

Kirchhoff's Voltage Law (KVL)

- The algebraic sum of the voltages around a closed loop (CW/CCW) is zero (Σ voltage rises = Σ voltage drops)
 - Applied to both instantaneous voltages or voltage phasors
- Loop 24312 (BCDA, CW):

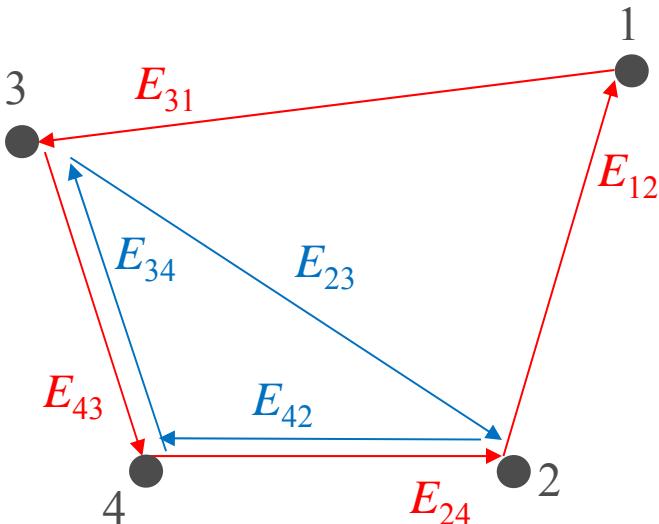
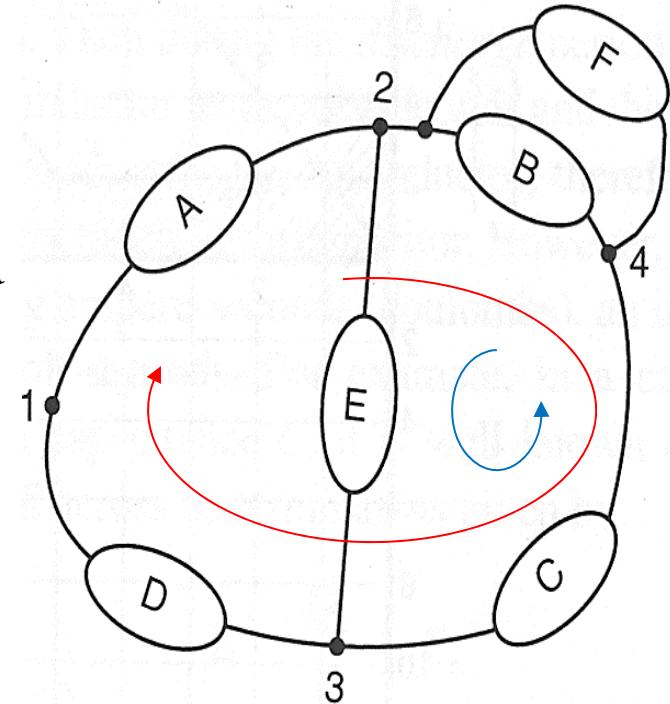
$$E_{24} + E_{43} + E_{31} + E_{12} = 0 \text{ or}$$

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- Loop 2342 (ECF, CCW):

$$E_{23} + E_{34} + E_{42} = 0 \text{ or}$$

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Kirchhoff's Current Law (KCL)

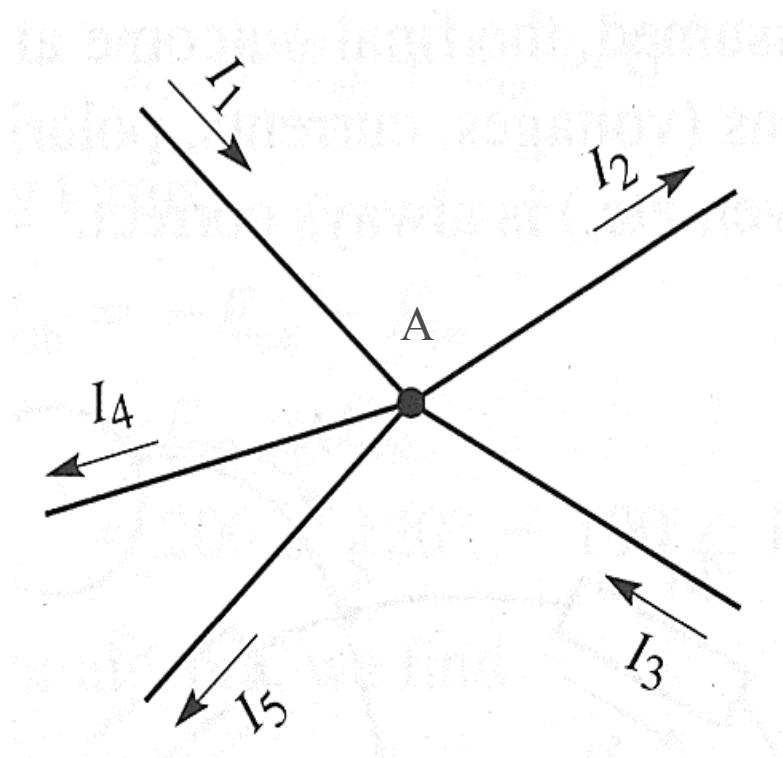
- The algebraic sum of the currents **arriving at** a node is equal to 0.
(Σ currents in = Σ currents out)
- Node A:

$$I_1 + I_3 = I_2 + I_4 + I_5$$

$$\text{or } i_1 + i_3 = i_2 + i_4 + i_5$$

$$I_1 + I_3 + (-I_2) + (-I_4) + (-I_5) = 0$$

$$\text{or } i_1 + i_3 + (-i_2) + (-i_4) + (-i_5) = 0$$



Kirchhoff's Laws and AC Circuits

- KVL

- Loop 24312, CW:

$$E_{24} + E_{43} + E_{31} + E_{12} = 0$$

$$I_2 Z_2 - I_3 Z_3 + E_b - I_1 Z_1 = 0$$

- Loop 2342, CCW:

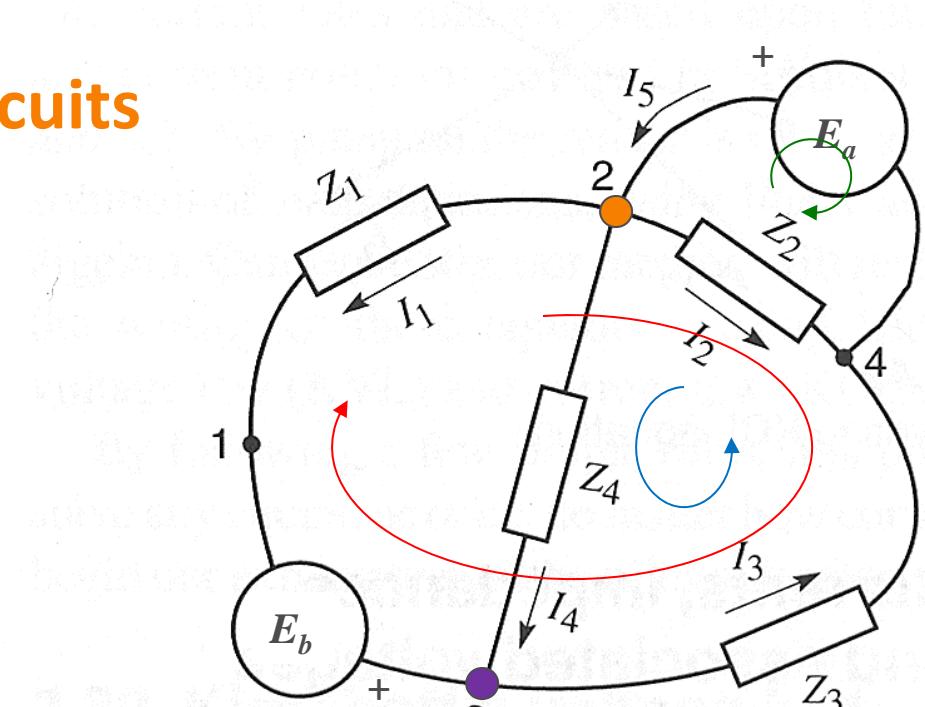
$$E_{23} + E_{34} + E_{42} = 0$$

$$I_4 Z_4 + I_3 Z_3 - I_2 Z_2 = 0$$

- Loop 242, CW:

$$E_{24} + E_{42} = 0$$

$$E_a - I_2 Z_2 = 0$$



- KCL

- Node 2:

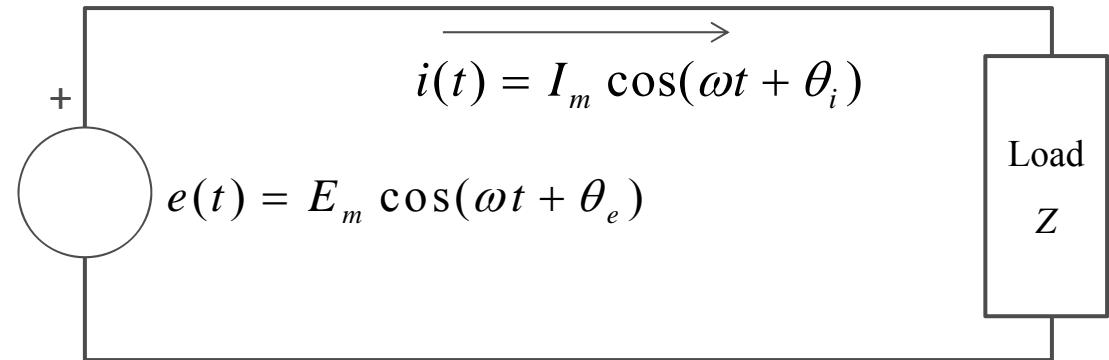
$$I_5 - I_2 - I_4 - I_1 = 0$$

- Node 3:

$$I_4 + I_1 - I_3 = 0$$

Examples 2-14 ~ 2-16

Instantaneous Power



$$\phi = \omega t$$

$$p(t) = e(t)i(t) = E_m I_m \cos(\phi + \theta_e) \cos(\phi + \theta_i)$$

Using trigonometric identity

$$\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$$

Impedance angle: $\theta = \theta_e - \theta_i$
 >0 for inductive load and
 <0 for capacitive load

$$\begin{aligned} p(t) &= E_m I_m [\cos(\theta_e - \theta_i) + \cos(2\phi + \theta_e + \theta_i)] \\ &= \frac{1}{2} E_m I_m \{\cos(\theta_e - \theta_i) + \cos[2(\phi + \theta_e) - (\theta_e - \theta_i)]\} \\ &= \frac{1}{2} E_m I_m \{\cos \theta + \cos[2(\phi + \theta_e) - \theta]\} \\ &= \frac{1}{2} E_m I_m [\cos \theta + \cos \theta \cos 2(\phi + \theta_e) + \sin \theta \sin 2(\phi + \theta_e)] \end{aligned}$$

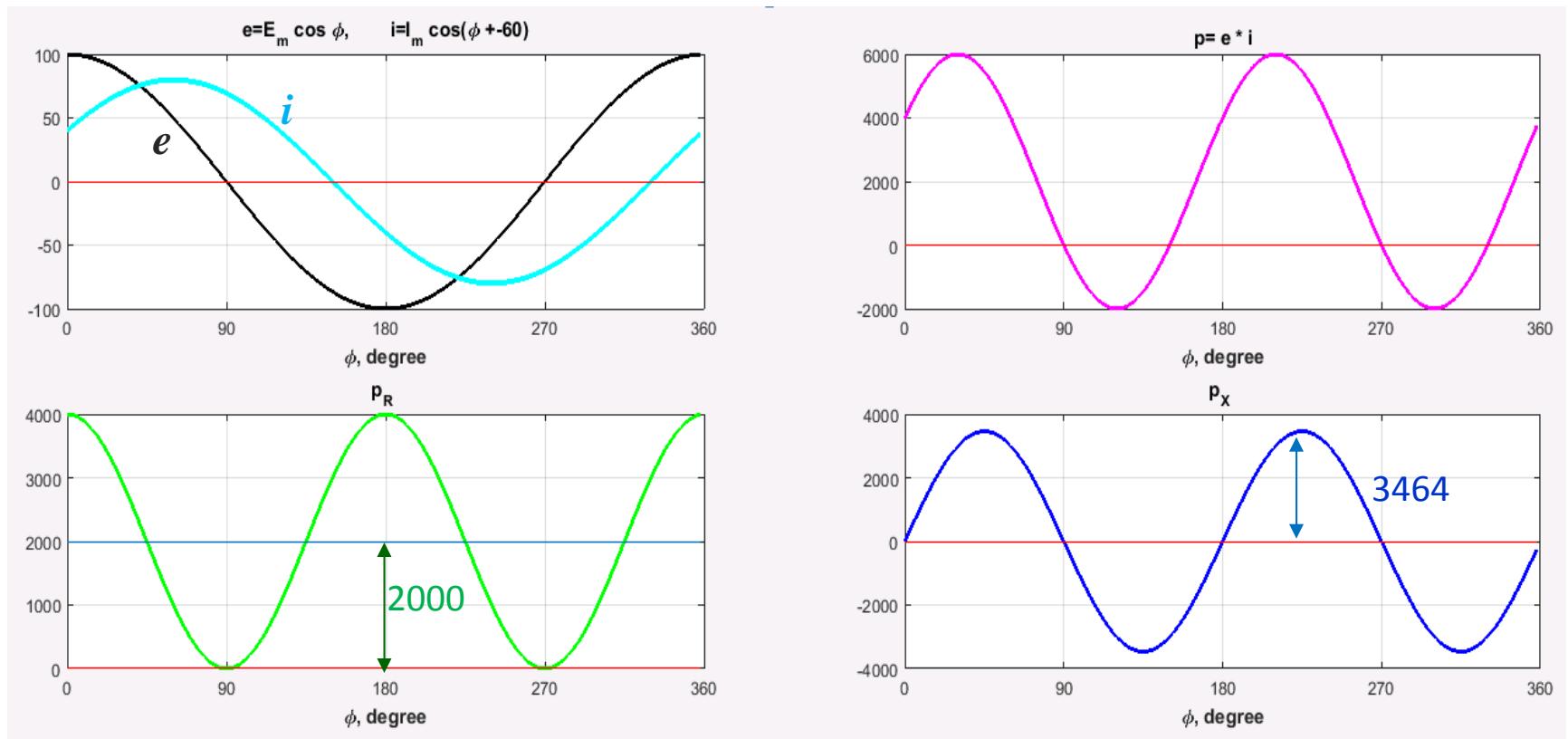
$$\begin{aligned} |E| &= E_m / \sqrt{2} \\ |I| &= I_m / \sqrt{2} \end{aligned}$$

$$p(\phi) = \underbrace{|E| \cdot |I| \cos \theta [1 + \cos 2(\phi + \theta_e)]}_{P_R} + \underbrace{|E| \cdot |I| \sin \theta \sin 2(\phi + \theta_e)}_{P_X}$$

Example: $e = 100 \cos \phi$ $E = \frac{100}{\sqrt{2}} \angle 0^\circ$
 $i = 80 \cos(\phi - 60^\circ)$ $I = \frac{80}{\sqrt{2}} \angle -60^\circ$

$$p(\phi) = \underbrace{|E| \cdot |I| \cos \theta [1 + \cos 2(\phi + \theta_e)]}_{p_R} + \underbrace{|E| \cdot |I| \sin \theta \sin 2(\phi + \theta_e)}_{p_X}$$

$$= \underbrace{2000(1 + \cos 2\phi)}_{p_R} + \underbrace{3464 \sin 2\phi}_{p_X}$$



$$\begin{aligned}
p(t) &= \underbrace{|E| \cdot |I| \cos \theta [1 + \cos 2(\phi + \theta_e)]}_{p_R(t)} + \underbrace{|E| \cdot |I| \sin \theta \sin 2(\phi + \theta_e)}_{p_X(t)} \\
&= \underbrace{2000(1 + \cos 2\phi)}_{p_R(t)} + \underbrace{3464 \sin 2\phi}_{p_X(t)}
\end{aligned}$$

Observations:

- The frequency of p , p_R and p_X is twice of e and i , which is $60 \times 2 = 120$ Hz
- $p_R(t)$
 - changes between 0 and 4000 ($= 2|E| \cdot |I| \cos \theta$)
 - always ≥ 0 , and has an average value of **2000** ($= |E| \cdot |I| \cos \theta$)
 - is the power consumed by the load
- $p_X(t)$
 - changes between **± 3464** ($= \pm |E| \cdot |I| \sin \theta$)
 - has an average value of 0
 - is the power borrowed & returned by the load.

Apparent, Active (Real) and Reactive Powers

$$p = \underbrace{[E \cdot |I| \cos \theta [1 + \cos 2(\phi + \theta_e)]]}_{p_R(t)} + \underbrace{[E \cdot |I| \sin \theta \sin 2(\phi + \theta_e)]}_{p_X(t)}$$

$$= P[1 + \cos 2(\phi + \theta_e)] + Q \sin 2(\phi + \theta_e)$$

$$|S| = |E| \cdot |I| \stackrel{\text{def}}{=}$$

- Apparent power
- Unit ~ volt ampere or VA (kVA or MVA)

$$P = |E| \cdot |I| \cos \theta \stackrel{\text{def}}{=}$$

- Real or active power (average power)
- Unit ~ watt or W
- Power factor (PF): $\cos \theta = \cos(\theta_e - \theta_i)$
 - The PF is said to be lagging if $\theta = \theta_e - \theta_i > 0$ or leading if $\theta < 0$

$$Q = |E| \cdot |I| \sin \theta \stackrel{\text{def}}{=}$$

- Reactive power
- Unit ~ var (volt-ampere reactive). Some people use Var, VAR or VAr
- $Q > 0$ if $\theta = \theta_e - \theta_i > 0$ (inductive) or $Q < 0$ if $\theta < 0$ (capacitive)

