ECE 325 – Electric Energy System Components
2- Fundamentals of Electrical Circuits

Instructor:

Kai Sun

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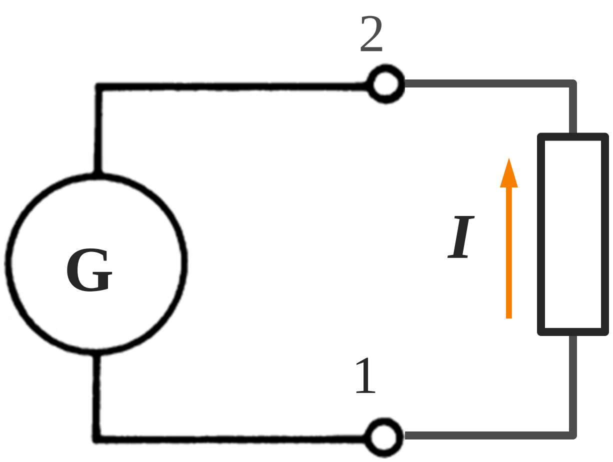
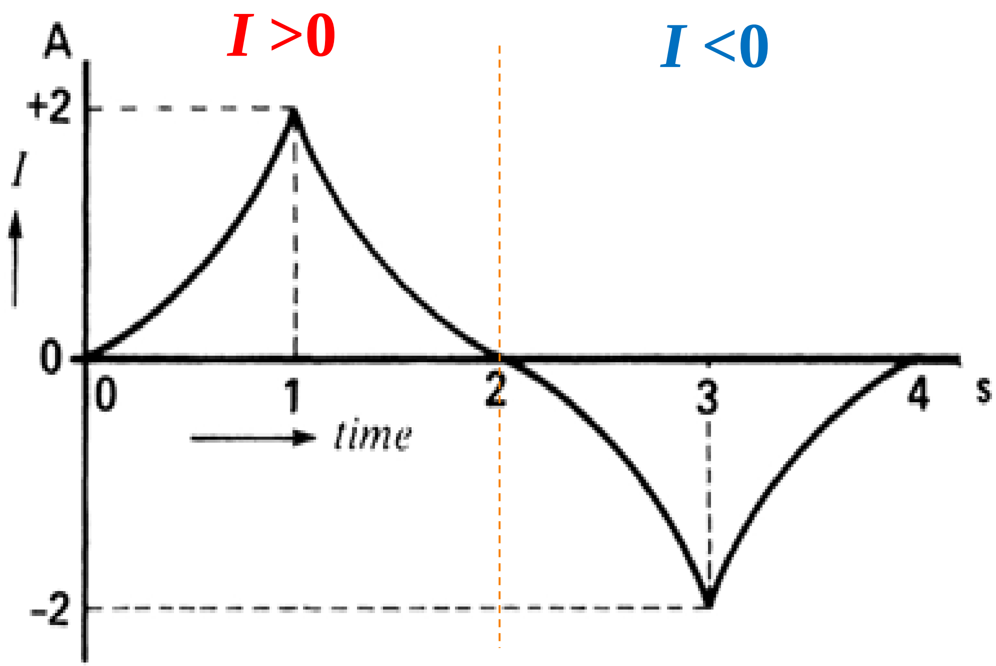
Content

- Fundamentals of electrical circuits (Ch. 2.0-2.15, 2.32-2.39)
- Active power, reactive power and apparent power (Ch. 7)
- Three-phase AC systems (Ch. 8)

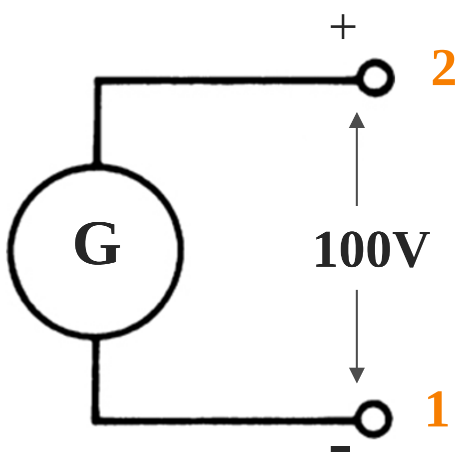
Notations: Current and Alternating Current





- **Arbitrarily** determine a positive direction, e.g. 1→2
  - If a current of 2A flows from 1 to 2, *I*=+2A
  - If a current of 2A flows from 2 to 1, *I*= -2A

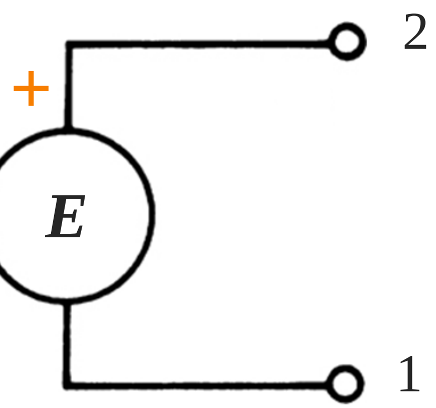
Notations: Voltage



1. **Double-subscript notation**:

   E_{21}= **+100V** (the voltage between 2 and 1 is 100V, and 2 is positive w.r.t 1)

   E_{12} **= -100V**

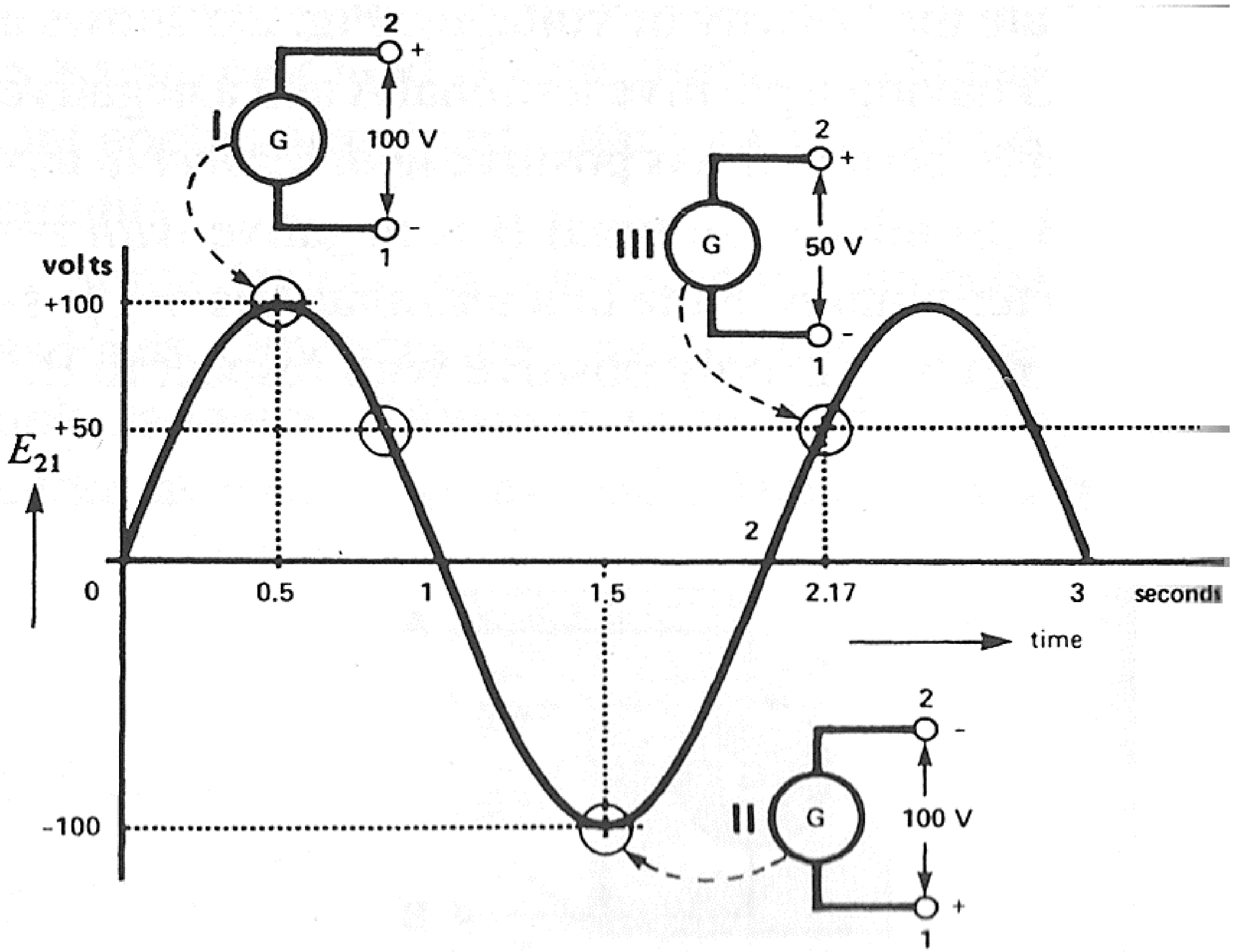


2. **Sign notation**:

   Arbitrarily mark a terminal with (+);
   E>0 if and only if that marked terminal is positive w.r.t the other.
   E.g. if E_{21}=+100V, $E=E_{21}$**=+100V**.

Both the double-subscript notation and sign notation apply to alternating voltage

Notations: Alternating Voltage



Notations: Sources and Loads

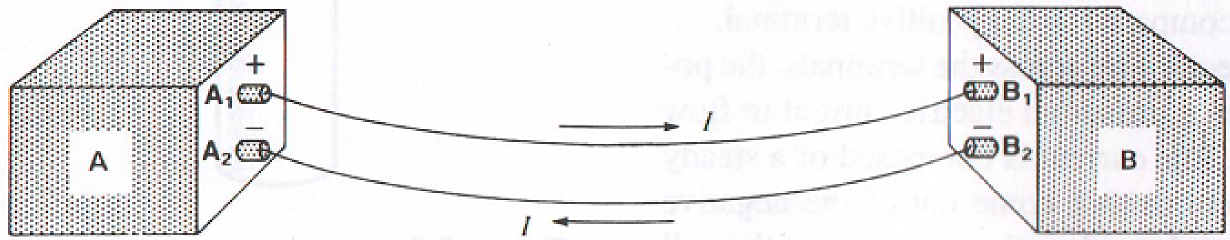
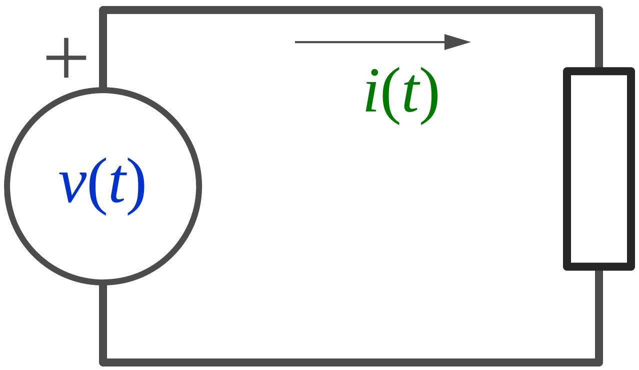
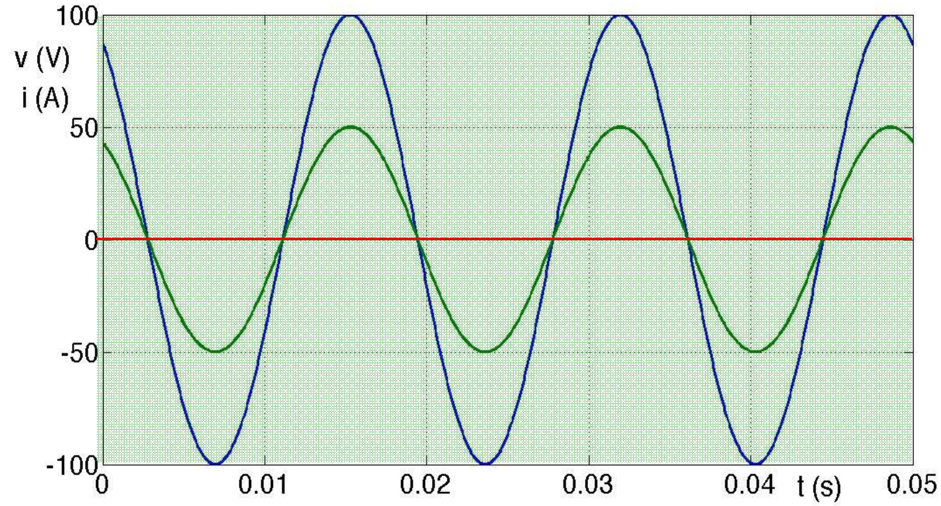


Figure 2.4
Distinction between a source and a load.

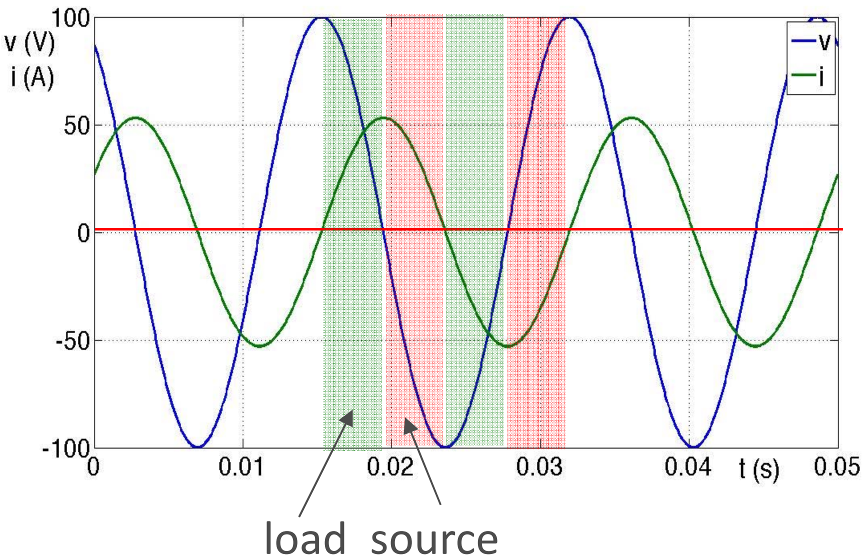
- **Definition**: given the instantaneous, **actual** polarity of voltage and **actual** direction of current
  - – Actual Source: whenever current flows out of the terminal (+)
  - – Actual Load: whenever current flows into the terminal (+)
- How about these?
  - – Resistor, battery cell, electric motor, capacitor and inductor



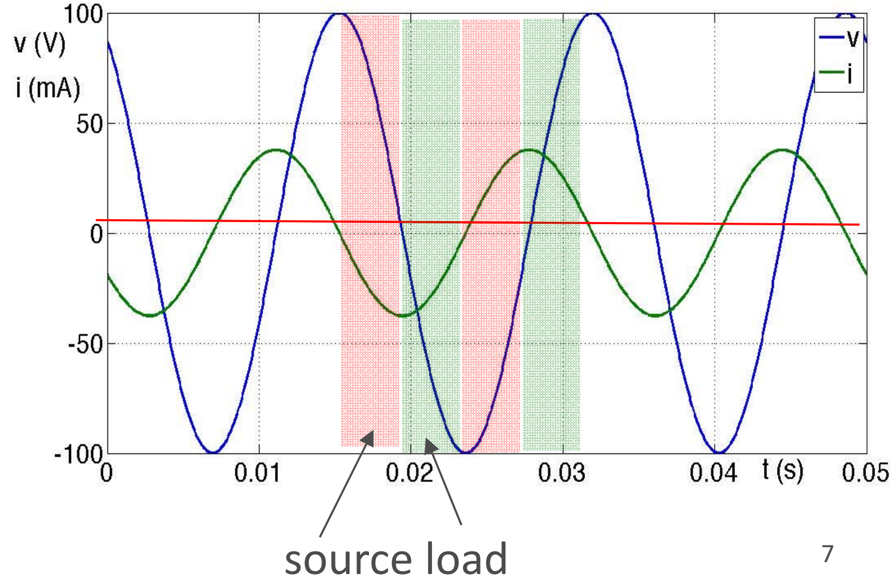
Resistor



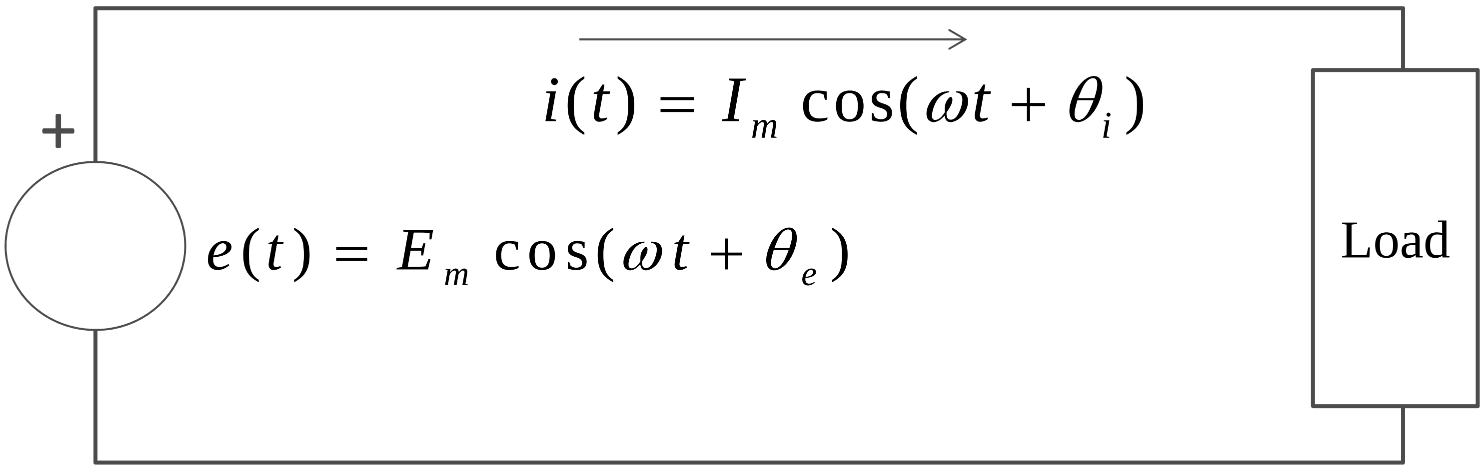
Inductor



Capacitor



1-Phase AC System with Sinusoidal Voltage and Current



- e , i: instantaneous voltage (V) and current (A)
- E_m , I_m : peak values of the sinusoidal voltage (V) and current (A)
- $\omega = 2\pi f$ (rad/s): angular frequency, which is assumed constant here
- θ_e , θ_i : constant phase angles (rad. or deg.) of voltage and current
- $E_m/\sqrt{2}$, $I_m/\sqrt{2}$: RMS (root-mean-square, effective) values

$$\int_{t-T}^{t} [i(t)]^2 R dt = \frac{I_m^2 RT}{2} = I_{dc}^2 RT$$

Equal heating effects

$$\rightarrow I_{dc} = \frac{I_m}{\sqrt{2}} \triangleq \text{RMS value}$$

Phasor Representation

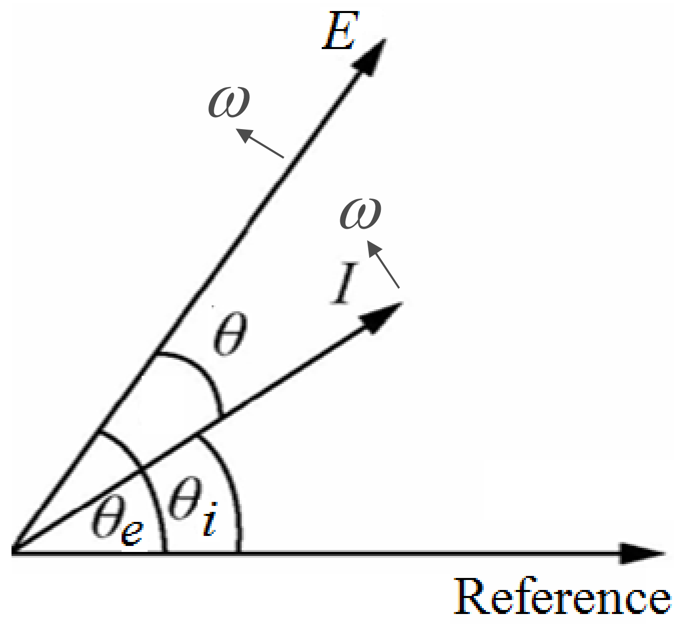
$$e(t) = E_m \cos(\omega t + \theta_e) = \sqrt{2}\,|E|\cos(\omega t + \theta_e)$$

$$i(t) = I_m \cos(\omega t + \theta_i) = \sqrt{2}\,|I|\cos(\omega t + \theta_i)$$

Constant ω

$$E = \frac{E_m}{\sqrt{2}}\angle\theta_e = |E|\angle\theta_e = |E|e^{j\theta_e}$$

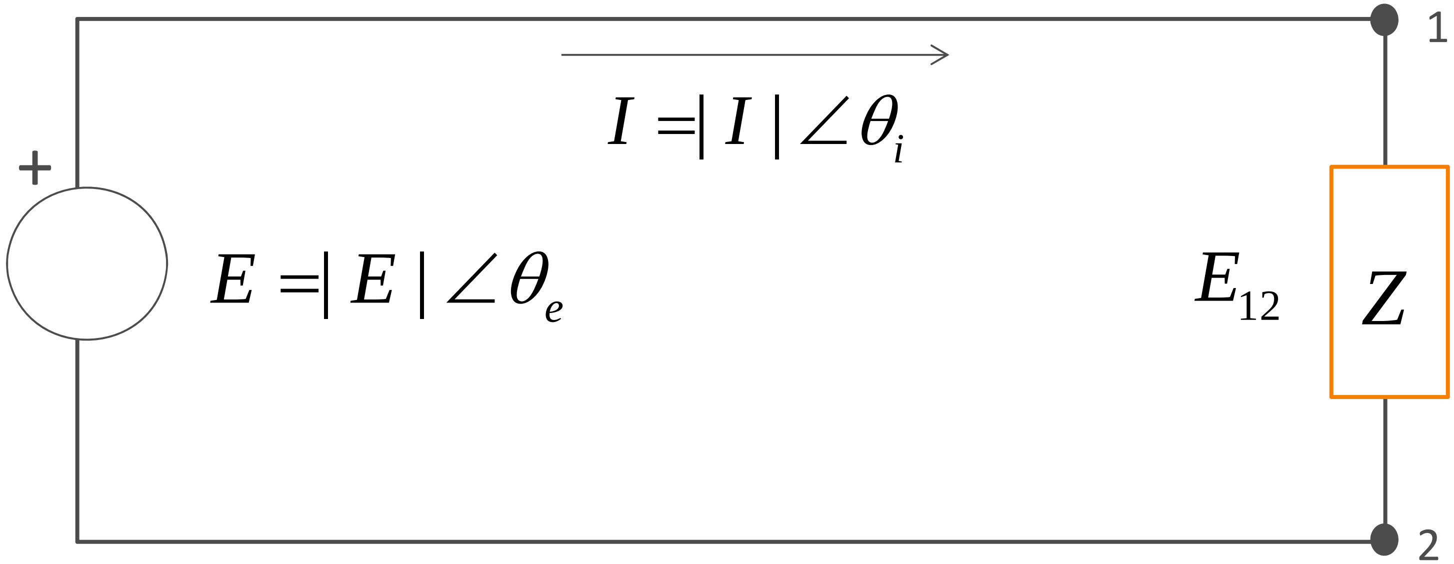
$$I = \frac{I_m}{\sqrt{2}}\angle\theta_i = |I|\angle\theta_i = |I|e^{j\theta_i}$$



- *E* and *I* are called RMS phasors of *e*(*t*) and *i*(*t*);

  E leads *I* by $\theta = \theta_e - \theta_i$ or in other words, *I* leads *E* by 2π-θ
- Phasor:
  - mapping a time-domain sinusoidal waveform (infinitely long in time) to a single **complex number**
  - carries the amplitude and phase angle information of a sinusoidal signal of a common frequency (ω) w.r.t. a chosen reference signal.

Impedance



$$I = |I| \angle \theta_i$$

$$E = |E| \angle \theta_e$$

$$E_{12}$$

$$Z$$

- Impedance is a complex number (in Ω) defined as

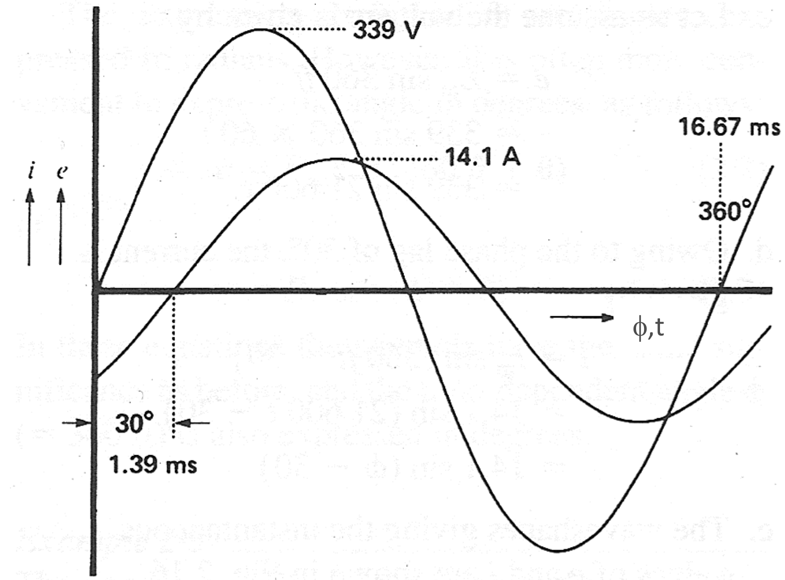
$$Z \overset{\text{def}}{=} \frac{E_{12}}{I} = \frac{E}{I} = \frac{|E| \angle \theta_e}{|I| \angle \theta_i} = |Z| \angle (\underline{\theta_e - \theta_i}) = R + jX$$

$$\theta_e - \theta_i \overset{\text{def}}{=} \theta \quad \text{(Impedance angle)}$$

Impedance of branch **1-2** = $\frac{\text{Voltage drop on } \mathbf{1\text{-}2}}{\text{Current flow into } \mathbf{1}}$

- Purely resistive: $Z = |Z| = R$
- Purely inductive: $Z = |Z| \angle 90^\circ = jX = jX_L = j\omega L$
- Purely capacitive: $Z = |Z| \angle -90^\circ = jX = -jX_C = -j\frac{1}{\omega C}$

Example 2-5



$$e(t) = 339\cos(21600t - 90^\circ)$$

$$i(t) = 14.1\cos(21600t - 120^\circ)$$

- Draw the phasor diagram of the voltage and current at a frequency of 60Hz. Calculate the time interval Δt between the positive peaks of E and I

Solution:

$\omega = 2\pi f = 377$ (rad/s)=21600 (deg/s)

$|E| = 339/\sqrt{2} = 240$ (V)

$|I| = 14.1/\sqrt{2} = 10$ (A)

Choose an arbitrary reference to draw phasors E and I

$\Delta t = \theta/\omega = 30/21600 = 0.00139$ (s)