ECE 325 – Electric Energy System Components
4- Transformers

Instructor:
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Content

• Ideal Transformer (Ch. 9)
• Practical Transformers (Ch. 10)
• Special Transformers (Ch. 11)
• Three-Phase Transformers (Ch. 12)
Power Transformer

• Ideal transformers
  – Winding resistance is negligible
  – No leakage flux
  – Permeability of the core is infinite (zero magnetizing current $I_p$ is needed to produce flux)
  – No core loss

• Real transformers:
  – Windings have resistance
  – Windings do not link the same flux (having leakage flux)
  – Permeability of the core is finite
  – Have core losses (hysteresis losses and eddy current losses due to time varying flux)
Ideal Transformer

• Primary winding (assuming sinusoidal flux):
  Because there is no flux leakage, 
  $\phi_1=\phi_2=\phi_m$ (mutual flux)
  $\phi_m = \Phi_{\text{max}} \cos \omega t$
  
  $e_g = e_1 = N_1 \frac{d\phi}{dt} = -\omega N_1 \Phi_{\text{max}} \sin \omega t$
  
  $= E_{1\text{max}} \cos(\omega t + 90^\circ)$

  $E_{1\text{max}} = 2\pi f N_1 \Phi_{\text{max}} = 6.28 f N_1 \Phi_{\text{max}} = \sqrt{2} \times 4.44 f N_1 \Phi_{\text{max}}$

• Secondary winding:
  Because there is no flux leakage
  $e_2 = N_2 \frac{d\phi_m}{dt} = E_{2\text{max}} \cos(\omega t + 90^\circ)$

  $E_{2\text{max}} = 2\pi f N_2 \Phi_{\text{max}} = \sqrt{2} \times 4.44 f N_2 \Phi_{\text{max}}$

• Voltage/turns ratio  $a=E_1/E_2=N_1/N_2$

$E_g = E_1 = 4.44 f N_1 \Phi_{\text{max}} \angle 90^\circ$

$E_2 = 4.44 f N_2 \Phi_{\text{max}} \angle 90^\circ$
No-load and Load Conditions

• Under no-load conditions, because of infinitely permeable core, no magnetizing current is required to produce flux $\phi_m$. There are

$$I_1 = I_2 = 0$$

• Under load conditions ($I_1 \neq 0, I_2 \neq 0$), because of the infinitely permeable core, there is an exact mmf balance

$$F_1 = I_1 N_1 = I_2 N_2 = F_2$$

$$a = \frac{E_1}{E_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

• Phasor diagram:
  – Assume power factor $\cos \theta$
Circuit Symbol for an Ideal Transformer

Figure 9.11
a. Symbol for an ideal transformer and phasor diagram using sign notation.
b. Symbol for an ideal transformer and phasor diagram using double-subscript notation.
Impedance Ratio

\[ Z = \frac{E_2}{I_2} \]

\[ E_1 = aE_2 \]

\[ I_1 = I_2 / a \]

\[ Z_x = \frac{E_1}{I_1} = \frac{aE_2}{I_2 / a} = a^2 Z \]

• Impedance transformation
  – The impedance seen by the source (primary side) is \( a^2 \) times the real impedance (secondary side)
  – Thus, an ideal transformer has the amazing ability to increase or decrease the value of an impedance

Figure 9.12
a. Impedance transformation using a transformer.
b. The impedance seen by the source differs from \( Z \).
Shifting Impedances (S → P)

\[ Z_p \xrightarrow{a^2} Z_s \]
\[ E_p \xrightarrow{a} E_s \]
\[ I_p \xrightarrow{1/a} I_s \]
Shifting Impedances (P→S)

\[ Z_p \rightarrow \frac{1}{a^2} \rightarrow Z_s \]
\[ E_p \rightarrow \frac{1}{a} \rightarrow E_s \]
\[ I_p \rightarrow a \rightarrow I_s \]
Examples 9-4&9-5
Considering an imperfect core

- $P_m + jQ_m$ and $I_o$ are the complex power and current outputs of source $E_g$ under no-loading conditions. $P_m$ is the iron loss.
- $R_m$ is a resistance causing the iron loss and resulting heat:
  \[ R_m = \frac{E_1^2}{P_m} \quad I_f = \frac{E_1}{R_m} \]
- $X_m$ is the magnetizing reactance that measures permeability of the core. A smaller $X_m$ means lower permeability and needs a higher magnetizing current $I_m$ (i.e. bigger reactive power $Q_m$) to set up mutual flux $\Phi_m$:
  \[ X_m = \frac{E_1^2}{Q_m} \quad I_m = \frac{E_1}{jX_m} \]
Considering loose coupling and resistances of windings

\[ E_1 = 4.44fN_1 \Phi_m \quad E_2 = 4.44fN_2 \Phi_m \]
\[ E_{f1} = 4.44fN_1 \Phi_{f1} \quad E_{f2} = 4.44fN_2 \Phi_{f2} \]
\[ E_1 + E_{f1} = E_p = E_g \quad E_2 + E_{f2} = E_s \]

- Define winding leakage reactances
  \[ X_{f1} = E_{f1}/I_1 \quad X_{f2} = E_{f2}/I_2 \]
- Add winding resistances \( R_1 \) and \( R_2 \) in series with \( X_{f1} \) and \( X_{f2} \), respectively

**Figure 10.5**
A transformer possesses two leakage fluxes and a mutual flux.

**Figure 10.6**
Separating the various induced voltages due to the mutual flux and the leakage fluxes.
Equivalent Circuit of a Practical Transformer

\[ I_1 = I_2 = 0 \]

- Ignore \( R_1, X_{f1}, R_2 \) and \( X_{f2} \)
- \( E_p \approx E_1 = aE_2 = aE_s \)

- No-load conditions

- 10%-100% load conditions
  - \( I_p \gg I_o \)
  - Ignore \( I_o \)

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**Table 10A: Actual Transformer Values**

<table>
<thead>
<tr>
<th>( S_n ) kVA</th>
<th>1</th>
<th>10</th>
<th>100</th>
<th>1000</th>
<th>400000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{np} ) V</td>
<td>2400</td>
<td>2400</td>
<td>12470</td>
<td>69000</td>
<td>13800</td>
</tr>
<tr>
<td>( E_{ns} ) V</td>
<td>460</td>
<td>347</td>
<td>600</td>
<td>6900</td>
<td>42400</td>
</tr>
<tr>
<td>( I_{np} ) A</td>
<td>0.417</td>
<td>4.17</td>
<td>8.02</td>
<td>14.5</td>
<td>29000</td>
</tr>
<tr>
<td>( I_{ns} ) A</td>
<td>2.17</td>
<td>28.8</td>
<td>167</td>
<td>145</td>
<td>943</td>
</tr>
<tr>
<td>( R_1 ) ( \Omega )</td>
<td>58.0</td>
<td>5.16</td>
<td>11.6</td>
<td>27.2</td>
<td>0.0003</td>
</tr>
<tr>
<td>( R_2 ) ( \Omega )</td>
<td>1.9</td>
<td>0.095</td>
<td>0.024</td>
<td>0.25</td>
<td>0.354</td>
</tr>
<tr>
<td>( X_{f1} ) ( \Omega )</td>
<td>32</td>
<td>4.3</td>
<td>39</td>
<td>151</td>
<td>0.028</td>
</tr>
<tr>
<td>( X_{f2} ) ( \Omega )</td>
<td>1.16</td>
<td>0.09</td>
<td>0.09</td>
<td>1.5</td>
<td>27</td>
</tr>
<tr>
<td>( X_m ) ( \Omega )</td>
<td>200000</td>
<td>29000</td>
<td>150000</td>
<td>505000</td>
<td>460</td>
</tr>
<tr>
<td>( R_m ) ( \Omega )</td>
<td>400000</td>
<td>51000</td>
<td>220000</td>
<td>432000</td>
<td>317</td>
</tr>
<tr>
<td>( I_o ) A</td>
<td>0.0134</td>
<td>0.0952</td>
<td>0.101</td>
<td>0.210</td>
<td>52.9</td>
</tr>
</tbody>
</table>
Simplified Equivalent Circuits

• No-load conditions
  – Ignoring winding leakage fluxes and losses

• 10%-100% load conditions
  – Ignoring core reluctance and losses

Figure 10.21
Simplified circuit at no-load.

Figure 10.22
Simplified equivalent circuit of a transformer at full-load.
Equivalent Circuits Referred to One Side

\[ R_1, X_{f1}, X_m, I_m, I_f, R_m, E_1, E_2, E_p, E_s, Z_p, Z_s, a, 1/a, 1/a^2, a^2 R_2, a^2 X_{f2}, a^2 Z, N_1, N_2 \]
Simplified Equivalent Circuits Referred to One Side

• Under load conditions

\[ a = \frac{N_1}{N_2} \]

\[ Z_p = R_p + jX_p \]

\[ = R_1 + a^2R_2 + j(X_{f1} + a^2X_{f2}) \]

Figure 10.23
Equivalent circuit with impedances shifted to the primary side.
Voltage Regulation

• With the primary voltage held constant at its rated value, the voltage regulation is % change of the secondary voltage from no-load to full load (rated)

\[
\text{Voltage Regulation} = \frac{|E_{NL}| - |E_{FL}|}{|E_{FL}|} \times 100
\]

- \(E_{NL}\): secondary voltage at no-load
- \(E_{FL}\): secondary voltage at full-load

• The voltage regulation depends on the power factor of the load on the secondary side
  – If the load is capacitive, the full-load voltage may exceed the no-load voltage, in which case the voltage regulation becomes negative
Measuring Transformer Impedances

• Open-circuit (no-load) test
  – Neglect $R_1$, $X_{f1}$, $R_2$, $X_{f2}$
  – Measure $P_m$ (core loss), $|E_p|$, $|I_o|$ and $|E_s|$

$$R_m = \frac{|E_p|^2}{P_m}, \quad |I_f| = \frac{|E_p|}{R_m}$$

$$|I_m| = \sqrt{|I_o|^2 - |I_f|^2}$$

$$X_m = \frac{|E_p|}{|I_m|}$$

$$a = \frac{N_1}{N_2} = \frac{|E_p|}{|E_s|}$$

Figure 10.27
Open-circuit test and determination of $R_m$, $X_m$, and turns ratio.
• Short-circuit test
  – Apply a low voltage $E_{SC}$ to the primary side to create $I_{SC}$ less than the nominal value to prevent overheating and rapid change in winding resistance
  – Neglect $R_m$ and $X_m$ due to low core flux

$$|Z_p| = \frac{|E_{sc}|}{|I_{sc}|}$$

$$R_p = \frac{P_{sc}}{|I_{sc}|^2}$$

$$X_p = \sqrt{|Z_p|^2 - R_p^2}$$

**Figure 10.28**
Short-circuit test to determine leakage reactance and winding resistance.
Construction of a power transformer

- **Core**: made of iron for high permeability; laminated and high resistive to reduce iron losses

- **Windings**: the primary and secondary coils are wound closely on top of each other with careful insulation for tight coupling; the HV winding has more turns but uses a smaller size of conductor, so copper/aluminum of two windings are about the same
Standard terminal markings and polarity tests

- High-Voltage winding \( (H_1&H_2) \) and Low-Voltage winding \( (X_1&X_2) \)
  \[ E_{H1,H2}/E_{X1,X2}=E_H/E_X=N_H/N_X \]

- Polarity test:
  1. Connect HV winding to a low voltage source \( E_g \)
  2. Connect a jumper J between any two adjacent HV and LV terminals
  3. Connect two voltmeters to as shown in Figure 10.11
  4. The polarity is additive if \(|E_x|>|E_p|\) or, otherwise, is subtractive.

Figure 10.10
Additive and subtractive polarity depend upon the location of the \( H_1-X_1 \) terminals.

Figure 10.11
Determining the polarity of a transformer using an ac source.
The Per-Unit System

- Quantity in **Per-Unit = Actual quantity / Base or nominal** value of quantity
- Why per-unit notations?

Choose 20kV, 345KV and 138kV as base voltages

- Neglecting different voltage levels of transformers, lines and generators
- Powers, voltages, currents and impedances are expressed as decimal fractions of respective base quantities

(Source: EPRI Dynamic Tutorial)
• Four base quantities are required to completely define a per-unit system

\[ S_{pu} = \frac{S}{S_n} \quad E_{pu} = \frac{E}{E_n} \quad I_{pu} = \frac{I}{I_n} \quad Z_{pu} = \frac{Z}{Z_n} \]

• We need to select two independent base quantities of the four and calculate the other two, e.g. selecting \( S_n \) and \( E_n \)

\[ I_n = \frac{S_n}{E_n} \quad Z_n = \frac{E_n}{I_n} = \frac{(E_n)^2}{S_n} \]

• For a transformer:
  – Usually, \( S_n, E_{pn}, E_{sn} \) and the total impedance \( Z_p \) (in p.u. or %) referred to the primary side are given.

\[ I_{np} = \frac{S_n}{E_{np}} \quad I_{ns} = \frac{S_n}{E_{ns}} \quad a = \frac{N_p}{N_s} = \frac{E_{np}}{E_{ns}} \]

\[ Z_{np} = \frac{E_{np}}{I_{np}} = \frac{(E_{np})^2}{S_n} \quad (\Omega) \quad Z_{ns} = \frac{E_{ns}}{I_{ns}} = \frac{(E_{ns})^2}{S_n} \quad (\Omega) \]

\[ Z_p (\Omega) = Z_p (\text{p.u.}) \times Z_{np} \]
Examples 10-5, 10-6, 10-7, 10-8 & 10-10
**Autotransformer**

- A conventional two-winding transformer can be changed to an autotransformer by connecting its two coils in series (note: 4 combinations).
- The connection may use a sliding contact to providing variable output voltage.

![Diagram of Autotransformer Evolution From a Two-Winding Transformer](source: EPRI Power System Dynamic Tutorial)

20MVA (115/69kV) McGraw-Edison Substation
Auto-Transformer (Y-Y) (Source: [http://www.tucsontransformer.com](http://www.tucsontransformer.com))
• The primary and secondary windings have a common terminal A, and hence are not isolated from each other

\[ \frac{E_2}{E_1} = \frac{E_{CA}}{E_{BA}} = \frac{N_2}{N_1} \] (There is always \( N_1 > N_2 \))

\[ I_1N_1 = I_2N_2 \iff I_1(N_1-N_2) = (I_2-I_1)N_2 \]

\[ E_1I_1 = E_2I_2 \]

• An autotransformer has a smaller size (or equivalently, a higher kVA rating for the same size) but loses insulation between primary and secondary windings
Conventional transformer connected as an autotransformer (Example 11-2)

• Reconnect the transformer to obtain
  – 600V / 480V
  – 600V / 720V
  – 120V / 480V
  – 120V / 720V

• \(I_1 = \frac{15000}{600} = 25\text{A}\)
• \(I_2 = \frac{15000}{120} = 125\text{A}\)

When a transformer connects a source (on the primary side) with a load (on the secondary side), it is the load (current flows into terminal +) of the primary circuit and is the source (current flows out of terminal +) of the secondary circuit, so two winding currents (\(I_1\) and \(I_2\)) have opposite directions.

Figure 11.6
Standard 15 kVA, 600 V/120 V transformer.
The maximum apparent power $|S_{\text{max}}| = \max(|E_1|, |E_2|) \times (|I_1| + |I_2|) = (|E_1| + |E_2|) \times \max(|I_1|, |I_2|)$

$|S_i| = 600 \times 100 = 60\text{kVA}$

$|S_o| = 480 \times 125 = 60\text{kVA}$

$|S_i| = 120 \times 100 = 12\text{kVA}$

$|S_o| = 480 \times 25 = 12\text{kVA}$

$|S_i| = 600 \times 150 = 90\text{kVA}$

$|S_o| = 720 \times 125 = 90\text{kVA}$
Voltage Transformers and Current Transformers

• Voltage transformers (VTs), also called potential transformers (PTs), are installed on HV lines to measure line-to-neutral voltage.

• Current transformers (CTs) are used to measure or monitor the current in a line for system measurement and protection.
Three-Phase Transformers

• A bank of three single-phase transformers connected in Y or Δ configurations
  – Four possible combinations: Y-Y, Δ-Δ, Y-Δ and Δ-Y

  – Y: lower insulation costs, with neutral for grounding, 3\textsuperscript{rd} harmonics
  – Δ: more insulation costs, no neutral, no 3\textsuperscript{rd} harmonics
3\textsuperscript{rd} harmonics problem with three-phase transformers

- A $\Delta$ configuration provides a closed path for 3\textsuperscript{rd} harmonics, or in other words, all triple harmonics are trapped in the $\Delta$ loop.
Three-phase transformers: \( \Delta-\Delta \) connection

- Each transformer, considered alone, acts as if it were placed in a single-phase circuit.
- Because \( E_{H1H2} \) and \( E_{X1X2} \) are in phase for each single transformer, line voltages \( E_{AB} \) with \( E_{12} \), \( E_{BC} \) with \( E_{23} \), and \( E_{CA} \) with \( E_{31} \) are in phase.
- A balanced three-phase load produces balanced line currents in line 1-2-3 and lines A-B-C.
- The power rating is 3x of a single transformer.
- One transformer may be absent (i.e. open-delta connection) under maintenance or emergency conditions.
Three-phase transformers: \( \Delta-Y \) (or \( Y-\Delta \)) connection

- \( X_2 \) terminals of 3 secondary windings are connected to a common neutral N.
- Because \( E_{H1H2} \) and \( E_{X1X2} \) are in phase for each single transformer, \( E_{AB} \) with \( E_{1N} \), \( E_{BC} \) with \( E_{2N} \), and \( E_{CA} \) with \( E_{3N} \) are in phase.
- There is a \( 30^o \) phase shift between the line voltages of the incoming (\( E_{AB} \)) and outgoing (\( E_{12} \)) lines.
- \( \Delta-Y \) is commonly used for voltage step-up transformers while \( Y-\Delta \) is commonly used for voltage step-down transformers.

\[ \sqrt{3} I_p \]
Three-phase transformers: Y-Y connection

- No phase shift between the incoming and outgoing line voltages
- Without a \( \Delta \) configuration to trap the 3\(^{rd} \) harmonic, it has seriously distorted voltages due to the 3\(^{rd} \) harmonics.
- Two solutions:
  - Ground the neutrals of the primary winding and the source
  - Provide each transformer with a tertiary winding, and connect 3 tertiary windings in delta

![Diagram](image1)

**Figure 12.6**
Wye-wye connection with neutral of the primary connected to the neutral of the source.

![Diagram](image2)

**Figure 12.7**
Wye-wye connection using a tertiary winding.
Phase-shifting Principle

• A 3-phase system can easily shift the phase angle of a voltage to create 2-, 6-, and 12-phase systems used in power electronic converters and controllers

• Phase shifting (performed by phase-shifting transformers) is also used in power flow control over long transmission lines

Figure 12.12
Voltage $E_{AP}$ can be phase-shifted with respect to $E_{AC}$ by means of a potentiometer.
Phase-shifting transformers

- A special type of 3-phase autotransformer that shifts the phase angle between the incoming and outgoing lines without changing the voltage ratio.
- Twist all incoming line voltages by angle $\alpha$ ($0^\circ \sim \pm 20^\circ$) without changing their magnitudes.

**Figure 12.18a**
Construction of a 3-phase-shift transformer. The incoming terminals are A, B, C; the outgoing terminals are 1, 2, 3.

**Figure 12.18b**
Schematic diagram of the transformer in Fig. 12.18a.
Calculations involving 3-phase transformers

• For any 3-phase transformer bank, assume that the primary and secondary windings are both connected in Y configuration (if not, transform it to Y by $|Z_Y| = |Z_\Delta|/3$)

• Consider only one transformer (single phase) of the assumed Y-Y transformer bank
  – Its primary and secondary voltages are both the line-to-neutral voltages of the incoming and outgoing lines
  – Its nominal power rating and load are 1/3 of the 3-phase transformer bank
Examples 12-1, 12-2, 12-3, 12-6 & 12-7
Homework Assignment #3

• Read Chapters 9&10
• Questions:
  – Prove the polarity test on slide #21
  – 9-4, 9-5, 9-6, 9-7, 10-19, 10-28, 10-29, 10-30, 10-31
• Due date:
  – hand in your solution in the class on 10/5 (Mon) or
  – to Denis at MK 205 or by email (dosipov@vols.utk.edu)
    before the end of 10/5