

- VR is a measure of line voltage drop and usually should not exceed  $\pm 5\%$  (or  $\pm 10\%$ )
- VR depends on the load power factor:
  - VR is often high (bad) for a low lagging power factor
  - Perhaps, VR<0 for a leading power factor (i.e.  $|E_1| \le |E_2|$ ).
  - If ignore  $X_{\rm C}$ , three typical loads with lagging, unity and leading power factors



### **Resistive line**

• There is an upper limit on the power transferred by the line to the load

$$P = |E_{R}| \cdot |I| = |I|^{2} (kR) = \left|\frac{E_{S}}{R + kR}\right|^{2} kR$$
$$= \frac{|E_{S}|^{2}}{R(1/k + k + 2)} \le \frac{|E_{S}|^{2}}{4R}$$

$$P = P_{\text{max}} = |E_{\text{S}}|^2 / 4R$$
 when  $k = 1$ , i.e.  $E_{\text{R}} = E_{\text{S}} / 2$ 

- If  $E_{\rm R} \ge 0.95 E_{\rm S}$  (around 5% VR) is required, the line can support a load that is only 19% of  $P_{\rm max}$
- The total power from the sender is  $P+|I|^2R$
- VR is a key factor that limit the power transmission capacity



Figure 25.21 Characteristics of a resistive line.

#### **Inductive line**

$$P = |I|^{2} (kX) = \left|\frac{E_{S}}{jX + kX}\right|^{2} kX$$
$$= \frac{|E_{S}|^{2}}{X | k - 1 / k + 2j |} \le \frac{|E_{S}|^{2}}{2X}$$

 $P_{\text{max}} = |E_{\text{S}}|^2 / 2X$  when k=1,  $|E_{\text{R}}| = |E_{\text{S}}| / \sqrt{2} = 0.707 |E_{\text{S}}|$ 

- A inductive line can deliver twice as much power as a resistive line (if *X*=*R*)
- If E<sub>R</sub>≥0.95E<sub>S</sub> (around 5% VR) is required, the line can support a load that is 60% of P<sub>max</sub>, i.e. 6x as much as power as a resistive line
- VR is a key factor that limit the power transmission capacity
- The total power from the sender is  $P+j|I|^2X$



Figure 25.22 Characteristics of an inductive line.

### Inductive line connecting two systems

$$S = E_{S}I^{*} = E_{S}\left(\frac{E_{S} - E_{R}}{jX}\right)^{*} = |E_{S}| \angle \delta\left(\frac{|E_{S}| \angle -\delta - |E_{R}| \angle 0^{\circ}}{X \angle -90^{\circ}}\right)$$
$$= \frac{|E_{S}|^{2}}{X} \angle 90^{\circ} - \frac{|E_{S}||E_{R}|}{X} \angle (\delta + 90^{\circ})$$
$$P = -\frac{|E_{S}||E_{R}|}{X} \cos(\delta + 90^{\circ}) = \frac{|E_{S}||E_{R}|}{X} \sin\delta \approx \frac{|E_{S}||E_{R}|}{X} \delta(\text{in rad})$$
$$Q = \frac{|E_{S}|^{2}}{X} - \frac{|E_{S}||E_{R}|}{X} \sin(\delta + 90^{\circ}) = \frac{|E_{S}|^{2}}{X} - \frac{|E_{S}||E_{R}|}{X} \cos\delta$$
$$= \frac{|E_{S}|}{X} (|E_{S}| - |E_{R}| \cos\delta) \approx \frac{|E_{S}|}{X} (|E_{S}| - |E_{R}|)$$
$$P = \frac{|E_{S}||E_{R}|}{X} \sin\delta$$

• The size of reactive flow depends on the voltage drop from the sending end to the receiving end.

• If  $|E_{\rm S}| = |E_{\rm R}| = E$ ,  $Q \approx 0$ , i.e. almost no reactive flow

$$P = \frac{E^2}{X} \sin \delta \le \frac{E^2}{X}$$







Figure 25.30a Power versus angle characteristic.

## Increasing the power transmission capacity

$$S = P + jQ = \frac{|E_S||E_R|}{X} \sin \delta + j|I|^2 X$$

#### 1. Use a shunt capacitor

• To have  $|E_{\rm R}| = |E_{\rm S}|$ , the line can fully be compensated by adding a shunt capacitor to the receiving end whose  $X_{\rm C}$  is adjustable so that

 $|E_{\rm S}|^2/X_{\rm C}=0.5|I|^2X$ 

while the other  $0.5|I|^2X$  is provided by the source or another capacitor with  $X_{\rm C}$  at the sending end.

• Thus, 
$$P_{\text{max}} = |E_{\text{S}}|^2 / X$$



To further increase  $P_{\text{max}}$ , an approach is to reduce line reactance X

#### 2. Use parallel lines:

 $X \rightarrow X/2 \dots X/N$ , so  $P_{\text{max}} \rightarrow 2P_{\text{max}} \dots N \times P_{\text{max}}$ Also improving security against a line trip.

#### 3. Use a series capacitor

 $P_{\text{max}} = |E_{\text{S}}|^2 / (X - X_{CS})$ 



• It may cause sub-synchronous resonance (SSR)

$$f_{SSR} = f \sqrt{\frac{X_{CS}}{X}} = f \sqrt{\frac{1}{LC_S}}$$

If  $X_{CS}/X=1/4$  (25% compensation),  $f_{SSR}=f/2=30$ Hz

# **Voltage Regulation for EHV lines**

A 3-phase 735kV 60Hz 600km line, operated at 727kV, has inductive reactance of 0.5  $\Omega$ /km and capacitive reactance of 300k $\Omega$ /km.

• At no-load (open-circuit) conditions,

for each phase,

 $|E_{\rm S}| = 727/\sqrt{3} = 420 {\rm kV},$ 

 $X_{\rm L}$ =0.5×600=300 $\Omega$ ,  $X_{\rm C}$ =300k/600=500 $\Omega$ ,

 $X_{C1} = X_{C2} = 2X_{C} = 1000 \Omega$   $E_{R} = E_{S} \times (-jX_{C2}) / (jX_{L} - jX_{C2})$  $= 420 \angle 0^{\circ} \times 1000 / (1000 - 300) = 600 \angle 0^{\circ} \text{ kV}$  To bring  $|E_{\rm R}|$  back to  $|E_{\rm S}|$ , add a shunt reactor of  $X_{\rm L2}$  at the receiving end:

If  $X_{L2} = X_{C2}$ , then  $-jX_{C2}//jX_{L2} = \infty$ and  $|E_R| = |E_S|$ .

The reactive power generated by  $X_{C2}$  is entirely absorbed by  $X_{L2}$  (cancelling each other)



Figure 25.35 EHV transmission line at no-load.



Figure 25.36 EHV reactor compensation.

## Surge-impedance load (SIL)

- When connected to a gradually increasing load with PF=1,  $|E_R|$  decreases from  $|E_{R,NL}|$  (open-circuit) to 0 (short-circuit).
- When  $|E_R| = |E_S|$ , the amount of load is called the surge-impedance load (SIL) and the corresponding load impedance is called the surge impedance, which has  $Z_Y \approx 400\Omega$  (line-to-neutral impedance) for aerial lines



#### **Inter-region power exchange: Example 25-8**



#### Figure 25.41a

An ordinary transmission line causes power to flow in the wrong direction.

# Calculate: (1) the power transmitted by the line and (2) the required phase-shift enabling transmitting 70MW from $E_a$ to $E_b$

(1)  $|E_a| = |E_b| = E$ , so  $P_{ab} = (E^2/X)\sin(\delta_a - \delta_b) = (100^2/20) \sin(-11^\circ) = -95.4$  W (in fact, a  $\leftarrow$  b) (2)  $P_{ab} = 70 = (100^2/20) \sin(\delta_a - \delta_b)$ , so  $\delta_a - \delta_b = 8^\circ$ , i.e. 8-(-11)=19° phase-shift of  $E_a$ 



#### Figure 25.41b

A phase-shift autotransformer can force power to flow in the desired direction (Example 25-8).

# Summary for Test #2

- Short answer questions will be from lecture slides
- Big calculation questions will be similar to some of the following:
  - 3 Electromagnetism & rotation motion:
    - All examples in lectures
    - Homework problems: 2-9, 3-19
  - 4 Transformers:
    - Examples: 9-4, 9-5, 10-5, 10-6, 10-7, 10-8, 10-10, 11-2, 12-1, 12-2, 12-3, 12-7
    - Homework problems: 11-7, 11-8, 11-9
  - 5 Transmission lines
    - Examples: 25-3, 25-4, 25-5, slide # 14-15, 25-8