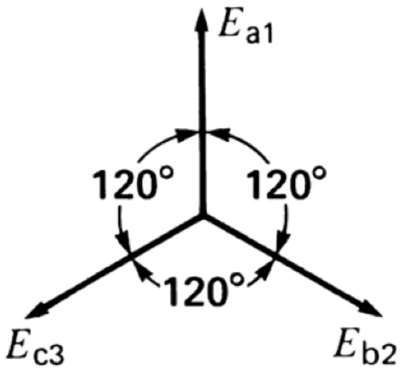
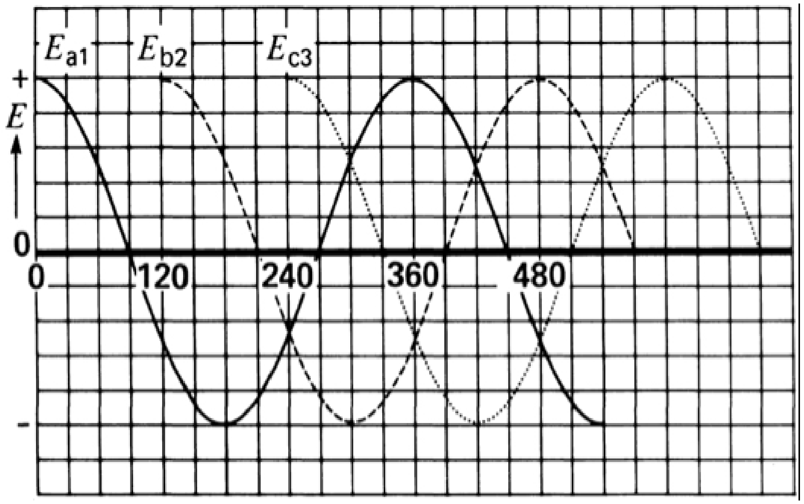
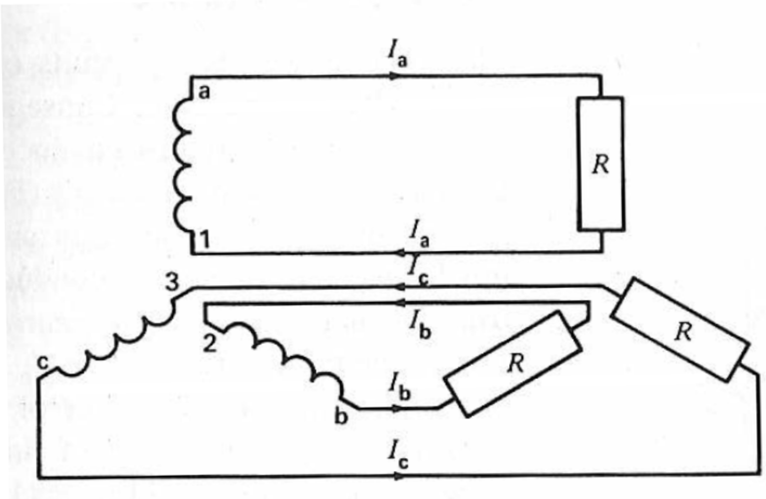
Three-Phase Systems

- Balanced 3-phase voltage:
- Sequence: a-b-c (CW).
- Phase voltages are displaced at 120° (a leads b, b leads c, and c leads a by 120°, respectively)
- Equal voltage magnitudes

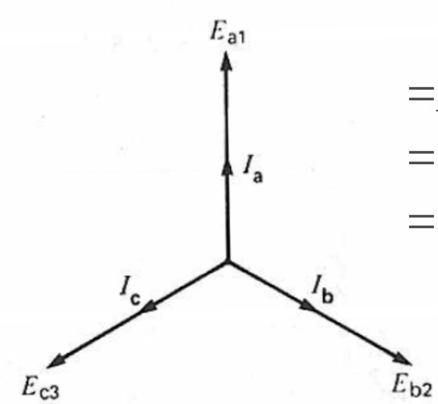




3-Phase, 4-Wire AC System

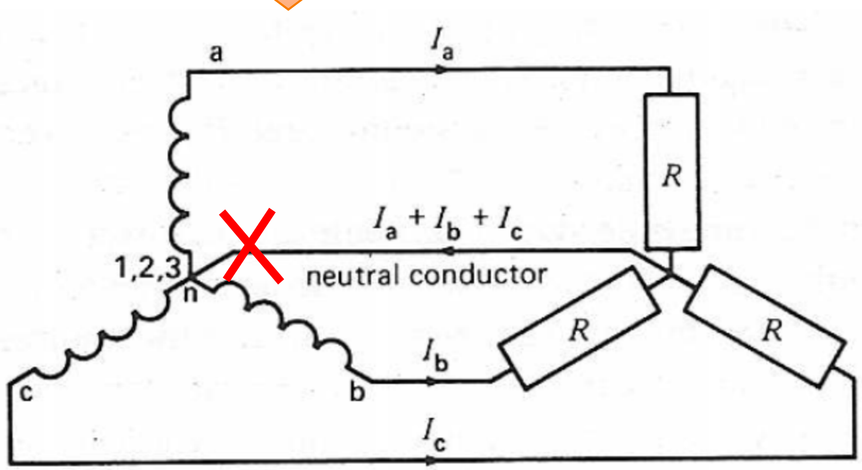


(a)

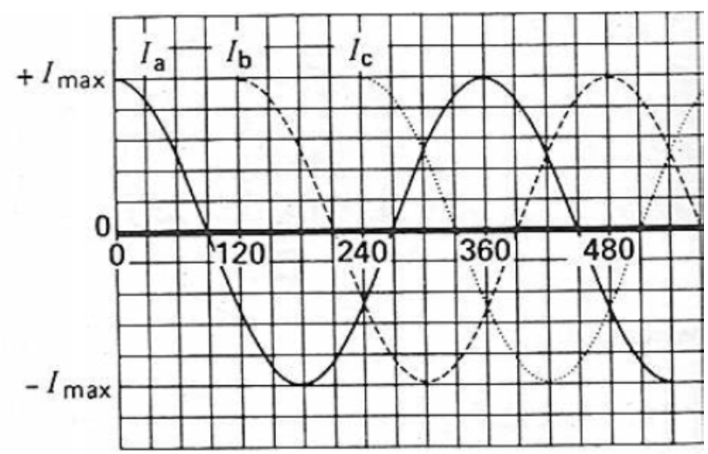


(b)

$$I_a + I_b + I_c$$
$$= E_{a1}/R + E_{b2}/R + E_{c3}/R$$
$$= (E_{a1} + E_{b2} + E_{c3})/R$$
$$= 0$$



(a)



(b)

3-Phase, 3-Wire AC System (balanced)

- Line currents:

  $|I_a|=|I_b|=|I_c|= I_L$

- Phase voltages (phase-to-neutral or line-to-neutral):

  Nodes 1, 2, 3 $\rightarrow$ Node n

  $|E_{an}|=|E_{bn}|=|E_{cn}|= E_{LN}$

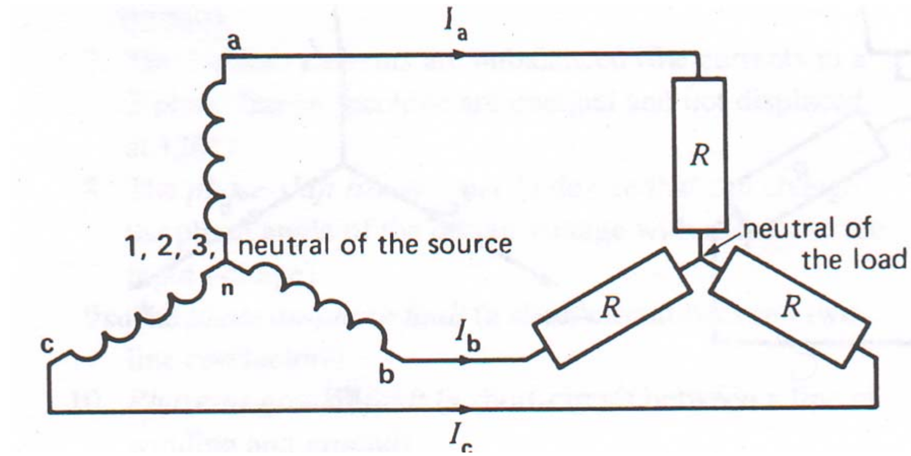
- From KVL, line voltages (line-to-line or phase-to-phase):

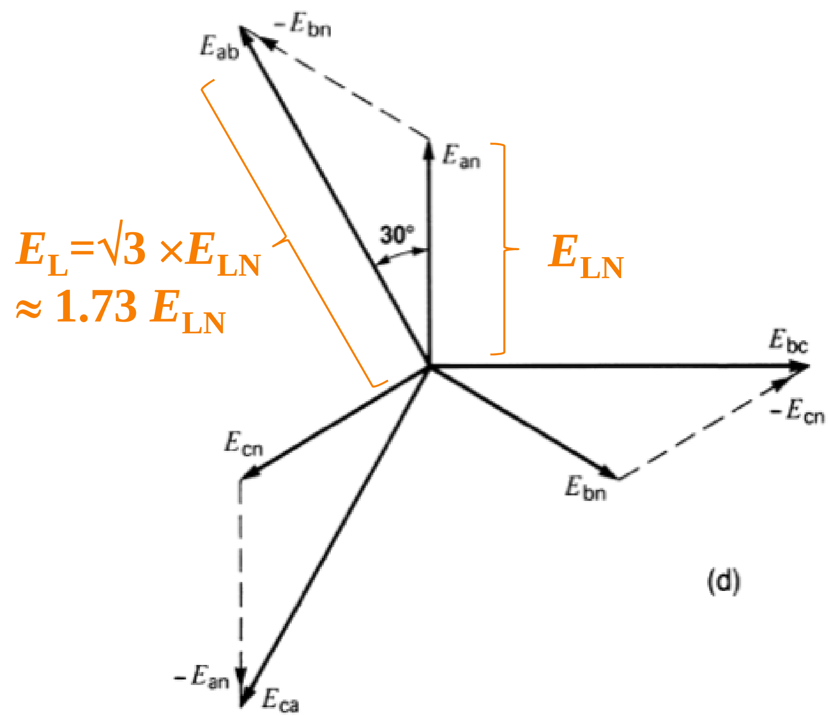
  $E_{ab}=E_{an}+E_{nb}=E_{an}-E_{bn}$

  $E_{bc}=E_{bn}+E_{nc}=E_{bn}-E_{cn}$

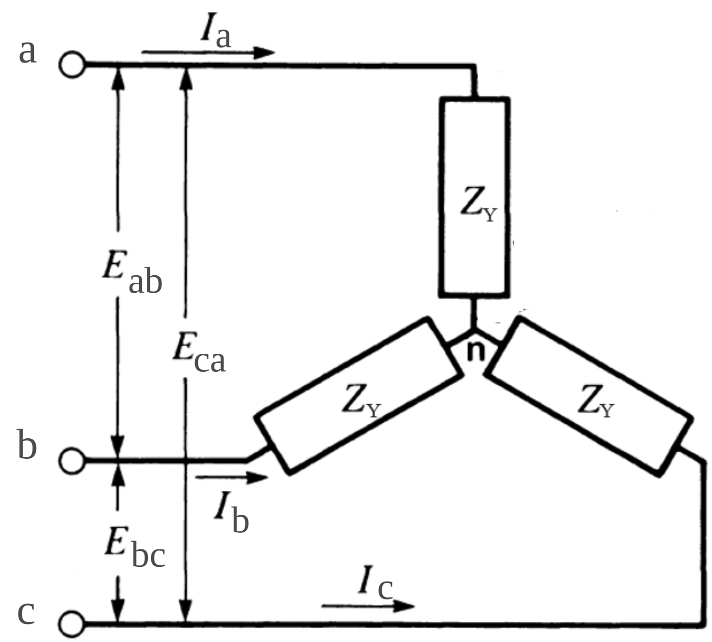
  $E_{ca}=E_{cn}+E_{na}=E_{cn}-E_{an}$

  $|E_{ab}|=|E_{bc}|=|E_{ca}|= E_L$





Wye (Y) Connection



- Each line current (I_a, I_b and I_c) equals the phase current.

$$I_L = I_Z$$

- Each line voltage is $\sqrt{3}=1.73$ times of a phase voltage in magnitude

$$E_L=\sqrt{3} \times E_{LN} \quad (\ |E_{ab}| = \sqrt{3}\ |E_{an}| \ )$$

- Apparent power of each phase:

$$|S_Z| = E_{LN} I_Z = \frac{E_L}{\sqrt{3}} I_L$$

- Total 3-phase apparent, active and reactive power:

$$|S_{3\phi}| = 3\frac{E_L}{\sqrt{3}} I_L = \sqrt{3} E_L I_L = \frac{E_L^2}{|Z_Y|} = 3 I_L^2 |Z_Y|$$

$$P_{3\phi} = \sqrt{3} E_L I_L \cos\theta \qquad Z_Y = |Z_Y| \angle\theta$$

θ is the power factor angle and load impedance angle

$$Q_{3\phi} = \sqrt{3} E_L I_L \sin\theta$$

Delta (Δ) Connection

- From KCL:

$$I_a = I_{ab} - I_{ca}$$
$$I_b = I_{bc} - I_{ab}$$
$$I_c = I_{ca} - I_{bc}$$

Because three equations are symmetric, and I_a leads I_b, I_b leads I_c and I_c leads I_a all by 120^o, we may easily conclude:

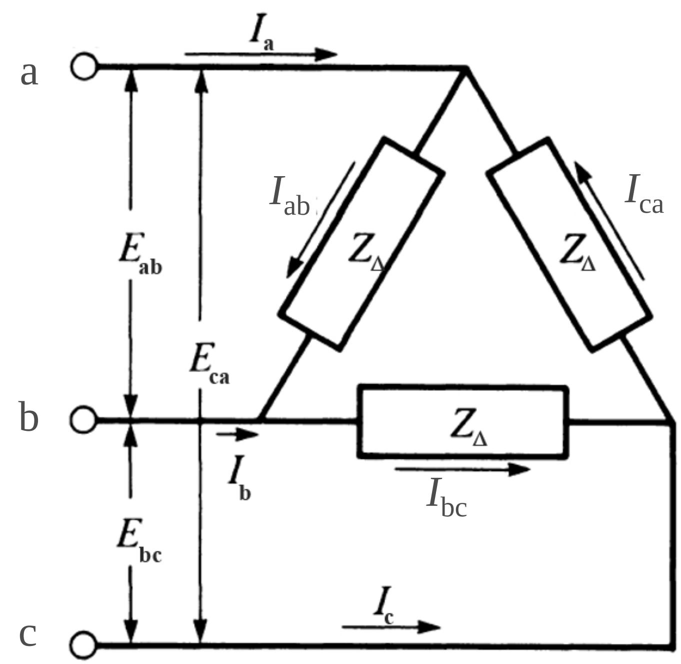
1. $|I_{ab}|=|I_{bc}|=|I_{ca}|=I_Z$
2. I_{ab} leads I_{bc}, I_{bc} leads I_{ca}, and I_{ca} leads I_{ab} by 120^o, respectively

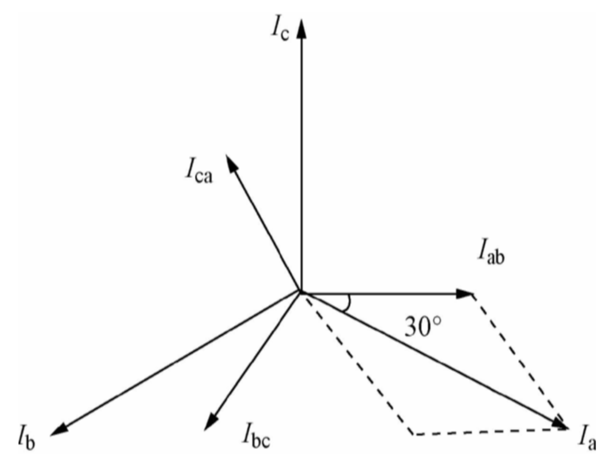
- Each line current is √3=1.73 times of a phase current in magnitude

$$\boldsymbol{I_L=\sqrt{3}\times I_Z} \quad (\ |I_a|= \sqrt{3}\ |I_{ab}| \ )$$

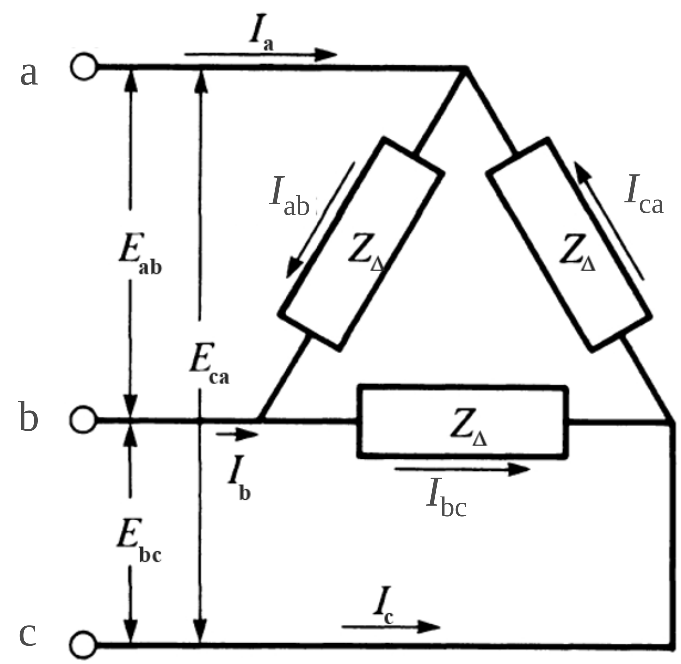
- Each phase voltage equals the line voltage

$$\boldsymbol{E_{LN} = E_L}$$





Delta (Δ) Connection



- Apparent power of each phase:

$$|S_Z| = E_L I_Z = E_L \frac{I_L}{\sqrt{3}}$$

- Total 3-phase apparent, active and reactive power:

$$|S_{3\phi}| = 3E_L \frac{I_L}{\sqrt{3}} = \sqrt{3}E_L I_L = \frac{3E_L^2}{|Z_\Delta|} = I_L^2 |Z_\Delta|$$

$|Z_Y|=|Z_\Delta|/3$

$$P_{3\phi} = \sqrt{3}E_L I_L \cos\theta$$

$$Z_\Delta = |Z_\Delta| \angle\theta$$

θ is the power factor angle and load impedance angle

$$Q_{3\phi} = \sqrt{3}E_L I_L \sin\theta$$

- Given line voltage E_L, line current I_L and power factor $\cos\theta$, calculation of power is independent of the connection (Y / Δ)

Examples 8-8 & 8-11

Summary

- Important questions: Exp 2-16, Prob 2-26, Exp 7-2&7-3, Prob 7-11, Exp 8-11
- $jX_L = j\omega L$, $jX_C = -1j/(\omega C)$
- KVL: $E_{14}=E_{12}+E_{23}+E_{34}$ or $E_{41}+E_{12}+E_{23}+E_{34}=0$
- How many KVL & KCL are needed for N-node B-branch network?
  - Keep adding new KVL and KCL equations until no new *E* or *I* can be introduced (# of KCLs: N-1; # of KVLs: B-N+1)
- Source or Load?
  1. Treat the branch with the (+) terminal receiving *I* as the load for calculation of power $S=P+jQ$
  2. $P>0 \rightarrow$ active load; $P<0 \rightarrow$ active source;
     $Q>0 \rightarrow$ reactive load = inductive load;
     $Q<0 \rightarrow$ reactive source = capacitive load
- Do calculations with all complex numbers to avoid confusion among RMS, phasor, peak and apparent values (Saadat's Example 2.2).
  - Most basic formulas $S=EI^*$ and $E=IZ$
  - Understand the power triangle for load ($S=P+jQ$, $Z=R+jX$, E leading I by θ)
- Three-phase system: here to put $\sqrt{3}$