

# **ECE 325 – Electric Energy System Components**

## **3- Fundamentals of Electromagnetism and Rotational Motion**

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Fall 2016

# Content

- Fundamentals of electromagnetism (Ch. 2.16-2.31)
- Fundamentals of rotational motion (Ch. 3.0-3.14)

# Magnetic Field Strength (Intensity) – $H$

- **Ampere's Circuital Law** states the relationship between the integrated magnetic field around a closed loop and the electric current passing through the loop.

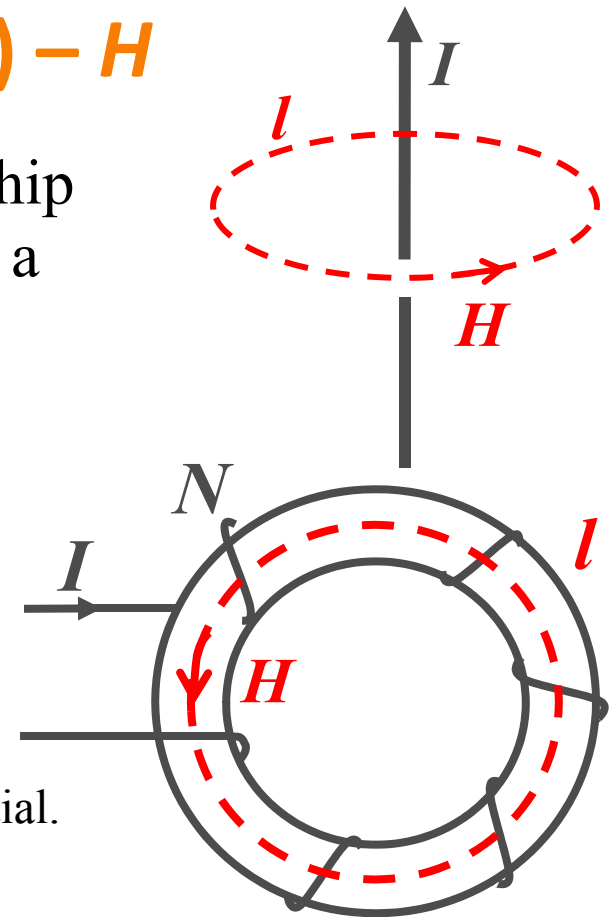
$$F_m = N \cdot I = \oint \vec{H} \cdot d\vec{l} = H \cdot l$$

$F_m$ : magnetomotive force (mmf) in A or A·turns

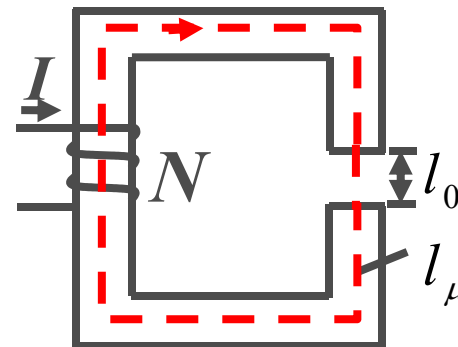
$H$ : magnetic field strength (intensity) in A/m

$H \cdot l$ : magnetic potential, analogous to the electric potential.

- Analogous to KVL with an electrical circuit.

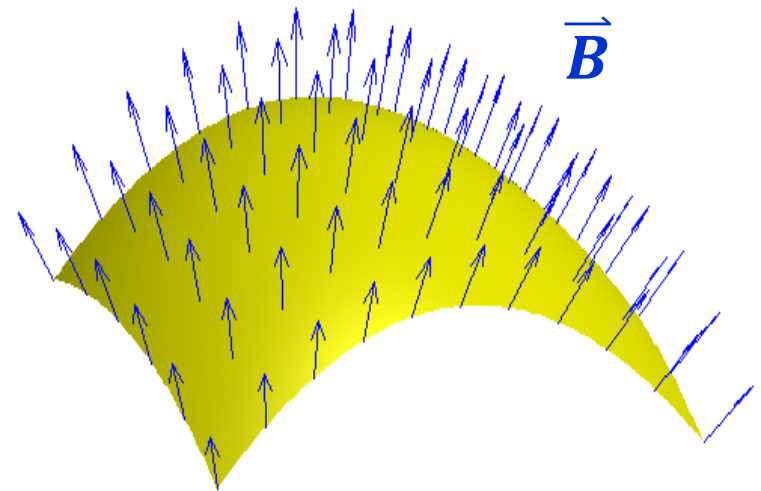
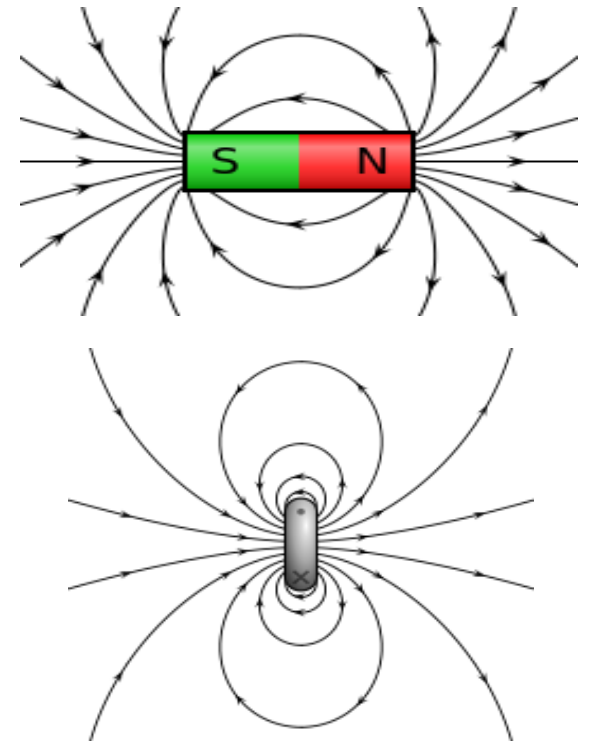


$$NI = H_\mu l_\mu + H_0 l_0$$



# Magnetic Flux Density - $B$

- Defined as the amount of magnetic flux  $\phi$  in an area  $A$  taken perpendicular to the magnetic flux's direction.
  - It is a vector field  $\vec{B}$  (i.e. the **B**-field) having its magnitude and direction at each point in space.
  - This course only concerns its magnitude  $B$ .
- Average  $B = \phi / A$  (unit: T=Wb/m<sup>2</sup>)  
where  $\phi$  - magnetic flux  
 $A$  - cross section area



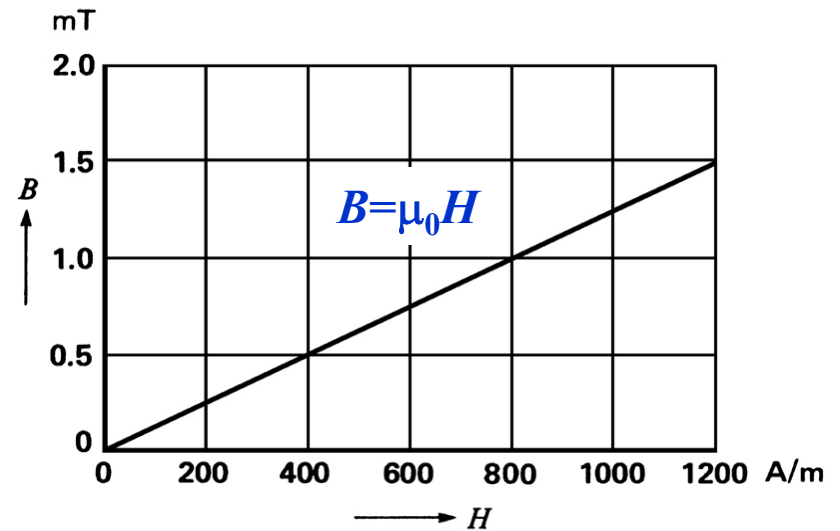
# B - H Curve

- Vacuum and nonmagnetic materials

$$B = \mu_0 H$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$\mu_0$  - magnetic constant or permeability of vacuum



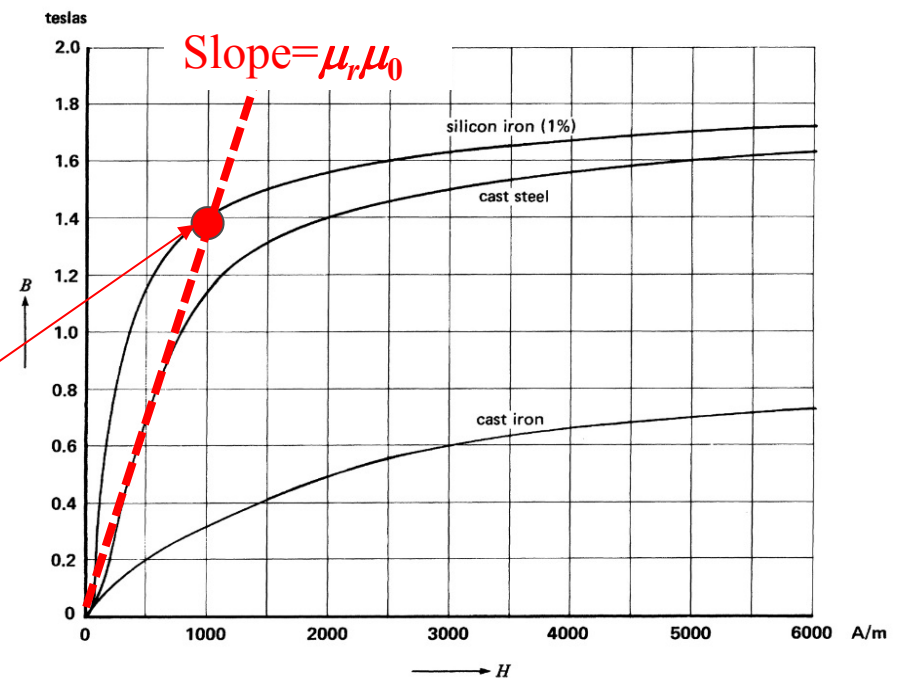
- Magnetic material

$$B = \mu H = \mu_r \mu_0 H \text{ (with saturation)}$$

$\mu = \mu_r \mu_0$  is called permeability

$\mu_r$  - relative permeability (not a constant)

$$\mu_r = \frac{B}{\mu_0 H} = \frac{1.4}{4\pi \times 10^{-7} \times 1000} = 1114$$



# Magnetic Reluctance (Resistance)

- Hopkinson's law (a counterpart to Ohm's law):

$$F_m = \phi R_m$$

$R_m$  - Magnetic reluctance (unit: A·turns/Wb), i.e. the ratio of the mmf in a magnetic circuit and the magnetic flux in this circuit.

(Note: it is not a constant.)

$$F_m = NI = \textcolor{red}{H}l = \phi R_m = B A R_m = \textcolor{red}{\mu} H A R_m = \mu_r \mu_0 H A R_m$$

$$R_m = \frac{l}{\mu A} \quad (\mu = \mu_r \mu_0: \text{permeability})$$

Analogous to the resistance in an electrical circuit

$$R = \rho \frac{l}{A} = \frac{l}{\sigma A} \quad (\rho: \text{resistivity}; \sigma: \text{conductivity})$$

## Example 1

The closed core is made of even magnetic material, and has flux leakage. Its average magnetic field line has length  $l=0.45$  m, and exciting winding has 300 turns.

Calculate the current  $I$  to have magnetic flux density  $B=1.4$  T respectively for two magnetic materials:

- silicon iron (1%)
- cast steel.

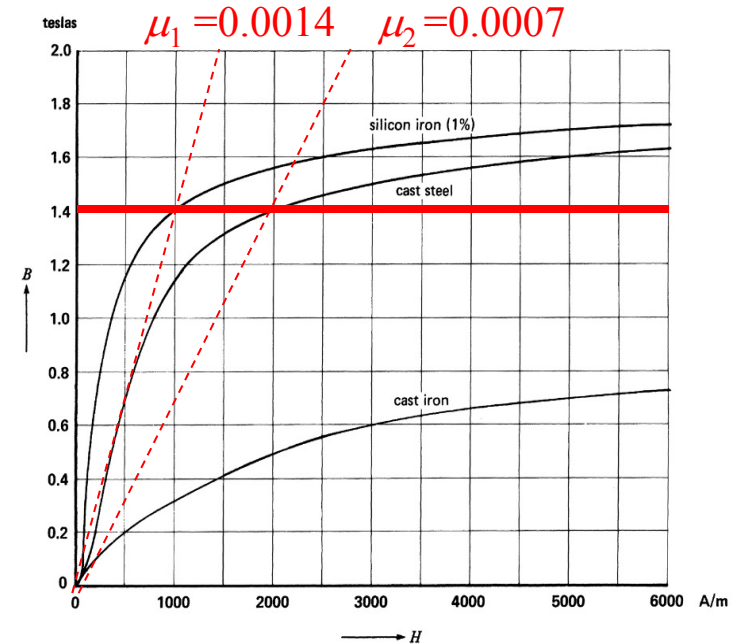
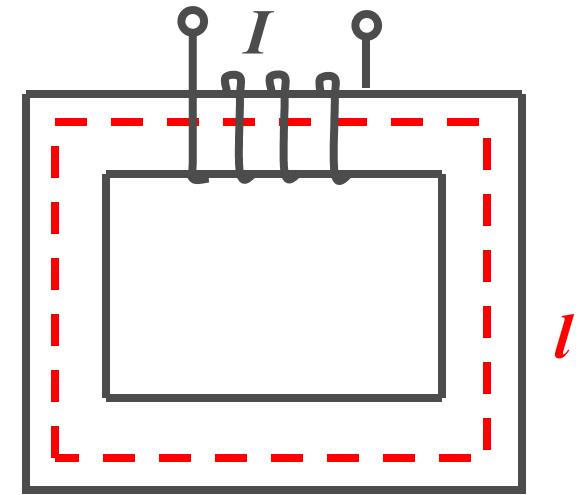
### Solution

From Figure 2.26,  $H_1=1000$  A/m and  $H_2=2000$  A/m

$$I_1 = \frac{H_1 l}{N} = \frac{1000 \times 0.45}{300} = 1.5 \text{ A}$$

$$I_2 = \frac{H_2 l}{N} = \frac{2000 \times 0.45}{300} = 3.0 \text{ A}$$

Observation: to have the same  $B$ , a larger current is needed for a material with lower permeability.



## Example 2

The toroid core made of magnetic material has an inner diameter of 10 cm and an outer diameter of 15 cm. There is an air gap of  $\delta=0.2$  cm wide. The current  $I$  in the conductor is 1 A. To have magnetic flux density  $B=0.9$  T, how many turns do we need for the exciting winding? Assume that the magnetic material has  $H=500$  A/m at  $B=0.9$  T.

### Solution

The magnetic field strength of the air gap:

$$H_0 = \frac{B}{\mu_0} = \frac{0.9}{4\pi \times 10^{-7}} = 7.2 \times 10^5 \text{ A/m}$$

The total average length of magnetic field lines:

$$l = \pi \times (0.1 + 0.15) / 2 = 0.393 \text{ m}$$

The average length of magnetic field lines in the core:

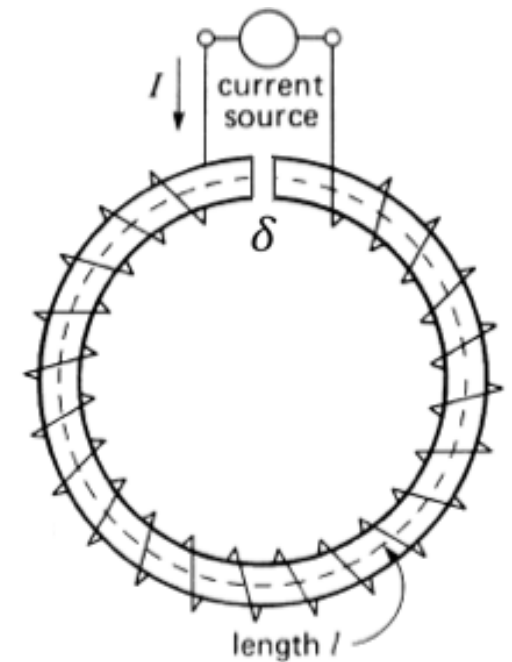
$$l_1 = l - \delta = 0.393 - 0.002 = 0.391 \text{ m}$$

The total mmf

$$\begin{aligned} F_m &= H_0 \delta + H_1 l_1 = 7.2 \times 10^5 \times 0.002 + 500 \times 0.391 \\ &= 1440 + 195.5 = 1635.5 \text{ (A} \cdot \text{turns)} \end{aligned}$$

The turns of the current:

$$N = F_m / I = 1636 \text{ (turns)}$$



The air gap has much smaller permeability, and hence much larger magnetic resistance, so the magnetic potential drop across the air gap is much larger than that in the magnetic material.