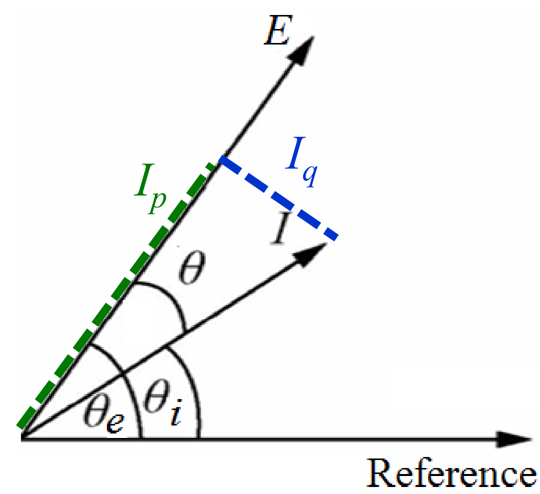
Complex Power and Power Triangle

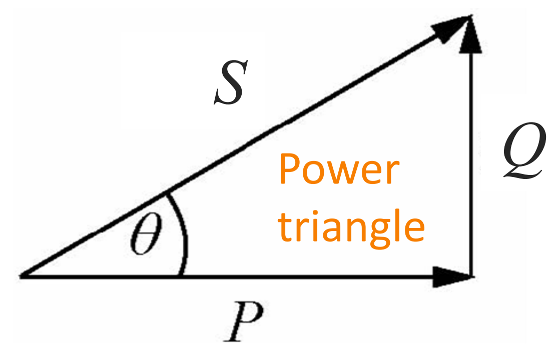
$$S \overset{\text{def}}{=} P + jQ \;= |E||I|\cos\theta + j|E||I|\sin\theta \;= |E|I_p + j|E|I_q$$

$$= |E||I|\angle\theta = |S|\angle\theta = \sqrt{P^2+Q^2}\angle\theta$$

$$= |E||I|\angle\theta_e - \theta_i = |E|\angle\theta_e\,|I|\angle-\theta_i = EI^*$$



- $\theta = \tan^{-1}(Q/P)$
  - If $\theta > 0$ ($\theta_i < \theta_e$, i.e. lagging PF), then $Q > 0$, which means **a reactive load** (inductive)
  - If $\theta < 0$ ($\theta_i > \theta_e$, i.e. leading PF), then $Q < 0$, which means **a reactive source** (capacitive)



- $PF = \cos\theta = P/|S|$ (+/- sign of Q tells a lagging/leading PF)

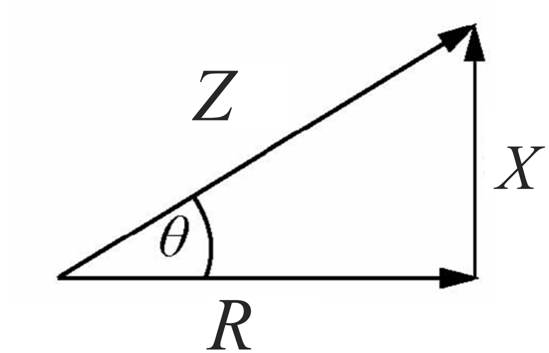
  $P = |S| \cdot \cos\theta = |S| \cdot PF$

- If the load impedance is $Z = E/I$, then

  $S = ZII^* = Z|I|^2 = R|I|^2 + jX|I|^2 = P + jQ$

  $P = R|I|^2$ $\qquad$ $Q = X|I|^2$

  θ is also called the load impedance angle



Other Useful Formulas

$$S = EI^* = \frac{EE^*}{Z^*} = \frac{|E|^2}{Z^*}$$
$$= II^* Z = |I|^2 Z$$

$$Z = \frac{|E|^2}{S^*} = \frac{S}{|I|^2}$$

- If Z is purely resistive

$$P = \frac{|E|^2}{R}$$

- If Z is purely reactive

$$Q = \frac{|E|^2}{X} = \frac{|E|^2}{\omega L} \text{ or } \frac{|E|^2}{-1/\omega C}$$

Theorem of Conservation of Complex Power

- For a power network supplied by independent sources all at the same frequency (all voltages and currents are assumed to be sinusoids), the sum of the complex power supplied by the independent sources equals the sum of the complex power received by all the other branches of the network

$$\Sigma(S_{\mathrm{G}} \text{ of all actual sources}) = \Sigma(S_{\mathrm{L}} \text{ of all the others})$$

- In other words, the total complex power supplied (received) by **all branches** is zero

$$\Sigma(S_{\mathrm{G}} \text{ of all braches}) = 0 \text{ or } \Sigma(S_{\mathrm{L}} \text{ of all braches}) = 0$$

  - For a single source with elements in series or parallel, it is proved by Kirchhoff's voltage or current law (KVL/KCL)
  - For a general case, it is proved by Tellegen's theorem.
  - Application of the theorem: any part of the power network can be replaced by an equivalent independent source

- Two examples from A. Bergen and V. Vittal, Power System Analysis, Prentice Hall, 2000

Example 2.4

For the circuit shown in Figure E2.4, find S_2 in terms of S_1, C, and V.

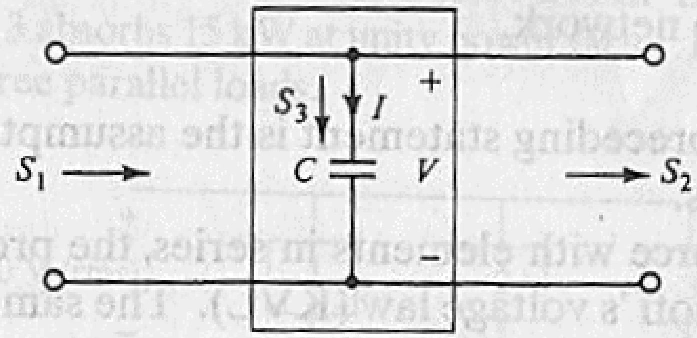


Figure E2.4

Solution Using the theorem of conservation of complex power, we have

$$S_1 - S_2 = S_3$$

where

$$S_3 = VI^* = VY^*V^* = -j\omega C|V|^2$$

Thus

$$S_2 = S_1 + j\omega C|V|^2$$

$$P_2 = P_1$$

$$Q_2 = Q_1 + \omega C|V|^2$$

We note that $Q_2 > Q_1$; thus, it is reasonable and convenient to consider C as a source of reactive power!

Example 2.5

Consider the circuit shown in Figure E2.5. Assume that $|V_2| = |V_1|$ and show that $S_2 = -S_1^*$.

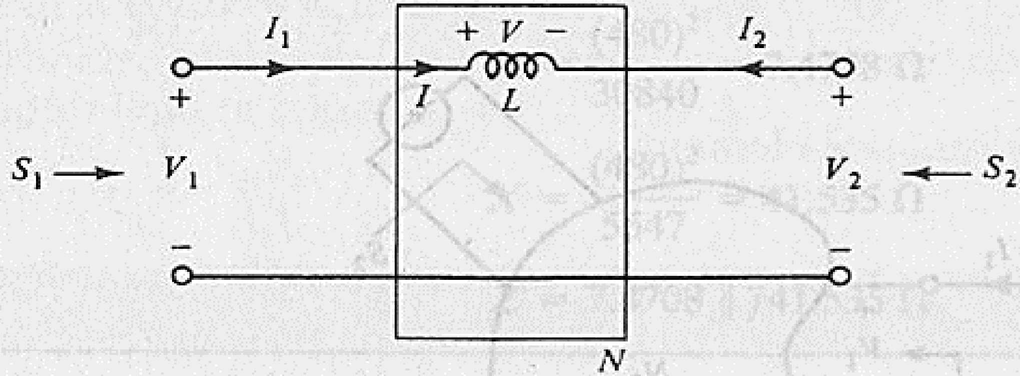


Figure E2.5

Solution Using the theorem of complex power, the complex power into the inductor is

$$S_1 + S_2 = VI^* = j\omega L|I|^2$$

Thus

$$P_1 + P_2 = 0$$
$$Q_1 + Q_2 = \omega L|I|^2$$

Now using $|V_2| = |V_1|$, we find

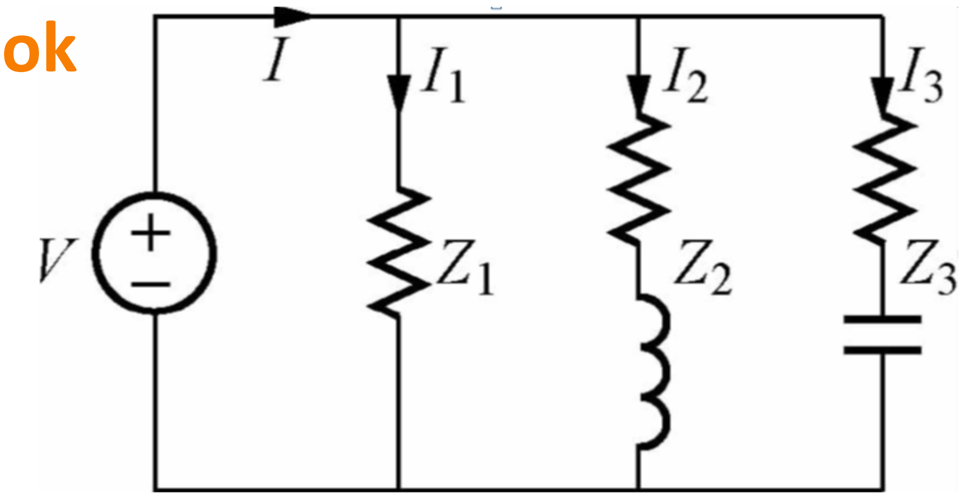
$$\left.\begin{aligned} S_1 &= V_1 I^* \\ S_2 &= -V_2 I^* \end{aligned}\right\} \Rightarrow |S_1| = |S_2| \Rightarrow P_1^2 + Q_1^2 = P_2^2 + Q_2^2$$

But since $|P_2| = |P_1|$, we get $|Q_2| = |Q_1|$. As a consequence, $Q_1 = Q_2 = \frac{1}{2}\omega L|I|^2$. We have finally

$$\left.\begin{aligned} P_2 &= -P_1 \\ Q_2 &= Q_1 \end{aligned}\right\} \Rightarrow S_2 = -S_1^*$$

Examples 7-2 & 7-3

Example 2.2 on Saadat's book



$$V = 1200\angle 0^\circ \text{ V}, \quad Z_1 = 60 + j0\Omega,$$
$$Z_2 = 6 + j12\Omega, \quad Z_3 = 30 - j30\Omega$$

Find the power absorbed by each load and the total complex power

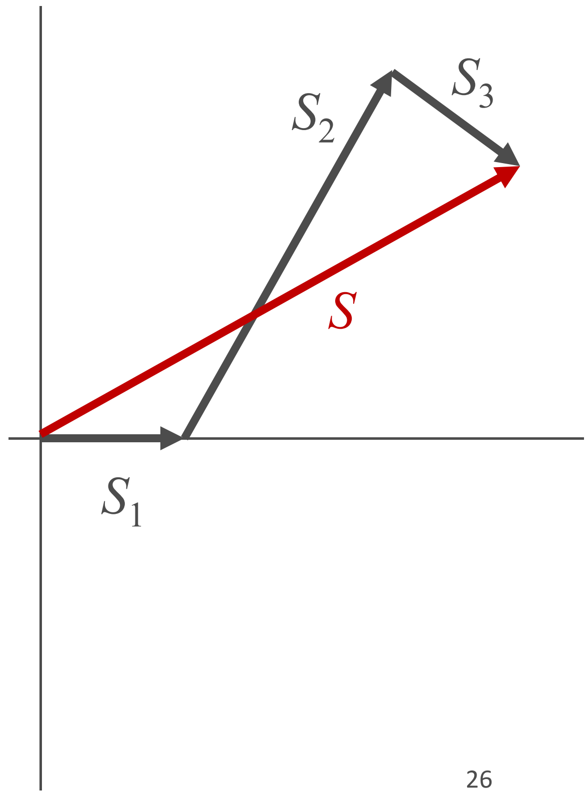
$$I_1 = \frac{V}{Z_1} = \frac{1200\angle 0^\circ}{60 + j0} = \frac{1200}{60} = 20 + j0 \text{ A}$$

$$I_2 = \frac{V}{Z_2} = \frac{1200\angle 0^\circ}{6 + j12} = \frac{200}{1 + j2} = \frac{200(1 - j2)}{5} = 40 - j80 \text{ A}$$

$$I_3 = \frac{V}{Z_3} = \frac{1200\angle 0^\circ}{30 - j30} = \frac{40}{1 - j} = \frac{40(1 + j)}{2} = 20 + j20 \text{ A}$$

$$S_1 = VI_1^* = 1200\angle 0^\circ (20 - j0) = 24{,}000 \text{ W} + j0 \text{ var}$$
$$S_2 = VI_2^* = 1200\angle 0^\circ (40 + j80) = 48{,}000 \text{ W} + j96{,}000 \text{ var}$$
$$S_3 = VI_3^* = 1200\angle 0^\circ (20 - j20) = 24{,}000 \text{ W} - j24{,}000 \text{ var}$$

$$S = S_1 + S_2 + S_3 = 96{,}000 \text{ W} + j72{,}000 \text{ var}$$



- Other approaches

$$I = I_1 + I_2 + I_3 = (20 + j0) + (40 - j80) + (20 + j20) = 80 - j60 = 100\angle -36.87° \text{ A}$$

$$S = VI^* = (1200\angle 0°)(100\angle 36.87°) = 120,000\angle 36.87° \text{ VA} = 96,000 \text{ W} + j72,000 \text{ var}$$

or

$$Z_{//} = Z_1 // Z_2 // Z_3 = \frac{1}{1/Z_1 + 1/Z_2 + 1/Z_3} = 9.6 + j7.2\Omega$$

$$S = \frac{|V|^2}{Z_{//}^*} = \frac{(1200)^2}{9.6 - j7.2} = 96,000 \text{ W} + j72,000 \text{ var}$$

$$S_1 = \frac{|V|^2}{Z_1^*} = \frac{(1200)^2}{60} = 24,000 \text{ W} + j0 \text{ var}$$

$$S_2 = \frac{|V|^2}{Z_2^*} = \frac{(1200)^2}{6 - j12} = 48,000 \text{ W} + j96,000 \text{ var}$$

$$S_3 = \frac{|V|^2}{Z_3^*} = \frac{(1200)^2}{30 + j30} = 24,000 \text{ W} - j24,000 \text{ var}$$