Rotational Motion of Electric Machines

- An electric machine rotates about a fixed axis, called the shaft, so its rotation is restricted to one angular dimension.
- Relative to a given end of the machine's shaft, the direction of counterclockwise (CCW) rotation is often assumed to be positive.
- Therefore, for rotation about a fixed shaft, all the concepts are scalars.

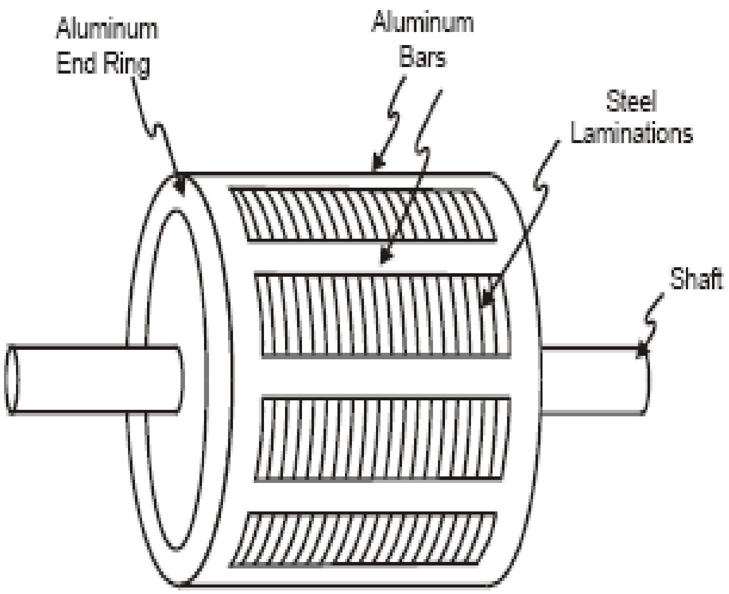
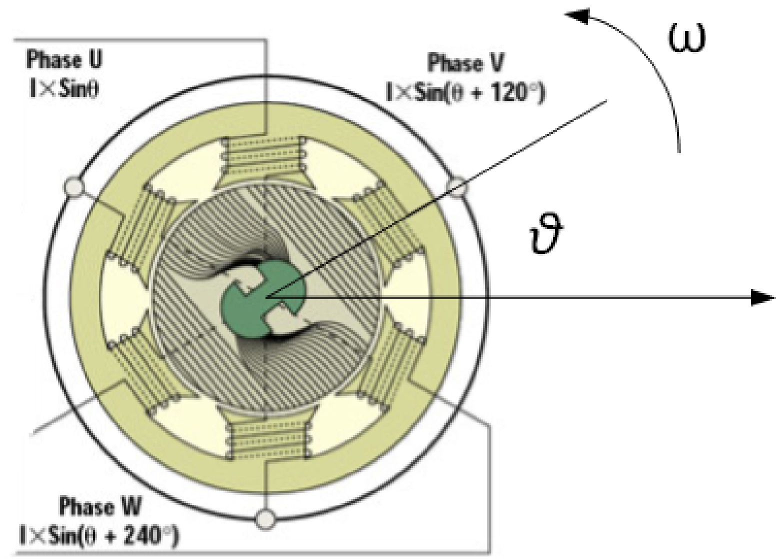


Figure 3-23. Squirrel cage induction motor rotor

Angular Position, Velocity and Acceleration

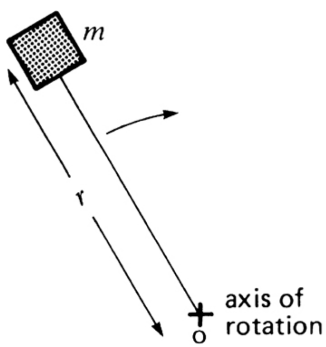


- **Angular position** θ
  - The angle at which an object is oriented, measured from some arbitrary reference point
  - Unit: rad or deg
  - Analogy of the linear concept of distance along a line.
- **Angular velocity** $\omega = d\theta/dt$
  - The rate of change in angular position with respect to time
  - Unit: rad/s or r/min (revolutions per minute or rpm for short)
  - Analogy of the concept of velocity on a straight line.
- **Angular acceleration** $\alpha = d\omega/dt$
  - The rate of change in angular velocity with respect to time
  - Unit: rad/s^2
- θ and $\omega > 0$ if the rotation is CCW
- $\alpha > 0$ if the absolute angular velocity is increasing in the CCW direction **or decreasing in the CW direction**

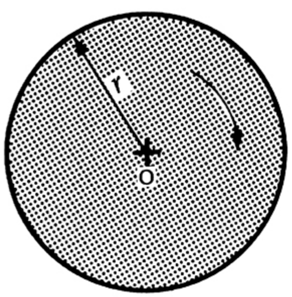
Moment of Inertia (or Inertia)

- Inertia depends on the mass and shape of the object (unit: kg·m^2)
- A complex shape can be broken up into 2 or more of simple shapes

Definition

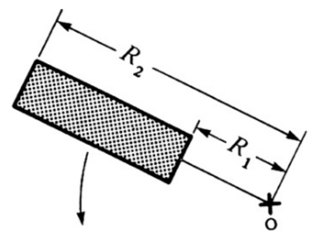


$$J = mr^2$$

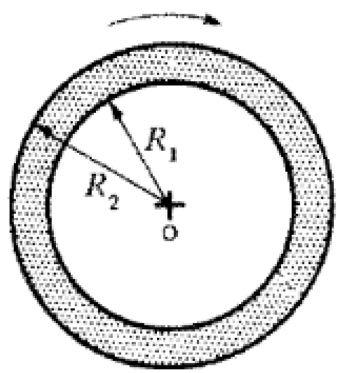


$$J = \frac{mr^2}{2}$$

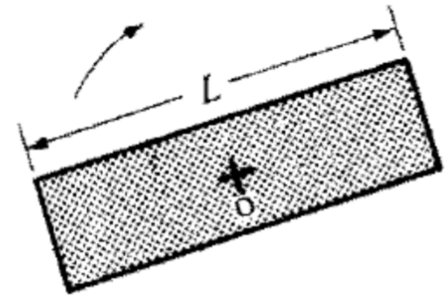
Two useful formulas



$$J = \frac{m}{3}(R_1^2 + R_2^2 + R_1 R_2)$$



$$J = \frac{m}{2}(R_1^2 + R_2^2)$$



$$J = \frac{mL^2}{12}$$

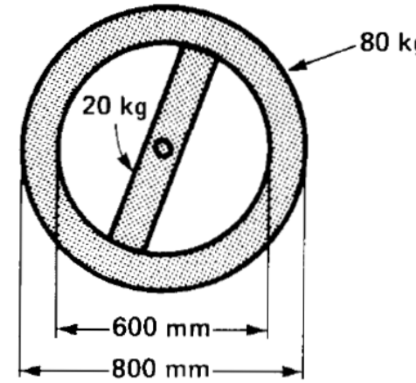
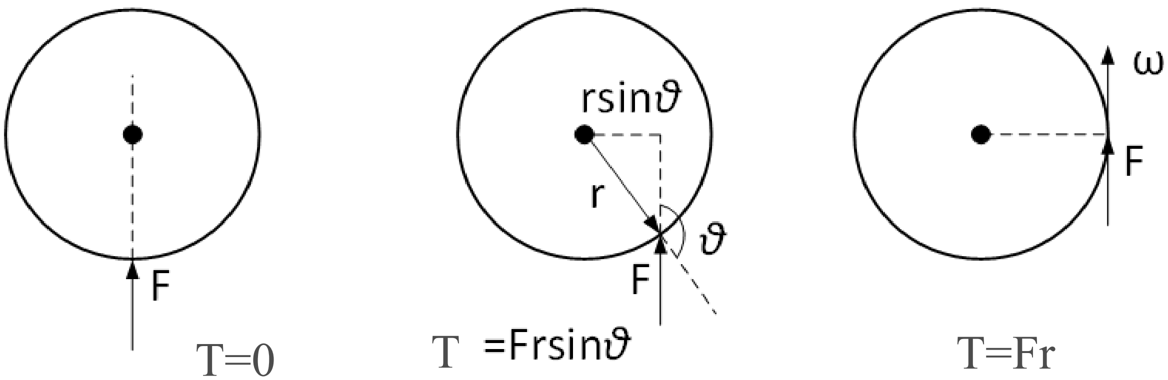


Figure 3.11
Flywheel in Example 3.10.

Torque and Change in Speed

- **Torque** is equal to the product of the force and the perpendicular distance between the axis of rotation and the point of application of the force.

  $T = Fr$ (N·m)



- **Newton's Law of Rotation**: Describes the relationship between the total torque applied to an object and its resulting angular acceleration.

$$T = J\alpha = J\frac{\Delta\omega}{\Delta t} = \frac{J}{9.55}\cdot\frac{\Delta n}{\Delta t} \qquad n[\text{r/min}] = \omega[\text{rad/s}]\times\frac{60[\text{min}]}{2\pi} = \frac{30}{\pi}\omega \approx 9.55\omega$$

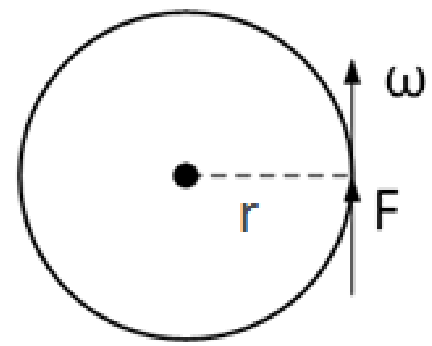
Δn: change in speed (**in r/min or rpm**)

Δt: interval of time during which the torque is applied (**in second**)

Mechanical Work, Power and Kinetic Energy of Rotational Motion

- Applying a constant torque T (N·m) to a motor
  - Work: $W = T\Delta\theta$ (J)

    $\Delta\theta$: angular distance (rad)



  - Power: $$P = T\omega = \frac{Tn}{9.55} \quad \text{(W)}$$

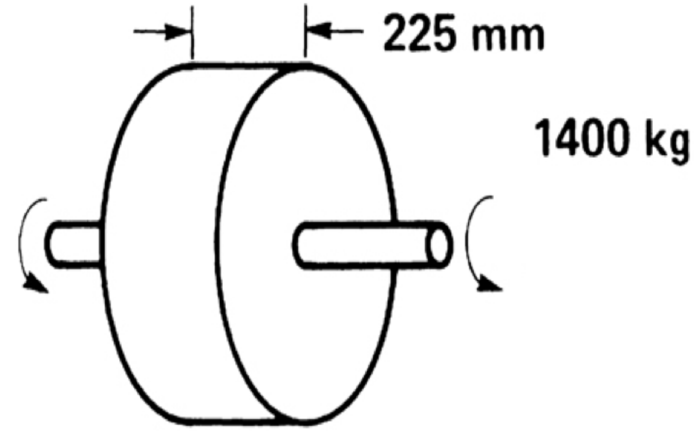
    ω: angular velocity or speed of rotation (rad/s)

    n: speed of rotation (r/min)

  - Kinetic Energy: $$E_k = \frac{1}{2}J\omega^2 = \frac{\pi^2}{1800}Jn^2 = 5.48 \times 10^{-3} Jn^2 \quad \text{(J)}$$

$$\omega[\text{rad/s}] = \frac{\pi}{30} n[\text{r/min}]$$

Example 4



A solid 1400 kg steel flywheel has diameter of 1 m and a thickness of 225 mm. Calculate:

- Its moment of inertia
- To increase its speed from 60 r/min to 600 r/min by applying a torque of 20 N·m. For how long must the torque be applied?
- The kinetic energy when the flywheel revolves at 1800r/min

Solution:

Ignoring the thickness of the flywheel, the moment of inertia:

$$J = \frac{mr^2}{2} = \frac{1400 \times 0.5^2}{2} = 175\ kg \cdot m^2$$

$$T = \frac{J}{9.55} \cdot \frac{\Delta n}{\Delta t} \qquad \Delta t = \frac{J}{9.55}\frac{\Delta n}{T} = \frac{175}{9.55} \times \frac{600 - 60}{20} = 494.7\ \text{s}$$

$$E_k = 5.48 \times 10^{-3} J n^2 = 5.48 \times 10^{-3} \times 175 \times 1800^2 = 3.1\text{MJ}$$

Mechanically Coupled System

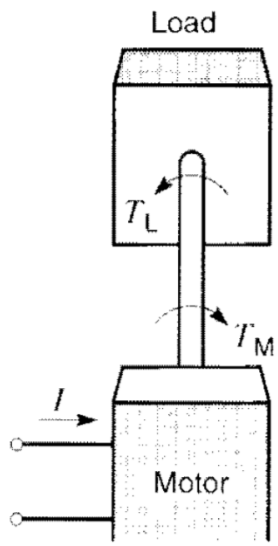


Figure 3.12
Shaft is stationary $T_M = T_L$.

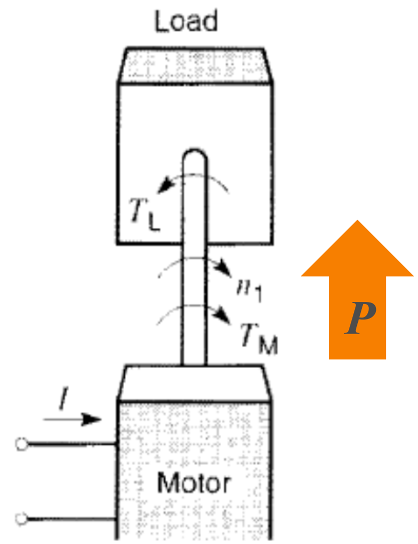


Figure 3.13
Shaft turns cw $T_M = T_L$.

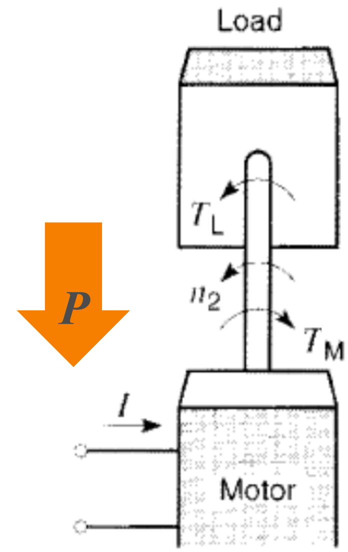


Figure 3.14
Shaft turns ccw $T_M = T_L$.

- Assume that under normal operating conditions, T_M (or n) is cw and T_L is ccw.
- When T_M=T_L and they have opposite directions, speed n (or ω) of the shaft is constant (note: n may be cw, ccw or even 0)

- If n is cw, to increase n to n_1 (cw)
  - current $I\uparrow \rightarrow T_M > T_L \rightarrow n\uparrow$
  - once $n=n_1$, $I\downarrow \rightarrow T_M=T_L$
- In order to decrease & reverse n to n_2 (ccw)
  - current $I\downarrow \rightarrow T_M < T_L \rightarrow n\downarrow \rightarrow 0 \rightarrow$ccw
  - once $n=n_2$, $I\uparrow \rightarrow T_M=T_L$

- What is the direction of power flow?

$$P = T\omega = \frac{Tn}{9.55} \quad \text{(W)}$$

Conclusions on a Mechanically Coupled System

- **Speed:**
  - When $T_M = T_L$ and they have opposite directions, the actual steady-state speed being cw (or ccw) depends on whether T_M was greater (or less) than T_L for a certain period of time before the actual steady-state condition was reached.
- **Power flow:**
  - When T_M and n have the same direction (i.e. $P>0$), the motor (in the motor mode) delivers power to the load; otherwise, the motor (in the generator mode) receives power from the load

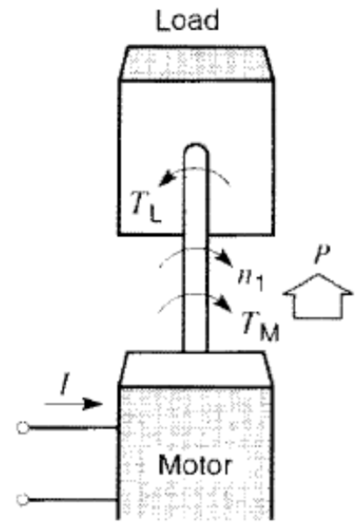


Figure 3.13
Shaft turns cw $T_M = T_L$.

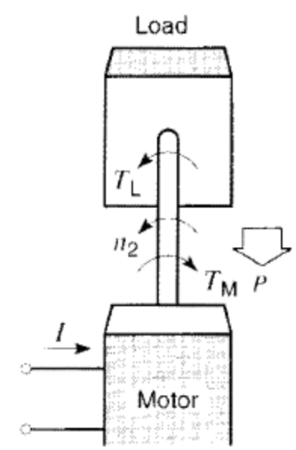


Figure 3.14
Shaft turns ccw $T_M = T_L$.

Question 3-11

A motor develops a cw torque of 60N·m, and the load develops a ccw torque of 50N·m.

- If this situation persists for some time, will the direction of rotation eventually be cw or ccw?

  cw since 60N·m (cw) > 50N·m (ccw)

- What value of motor torque is needed to keep the speed constant?

  50N·m

Electric Motors Driving Linear Motion Loads

- Power output

  $P_o = F \cdot v$

  F in N and v in m/s

- Power input

  $P_i = T \cdot n / 9.55$

- Assuming no losses

  $P_i = P_o$

  $T \cdot n = 9.55 F \cdot v$

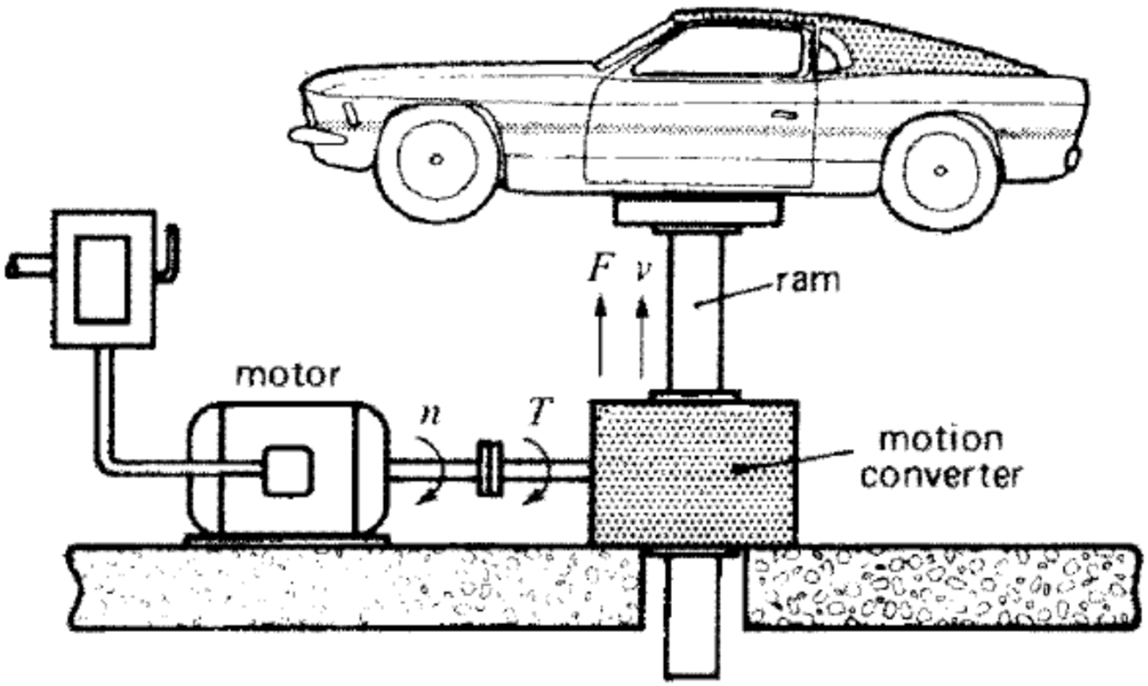


Figure 3.15
Converting rotary motion into linear motion.

Homework #3

- Read Ch. 2.16-2.31, 3.0-3.14
- Questions 2-5, 2-8, 2-9, 3-12, 3-17, 3-18, 3-19
- Due date: 9/30 (Friday)
  - hand in your solution in the class or
  - to Wenyun Ju at MK207