Example 13-3

- A 0.5 hp, 6-pole induction motor is excited by a 3-phase, 60 Hz source. Calculate the slip, and the frequency of the rotor current under the following conditions:
  a. At standstill
  b. Motor turning at 500r/min in the same direction as the revolving field
  c. Motor turning at 500r/min in the opposite direction to the revolving field
  d. Motor turning at 2000r/min in the same direction as the revolving field

Solution:

n_s=120f/p=120×60/6 = 1200 r/min

a. s= (n_s-n)/n_s=(1200-0)/1200 =1 pu

f_2=sf=60Hz

b. s= (1200-500)/1200 = 0.583 pu

f_2=sf=0.583 × 60Hz = 35 Hz

c. s= [1200- (-500)]/1200 =1.417 pu

f_2=sf=1.417 × 60Hz = 85 Hz

d. s= [1200- 2000)]/1200 = -0.667 pu

f_2=sf=-0.667 × 60Hz = -40 Hz

Slip	Rotor speed & freq	Operating mode
s=1	n=0 f_2=f	Transformer (standstill)
0<s<1	0<n<n_s 0<f_2<f	Motor
s>1	n<0 f_2>f	Brake
s<0	n>n_s f_2<0	Generator (reversed phase sequence A-C-B)

Equivalent circuits: wound-rotor induction motor

1. **When the motor is at standstill (i.e. n=0, s=1)**, it acts exactly like a conventional transformer. Assume a Y-connection for both the stator and rotor, and a turns ratio of **1:1**. Consider the per-phase equivalent circuit:

   E_g: source voltage, line to neutral,

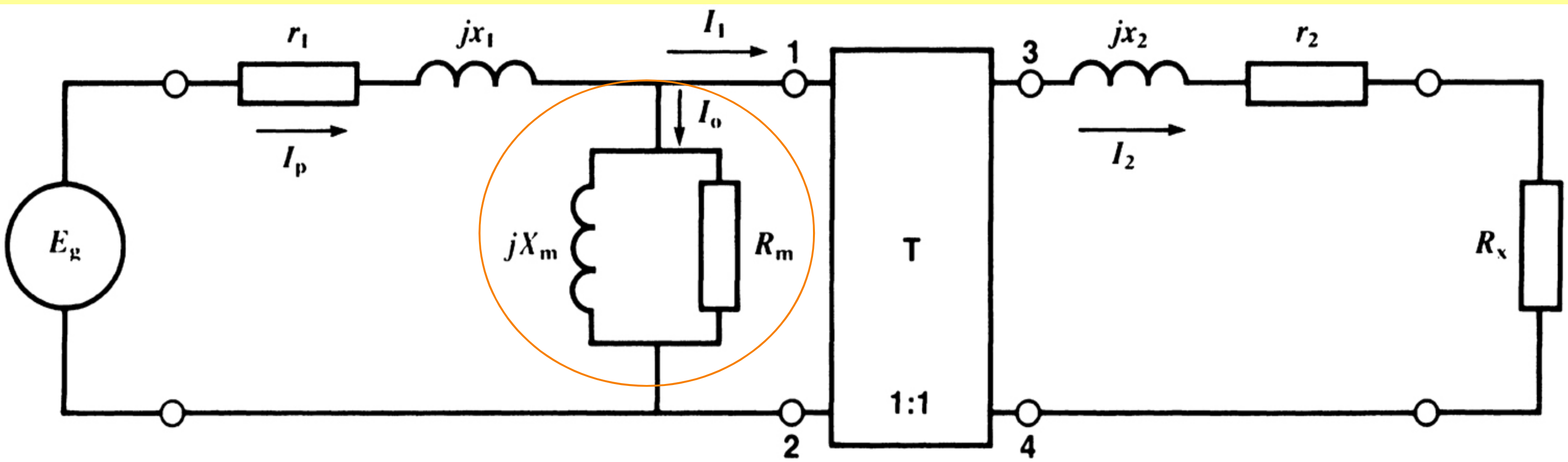
   x_1, r_1: stator leakage reactance and winding resistance

   x_2, r_2: rotor leakage reactance and winding resistance

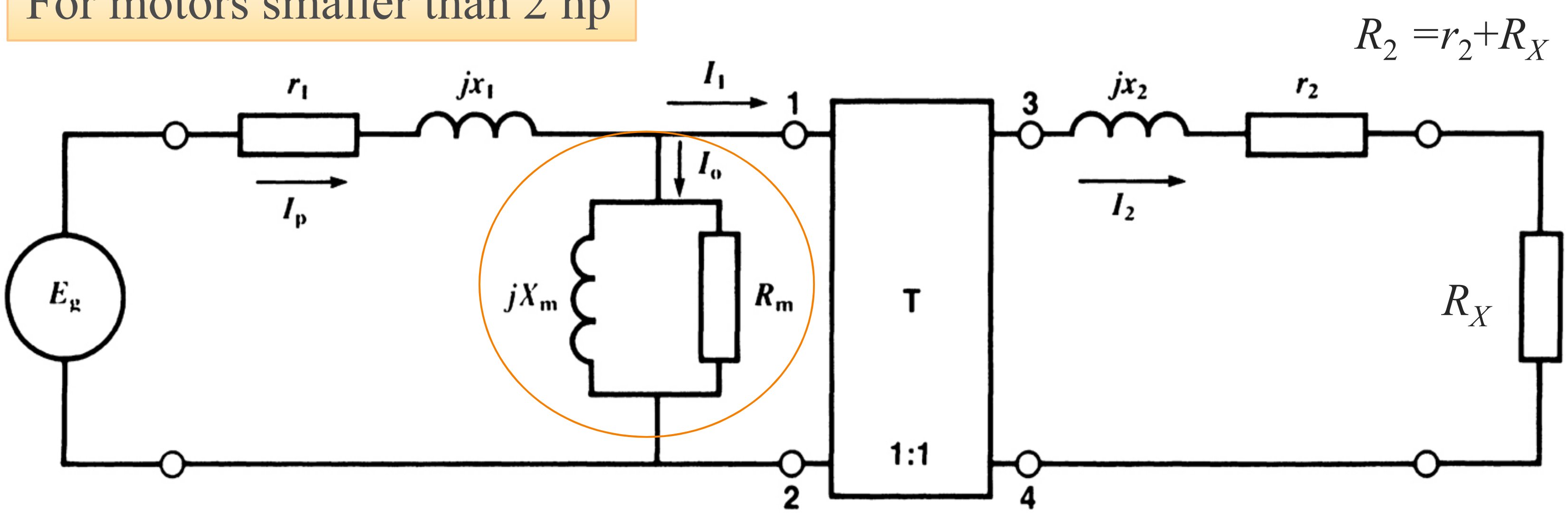
   X_m, R_m: magnetizing reactance and resistance modeling losses of iron, windage and friction

   R_x: external resistance, connecting one slip-ring to the neutral of the rotor

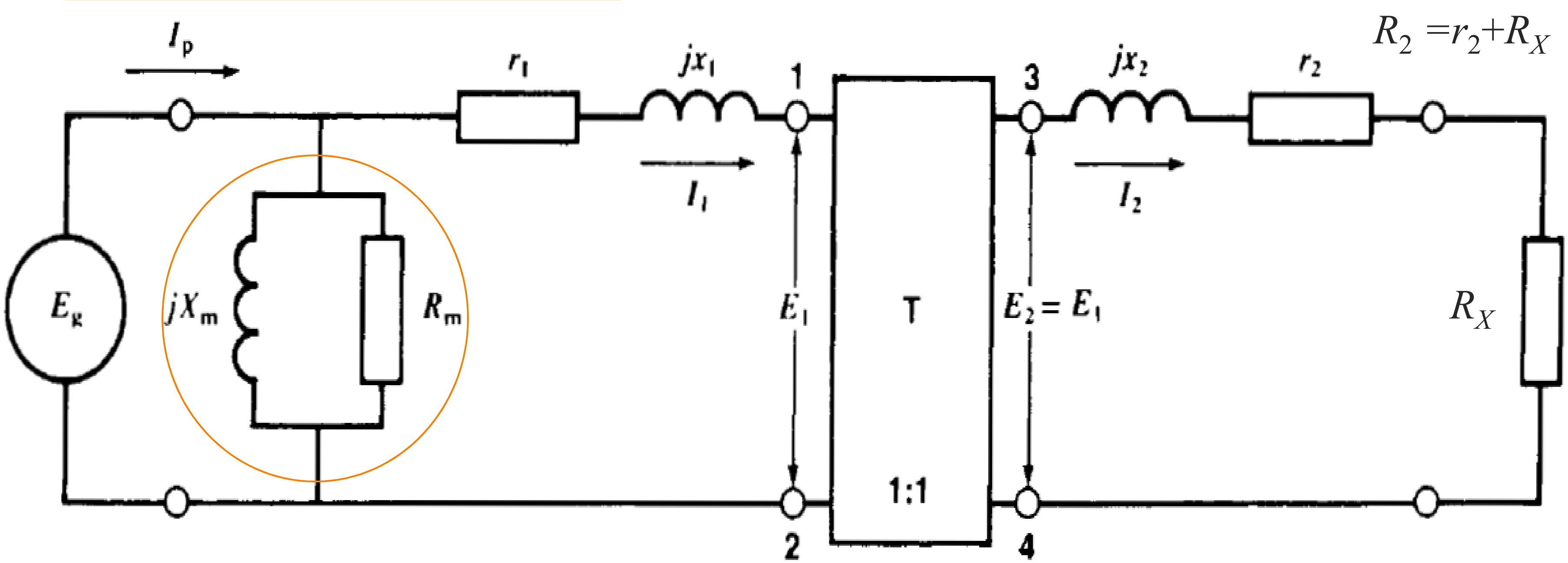
Note: I_o may reach 40% of I_p due to the air gap between the stator and rotor (much bigger than the air gap between the P and S windings of a transformer), so we cannot eliminate the magnetizing branch.



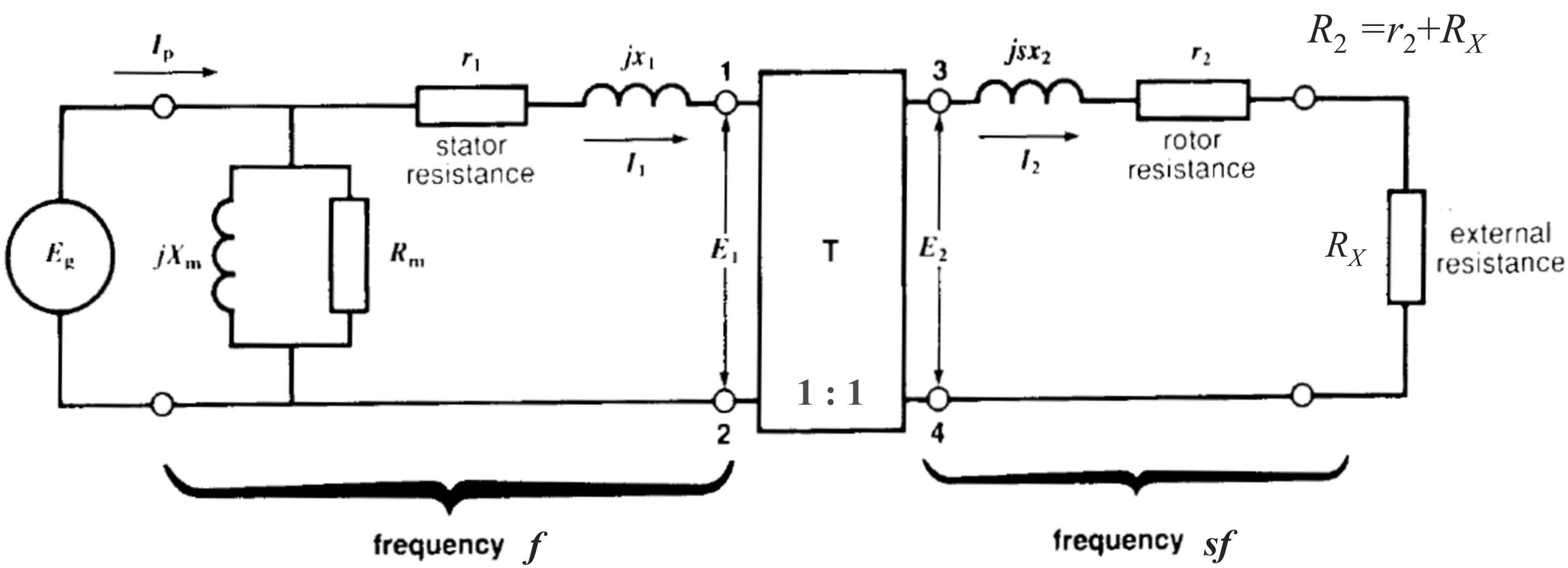
For motors smaller than 2 hp



For motors bigger than 2 hp



2. When the motor runs at a slip s, i.e. n=(1-s)n_s



$|E_2|=|sE_1|$, $|I_2|=|I_1|$, $jx_2 \to jsx_2$ r_2 and R_X are the same

$$I_2 = \frac{E_2}{R_2 + jsx_2} = \frac{|sE_1|\angle 0}{|R_2 + jsx_2|\angle\beta} = \frac{|sE_1|\angle -\beta}{\sqrt{R_2^2 + (sx_2)^2}} \quad \beta = \arctan\frac{sx_2}{R_2}$$

Note: the phasors on the primary side (E_1 and I_1) and the secondary side (E_2 and I_2) cannot be drawn in one phasor diagram because they have different frequencies

Phasor diagrams on the rotor and stator

- I_1 and I_2 have the same effective value even though they have different frequencies
  $|I_1|=|I_2|$
- sE_1 and E_2 have the same effective values
  $|sE_1|=|E_2|$
- E_1 leads I_1 and E_2 leads I_2 both by β

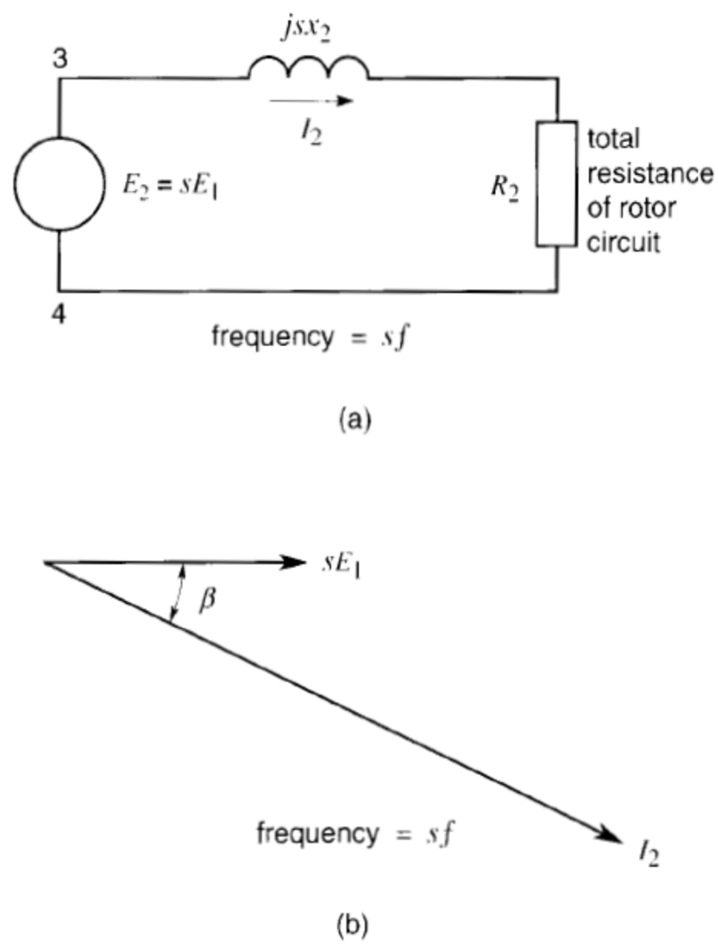


Figure 15.4
a. Equivalent circuit of the rotor; E_2 and I_2 have a frequency sf.
b. Phasor diagram showing the current lagging behind the voltage by angle β.

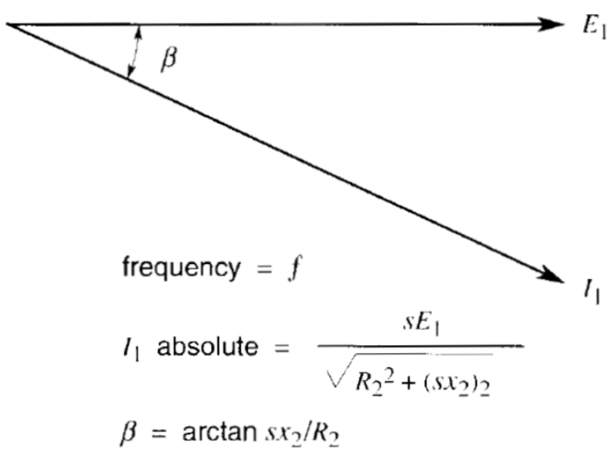
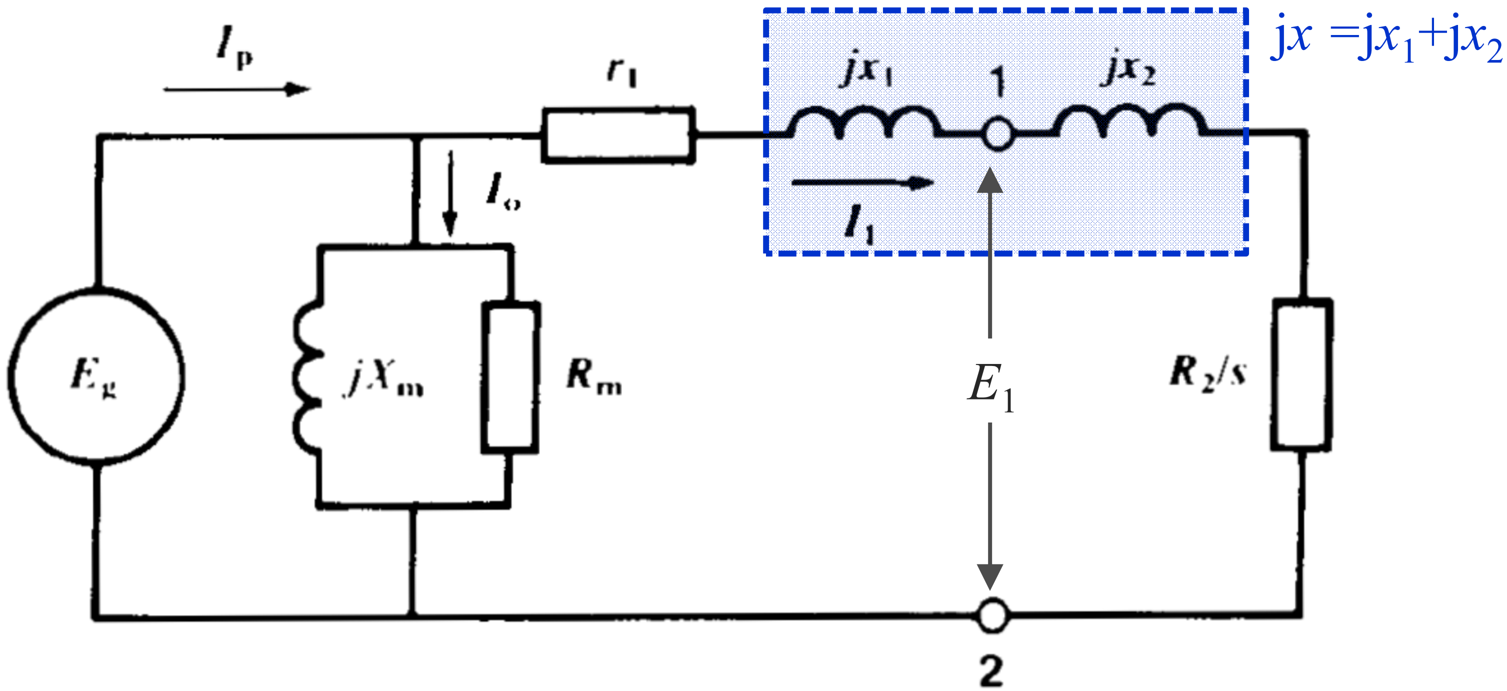


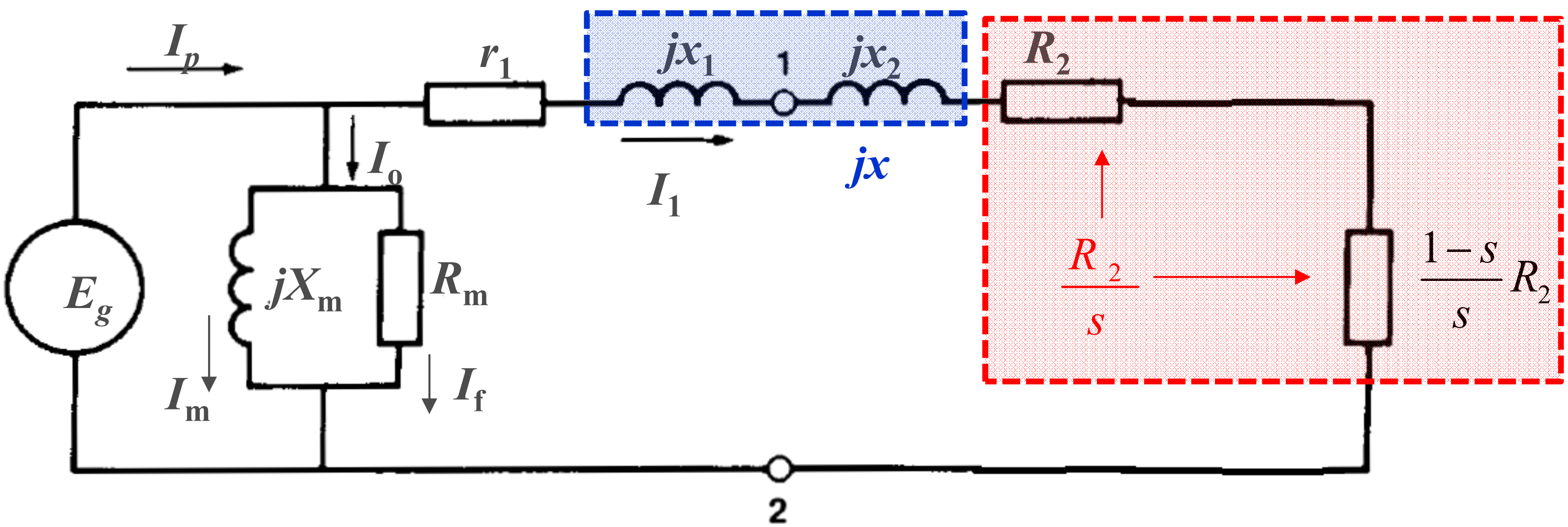
Figure 15.5
The voltage and current in the stator are separated by the same phase angle β, even though the frequency is different.

Simplified equivalent circuit: referred to the stator side

$$I_1 = I_2 = \frac{sE_1}{R_2 + \mathrm{j}sx_2} \Rightarrow I_1 = \frac{E_1}{\frac{R_2}{s} + \mathrm{j}x_2} = \frac{E_1}{Z_2} \qquad Z_2 \overset{\mathrm{def}}{=} \frac{R_2}{s} + \mathrm{j}x_2 = \frac{E_1}{I_1}$$



Note: The value of R_2/s will vary from R_2 to ∞ as the motor goes from start-up (s=1) to n_s (s=0)



- Total power absorbed by the motor:

$$S = \frac{|E_g|^2}{R_m} + j\frac{|E_g|^2}{X_m} + |I_1|^2 (r_1 + jx + \frac{R_2}{s})$$

$$P = \frac{|E_g|^2}{R_m} + |I_1|^2 r_1 + |I_1|^2 \frac{R_2}{s}$$

- Stator iron losses

$$P_f = |E_g|^2 / R_m$$

- Stator copper losses

$$P_{js} = |I_1|^2 r_1$$

- Active power supplied to the rotor:

$$P_r = |I_1|^2 R_2 / s$$

- Total I^2R losses in the rotor circuit

$$P_{jr} = |I_1|^2 R_2 = sP_r$$

- Mechanical power developed by the motor

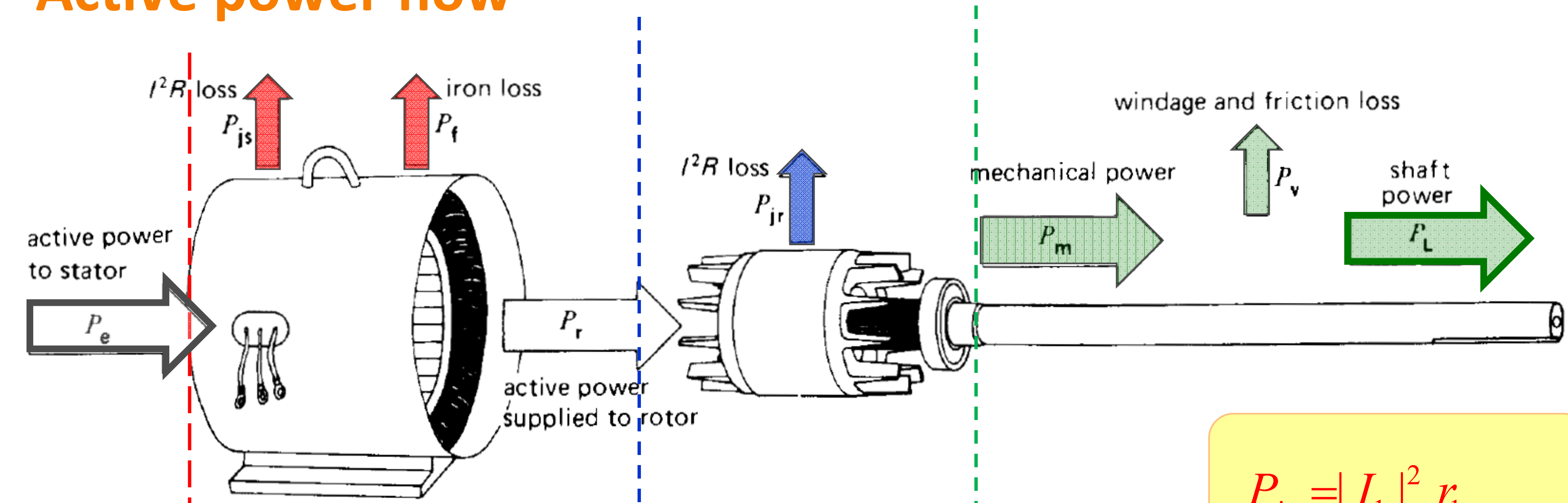
$$P_m = P_r - P_{jr} = P_r(1-s) = |I_1|^2 \frac{1-s}{s} R_2$$

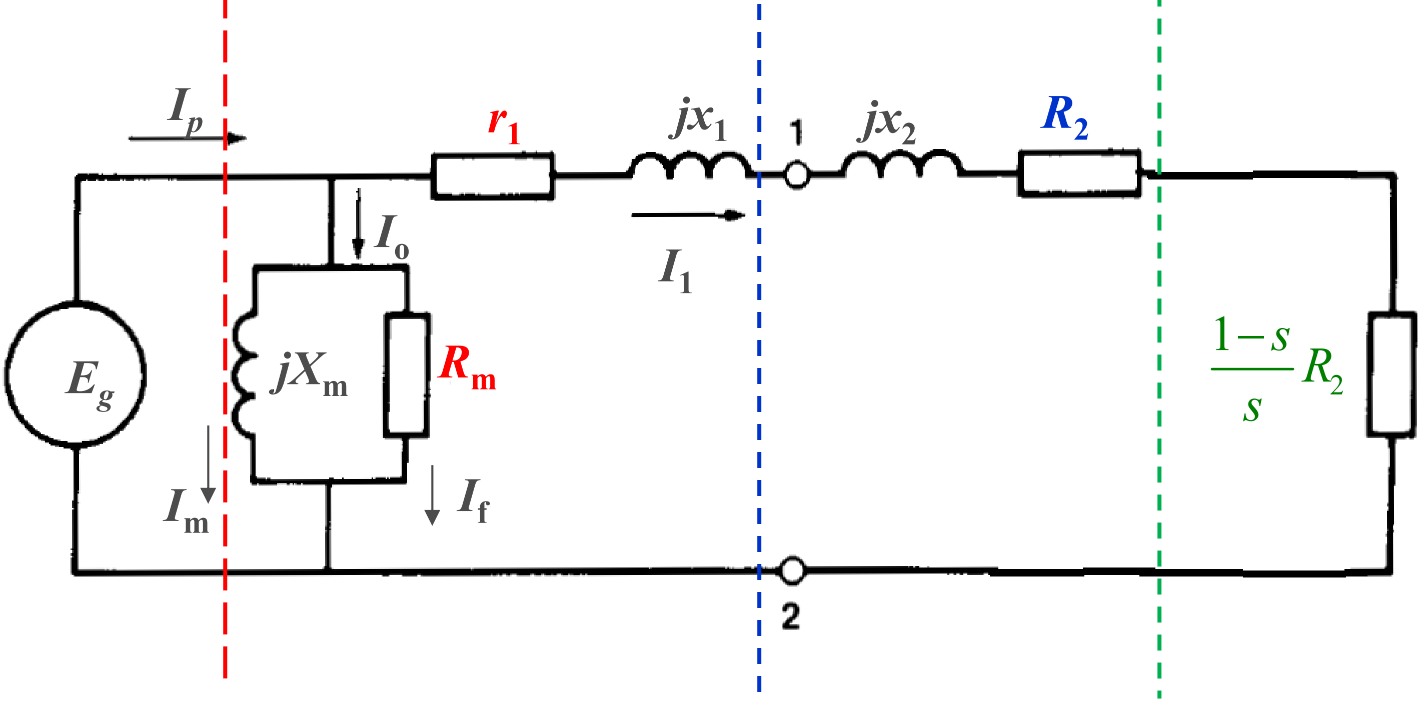
- Torque developed by the motor

$$T = \frac{9.55P_m}{n} = \frac{9.55P_r(1-s)}{n_s(1-s)} = \frac{9.55P_r}{n_s}$$

Note: The torque only depends on P_r

Active power flow





$$P_{js} = |I_1|^2 r_1$$

$$P_f = |E_g|^2 / R_m$$

$$P_{jr} = |I_1|^2 R_2$$

$$P_m = |I_1|^2 \frac{1-s}{s} R_2$$

$$\eta = P_L / P_e$$