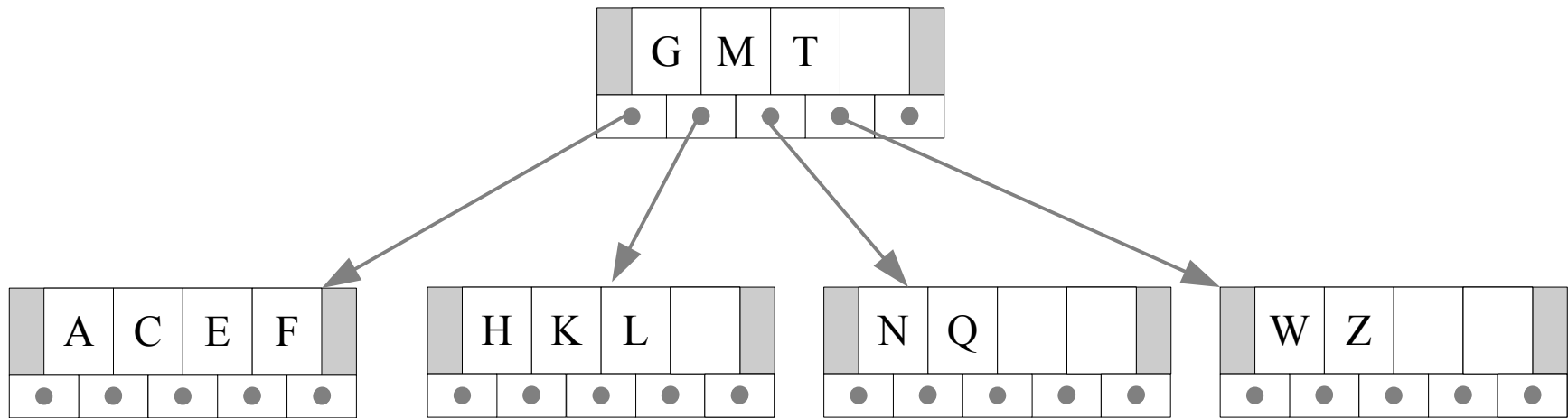
- Next, we'll insert E, K and Q – these all go into leaf nodes.
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- Insert F, W, L, T
- Insert Z: Another split





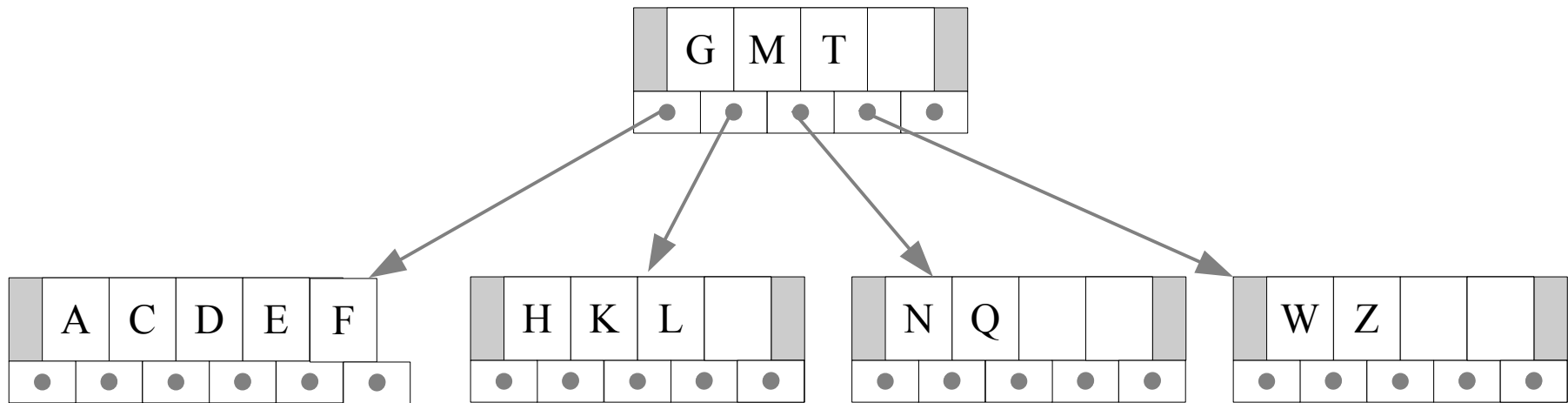
# To Construct a B-Tree

- Insert D – another split:.



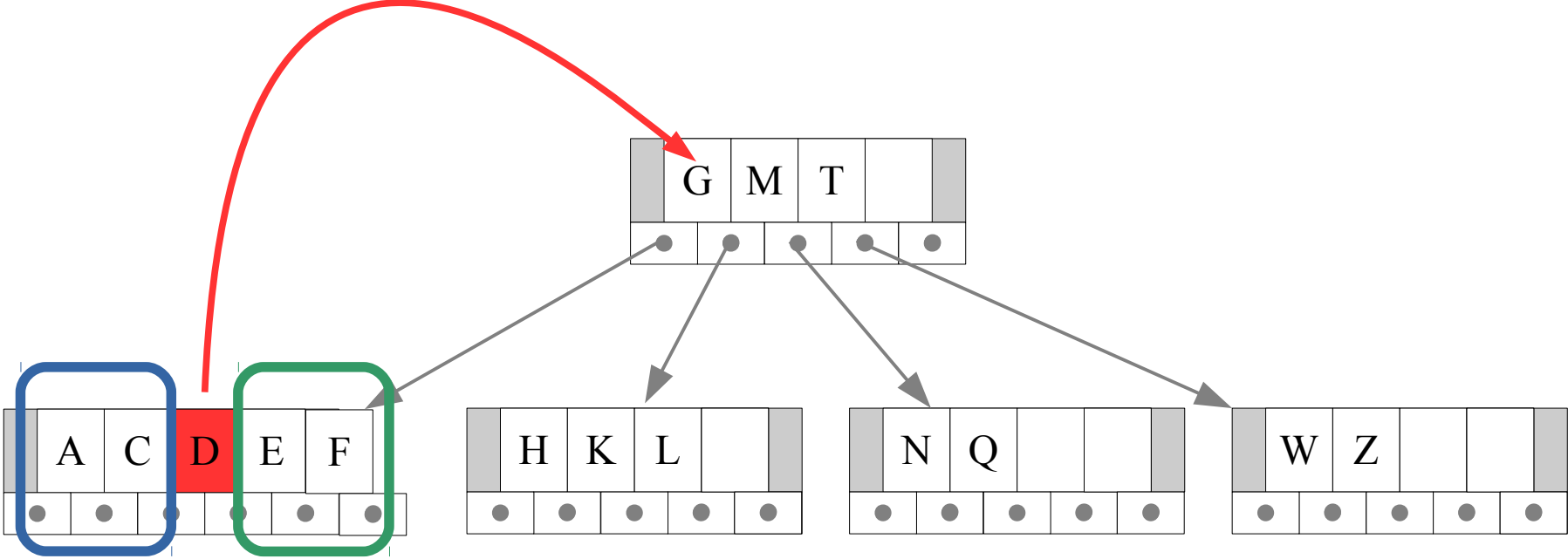
# To Construct a B-Tree

- Insert D – another split:



# To Construct a B-Tree

- Insert D – another split:

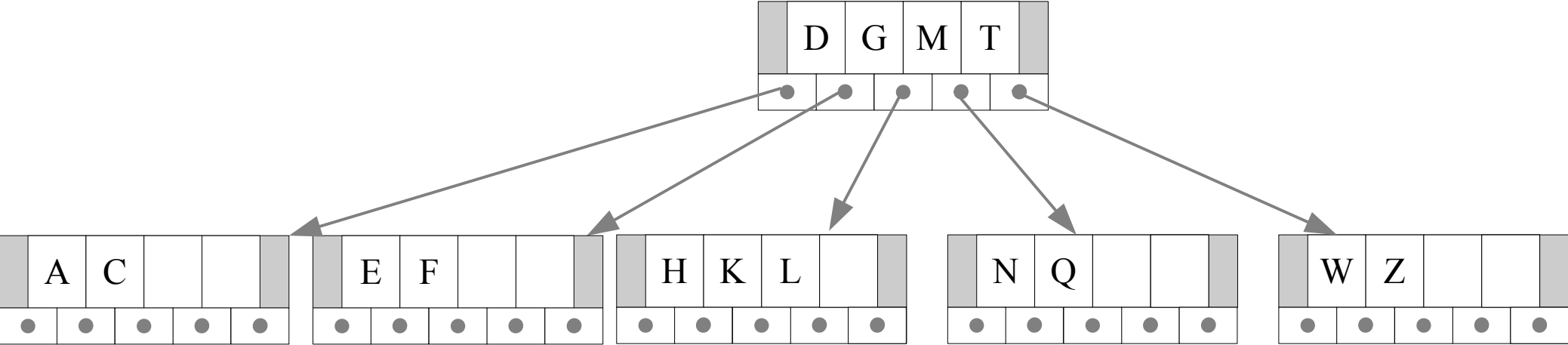


These keys stay

These keys go into a new external node

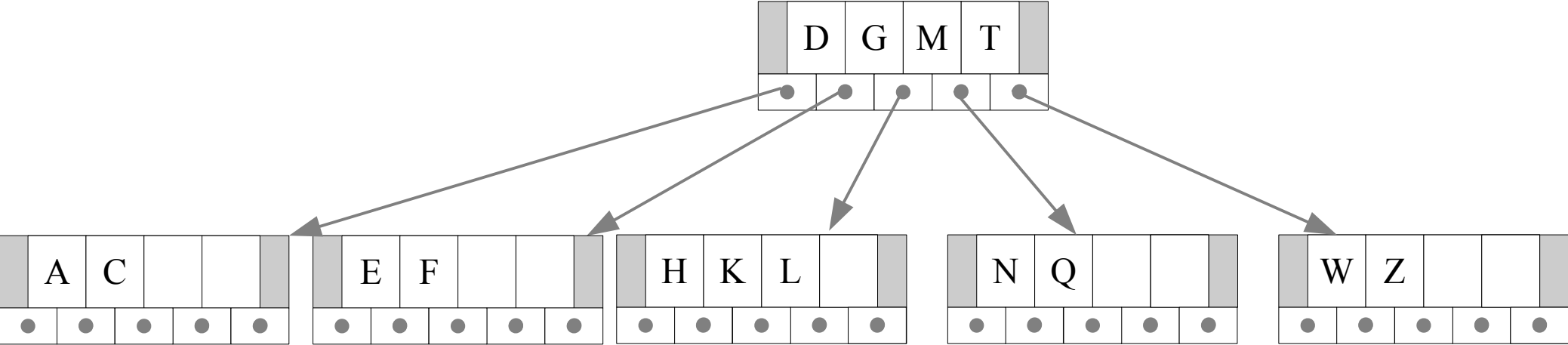
# To Construct a B-Tree

- Insert D – another split:



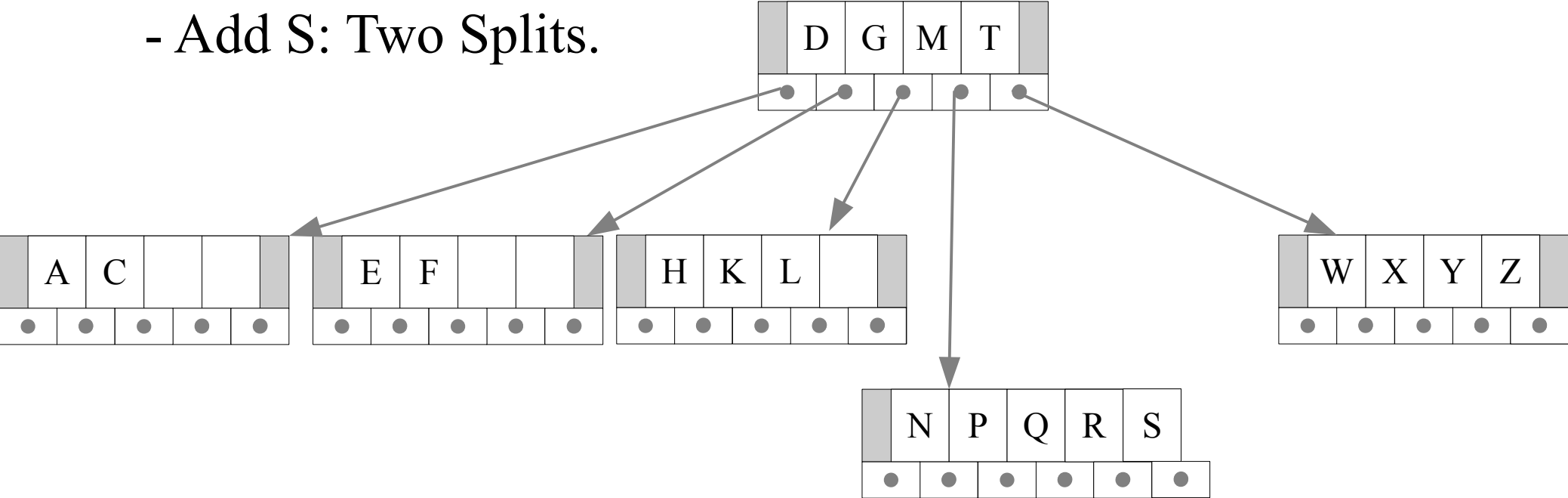
# To Construct a B-Tree

- Insert D – another split:
- Add P, R, X, Y



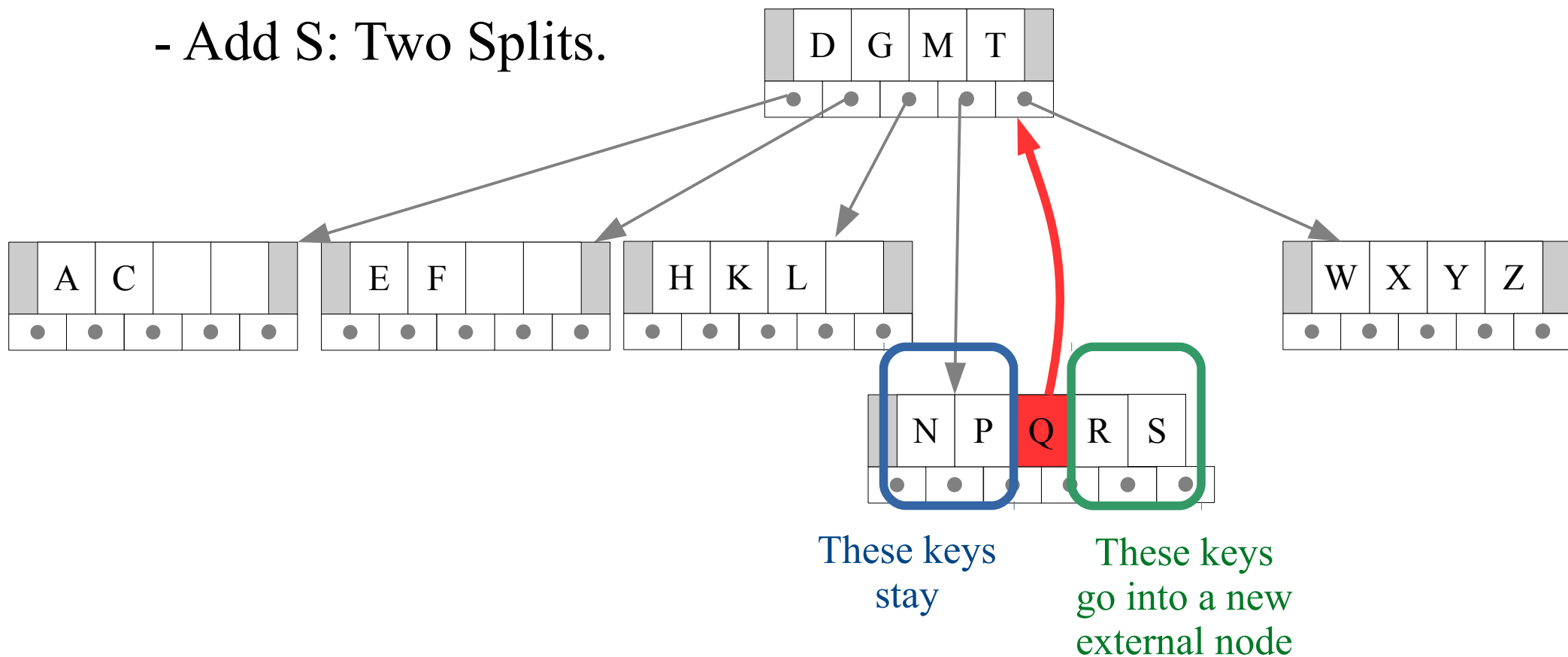
# To Construct a B-Tree

- Insert D – another split:
- Add P, R, X, Y
- Add S: Two Splits.



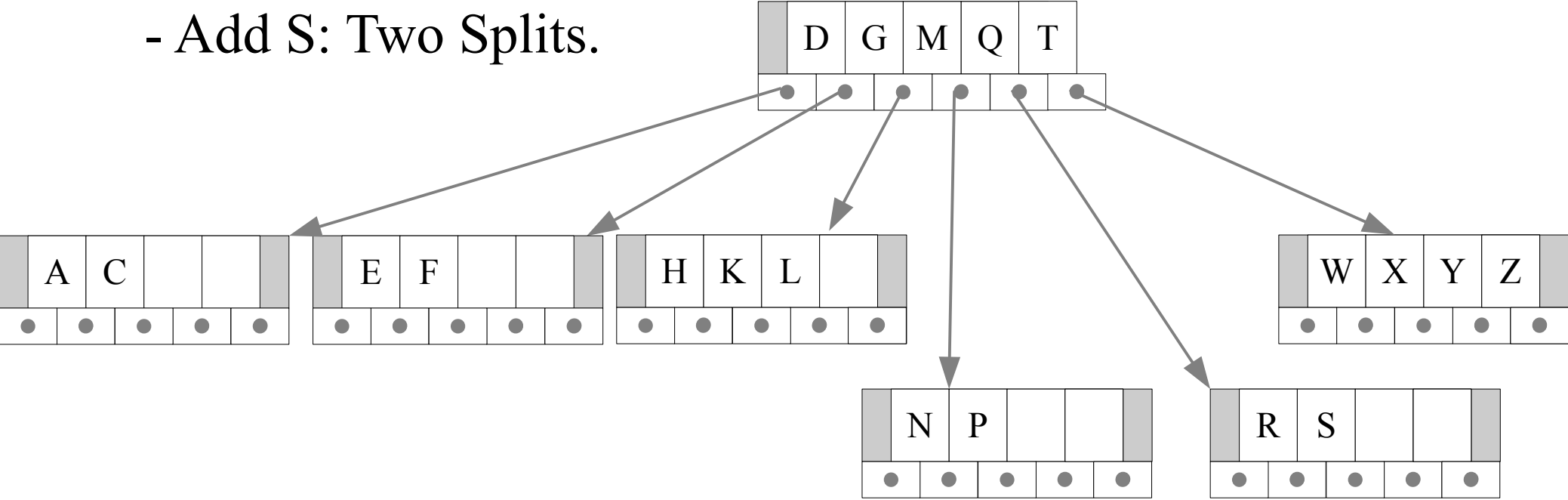
# To Construct a B-Tree

- Insert D – another split:
- Add P, R, X, Y
- Add S: Two Splits.



# To Construct a B-Tree

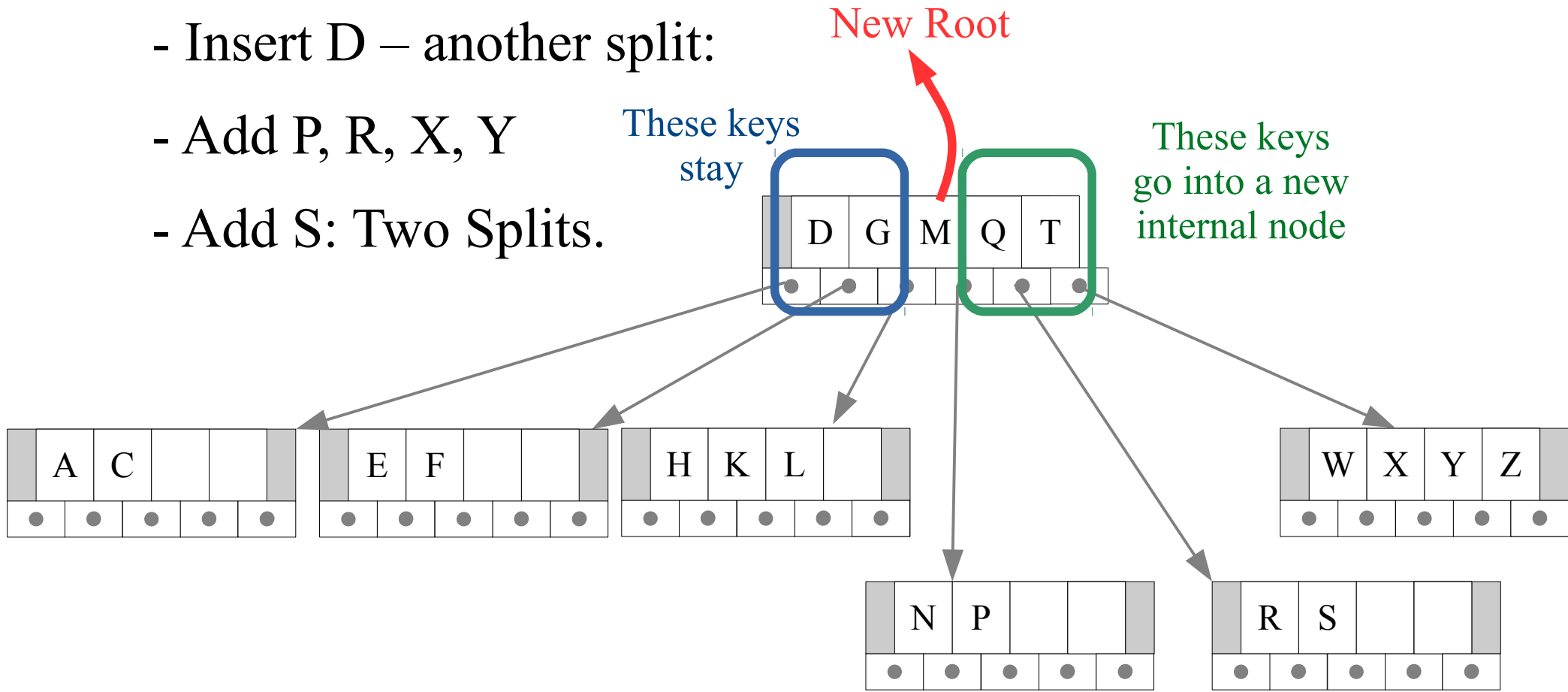
- Insert D – another split:
- Add P, R, X, Y
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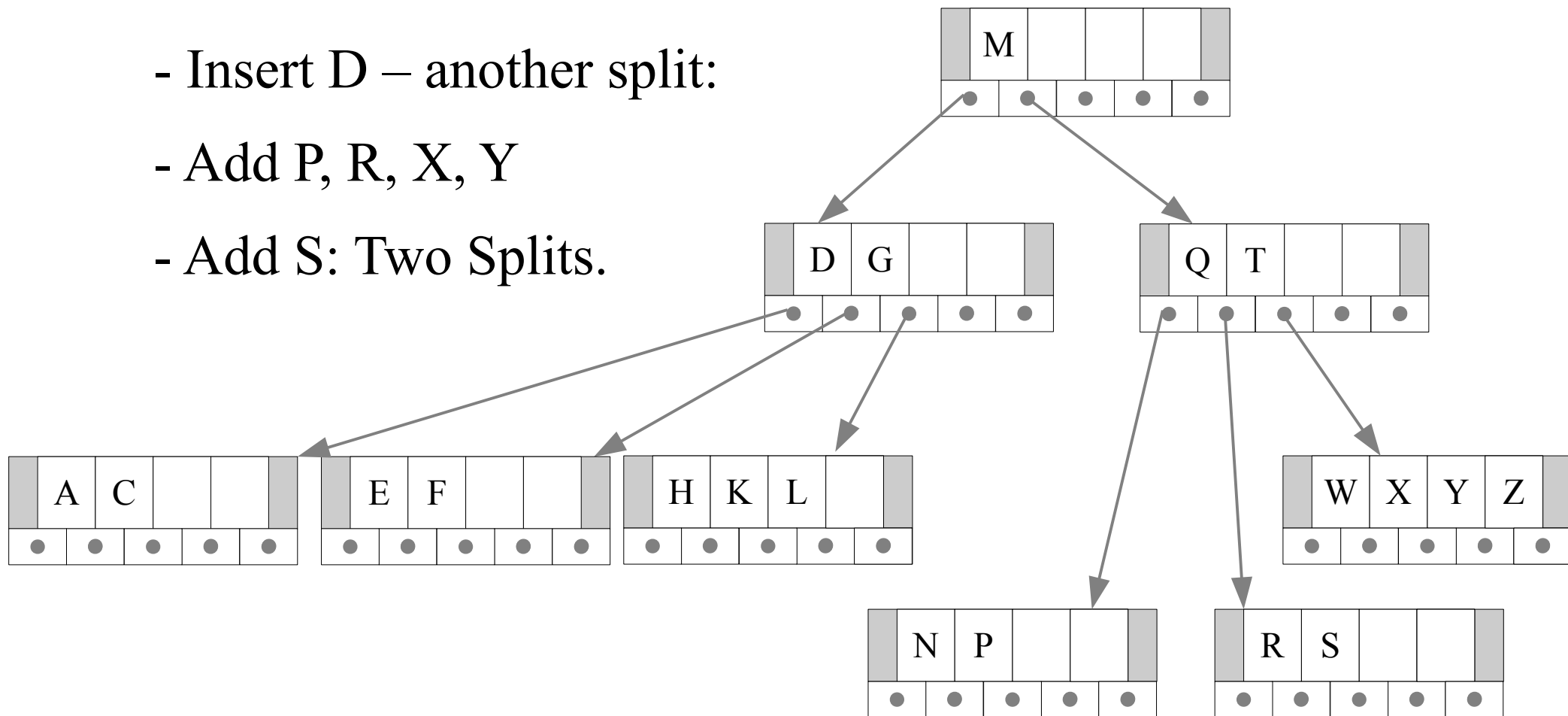
# To Construct a B-Tree

- Insert D – another split:
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- Add S: Two Splits.



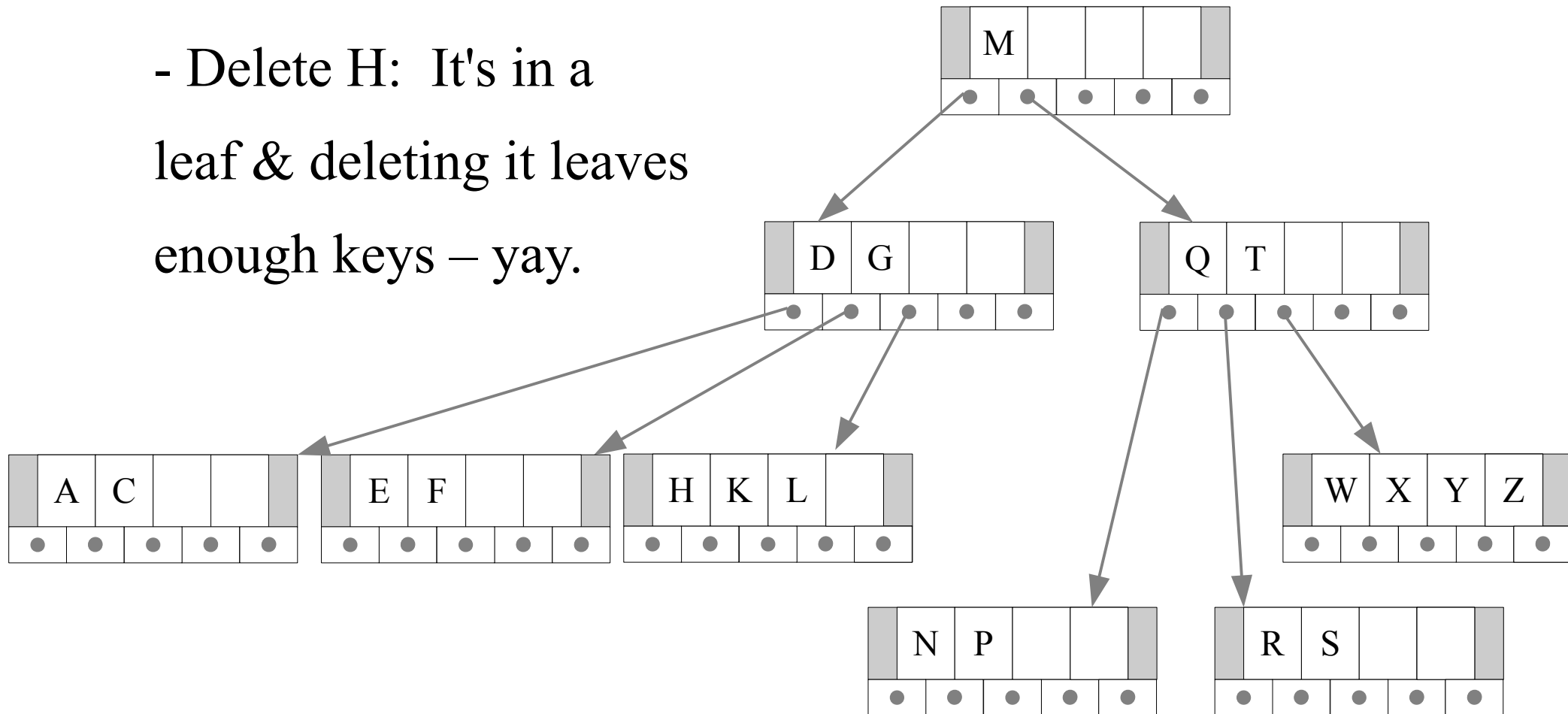
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- Insert D – another split:
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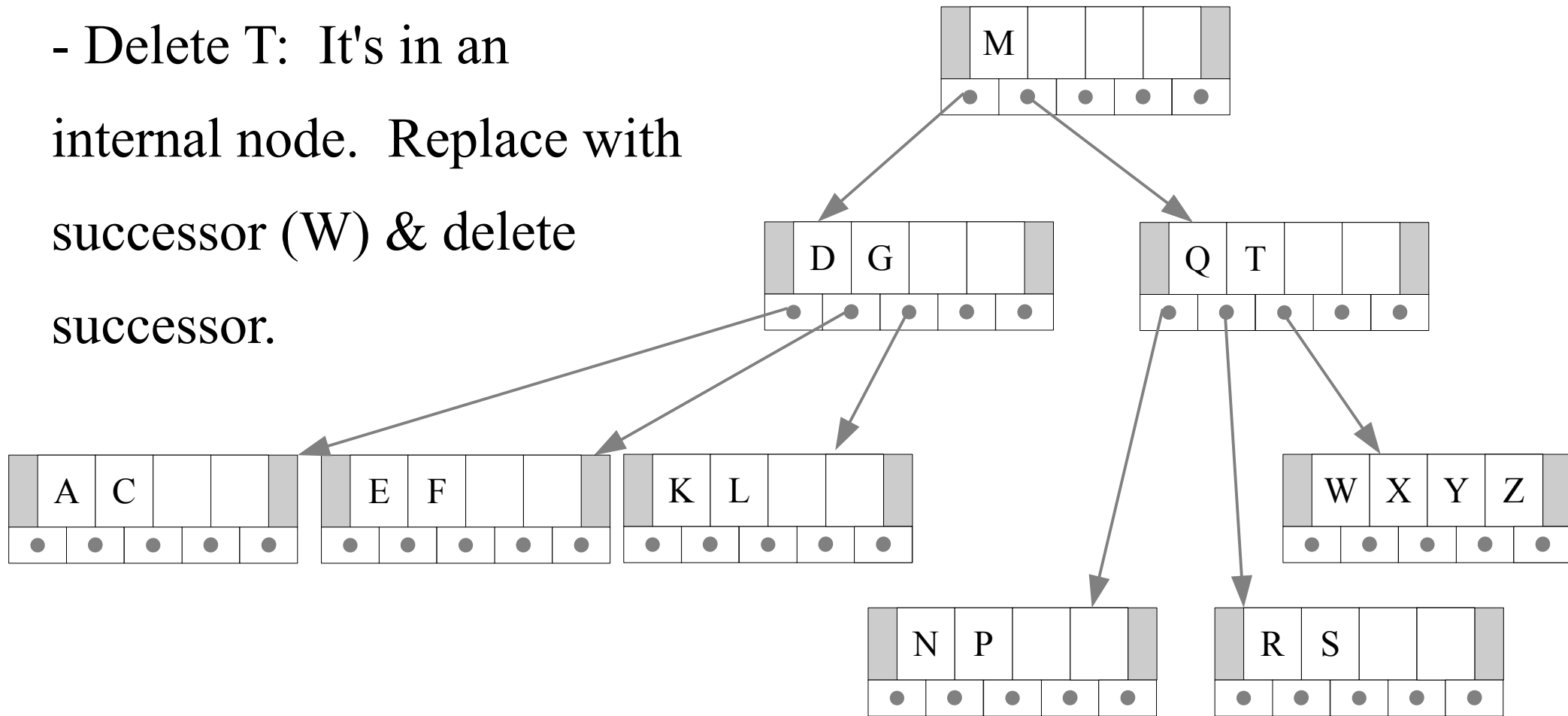
# Onto Deletion

- Delete H: It's in a leaf & deleting it leaves enough keys – yay.



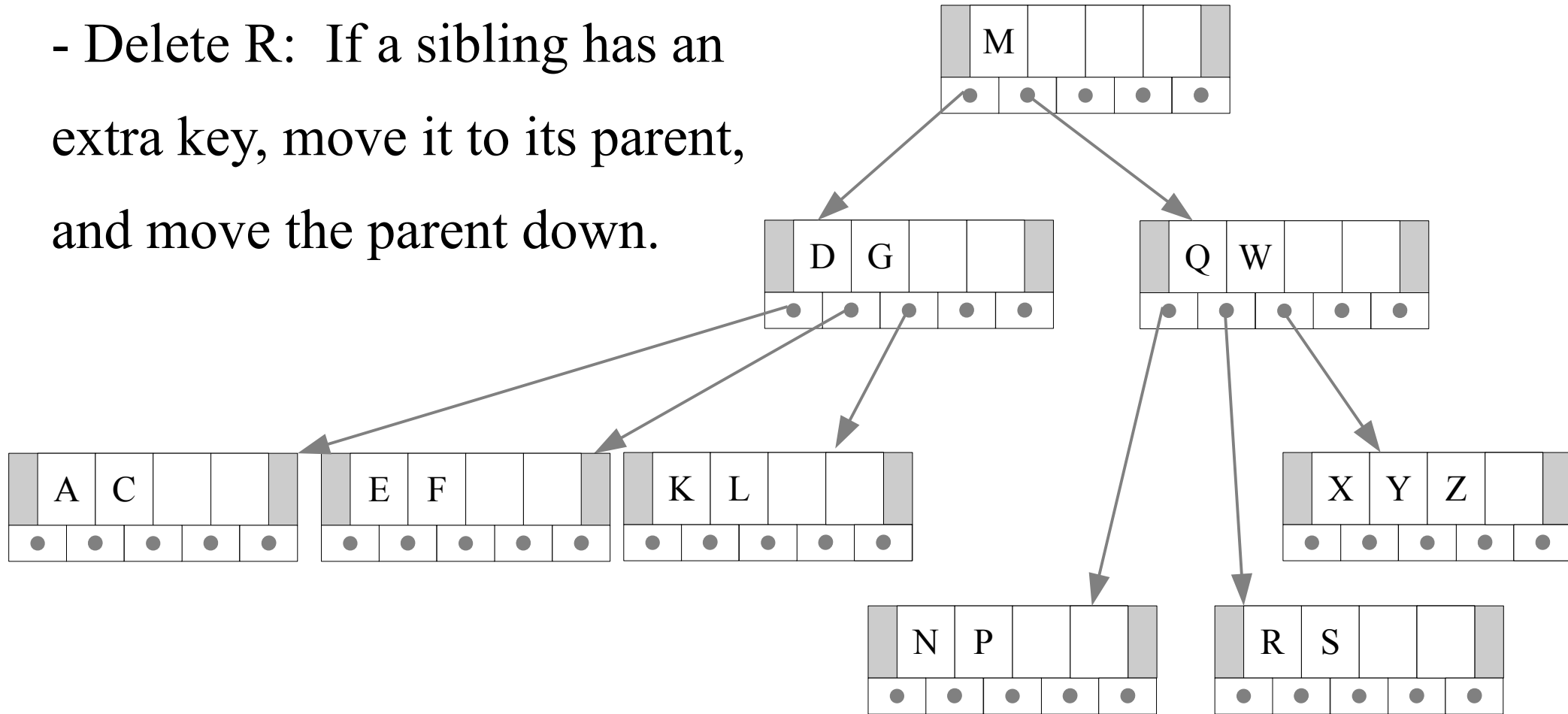
# Onto Deletion

- Delete T: It's in an internal node. Replace with successor (W) & delete successor.



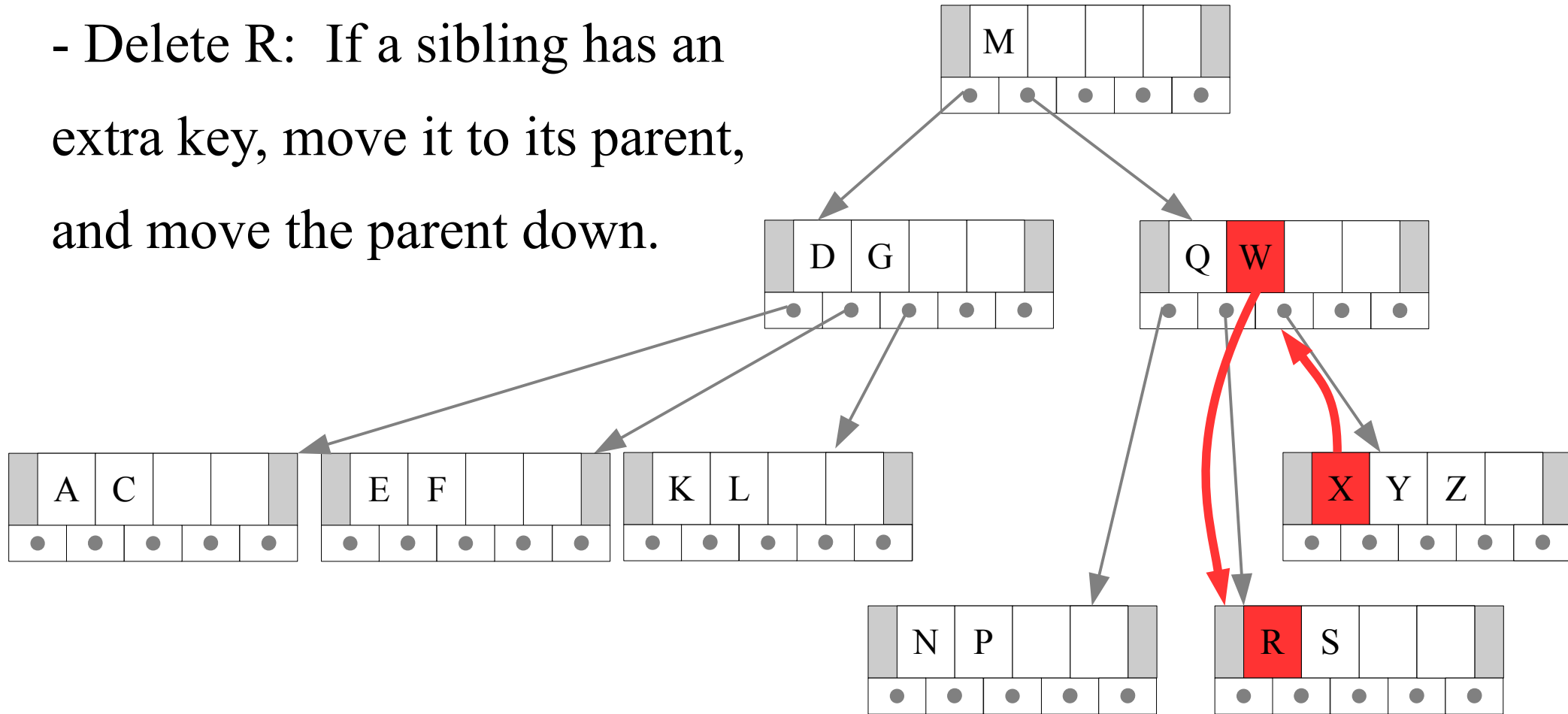
# Onto Deletion

- Delete R: If a sibling has an extra key, move it to its parent, and move the parent down.



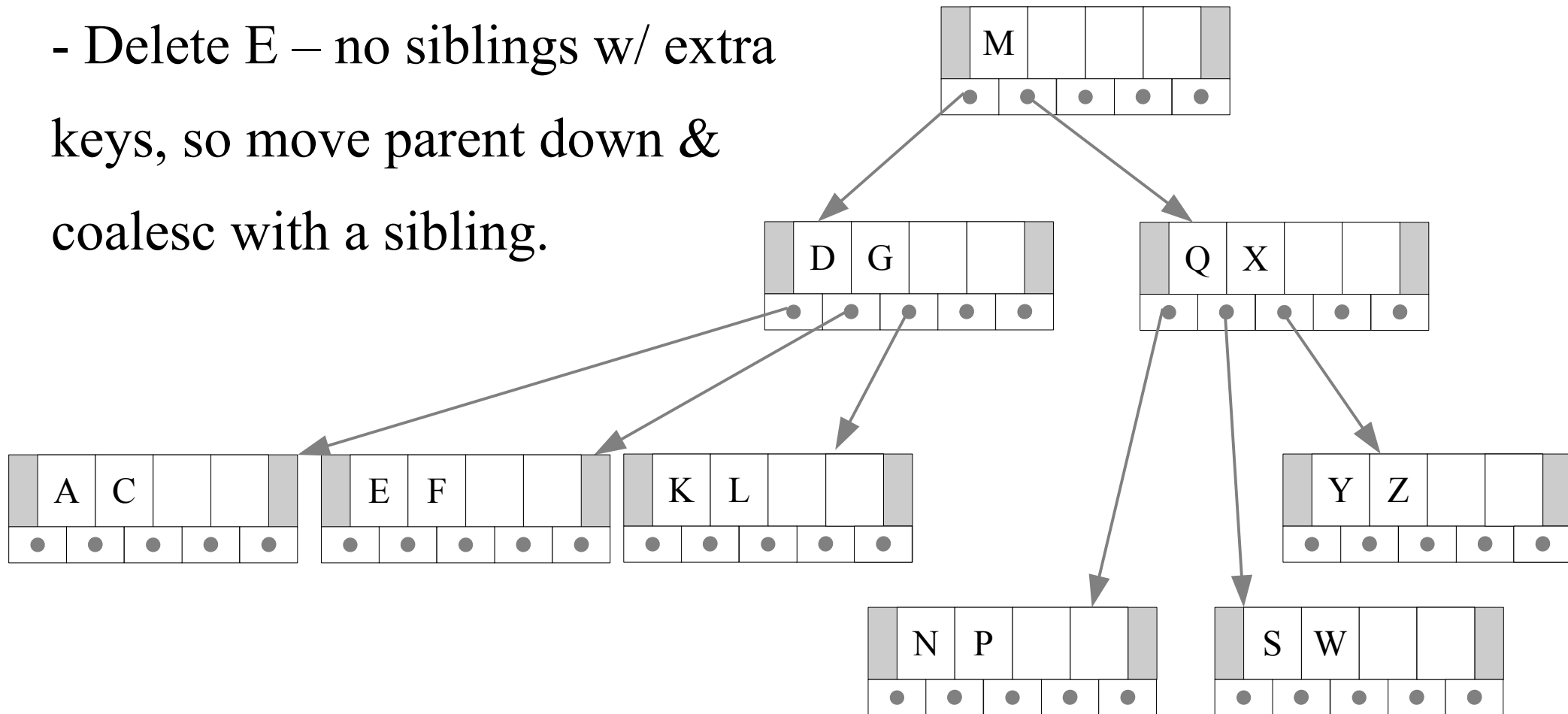
# Onto Deletion

- Delete R: If a sibling has an extra key, move it to its parent, and move the parent down.



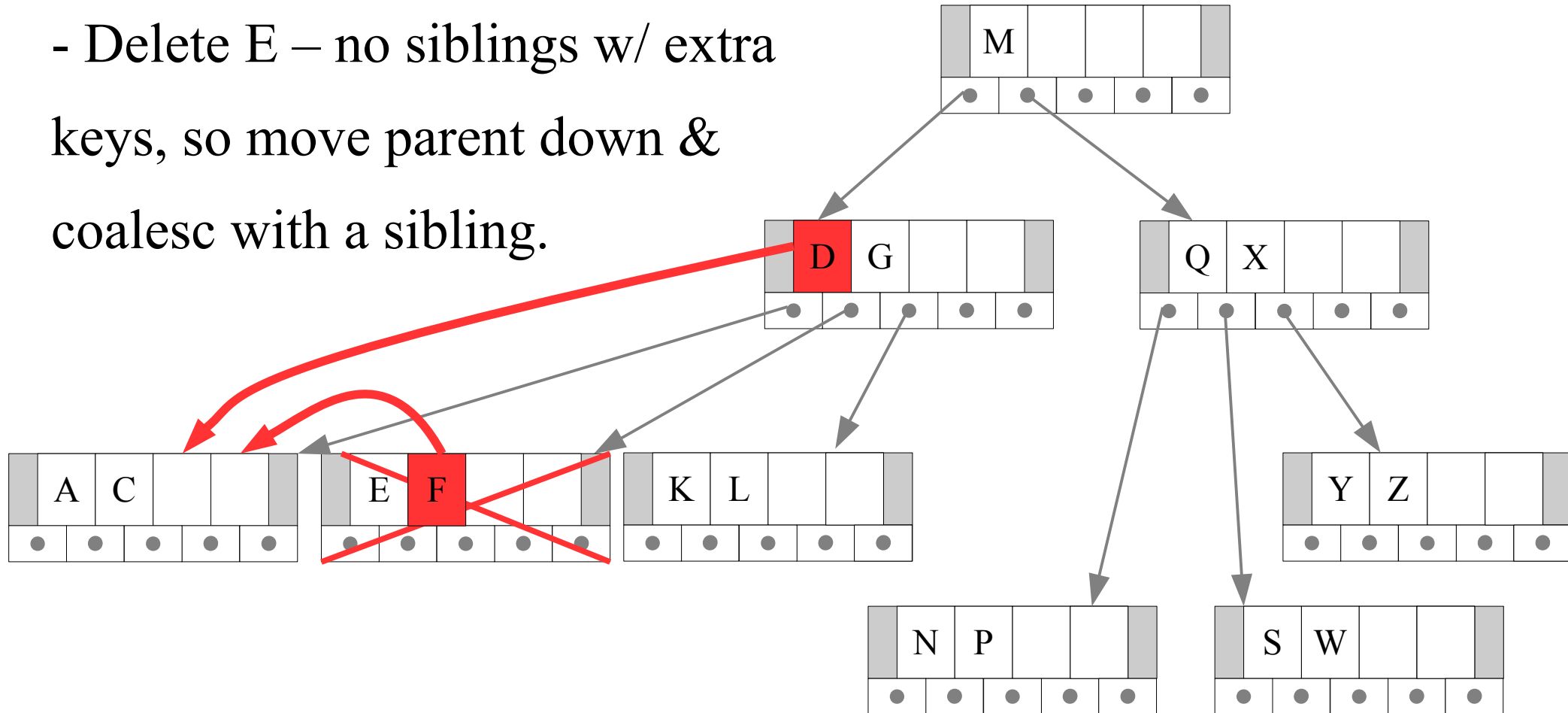
# Onto Deletion

- Delete E – no siblings w/ extra keys, so move parent down & coalesce with a sibling.



# Onto Deletion

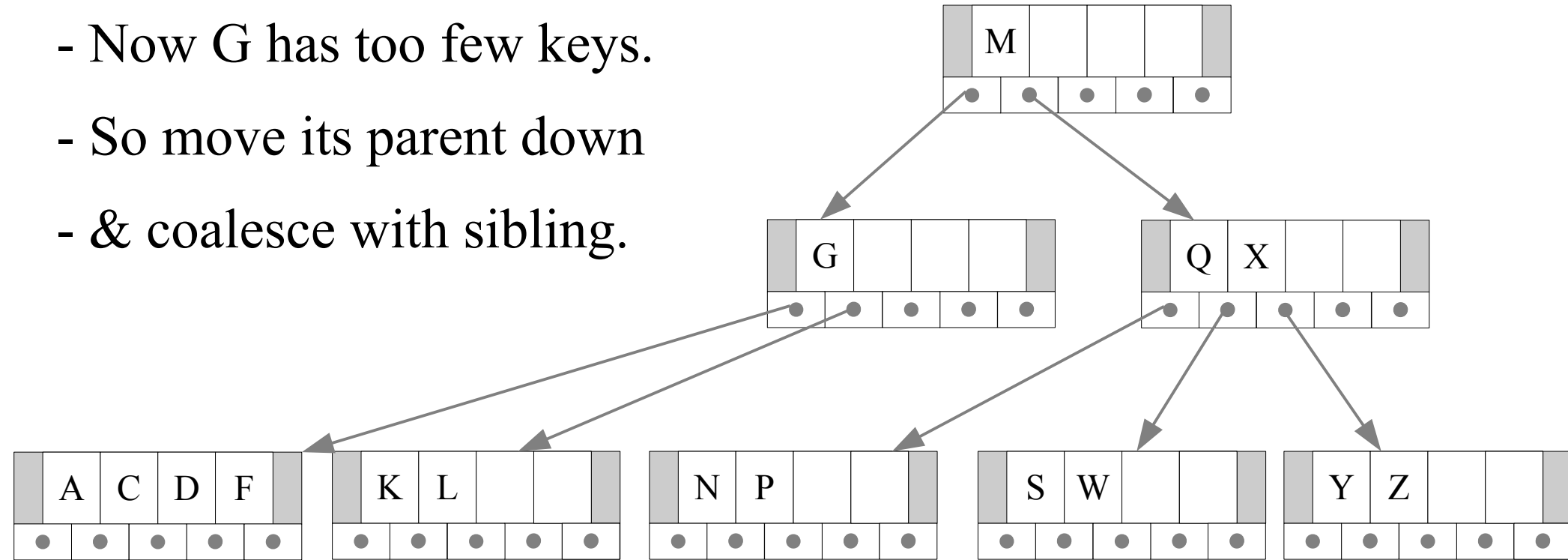
- Delete E – no siblings w/ extra keys, so move parent down & coalesce with a sibling.





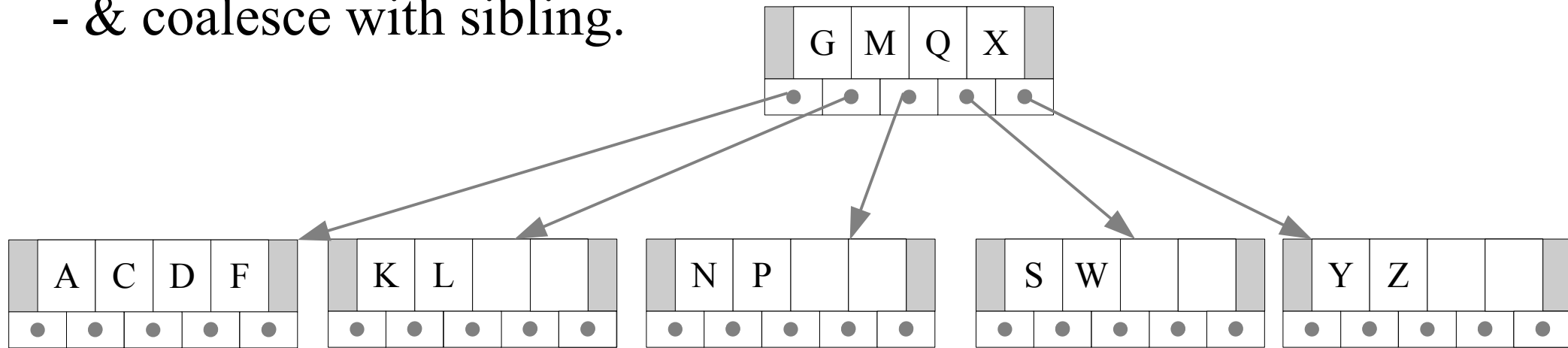
# Onto Deletion

- Now G has too few keys.
- So move its parent down
- & coalesce with sibling.



# Onto Deletion

- Now G has too few keys.
- So move its parent down
- & coalesce with sibling.



# B+ Tree (Backward from stvincent's explanation)

- External nodes have pointers too.
- These are to data associated w/ key.
- Internal key's data in predecessor's rightmost rightmost empty val.

