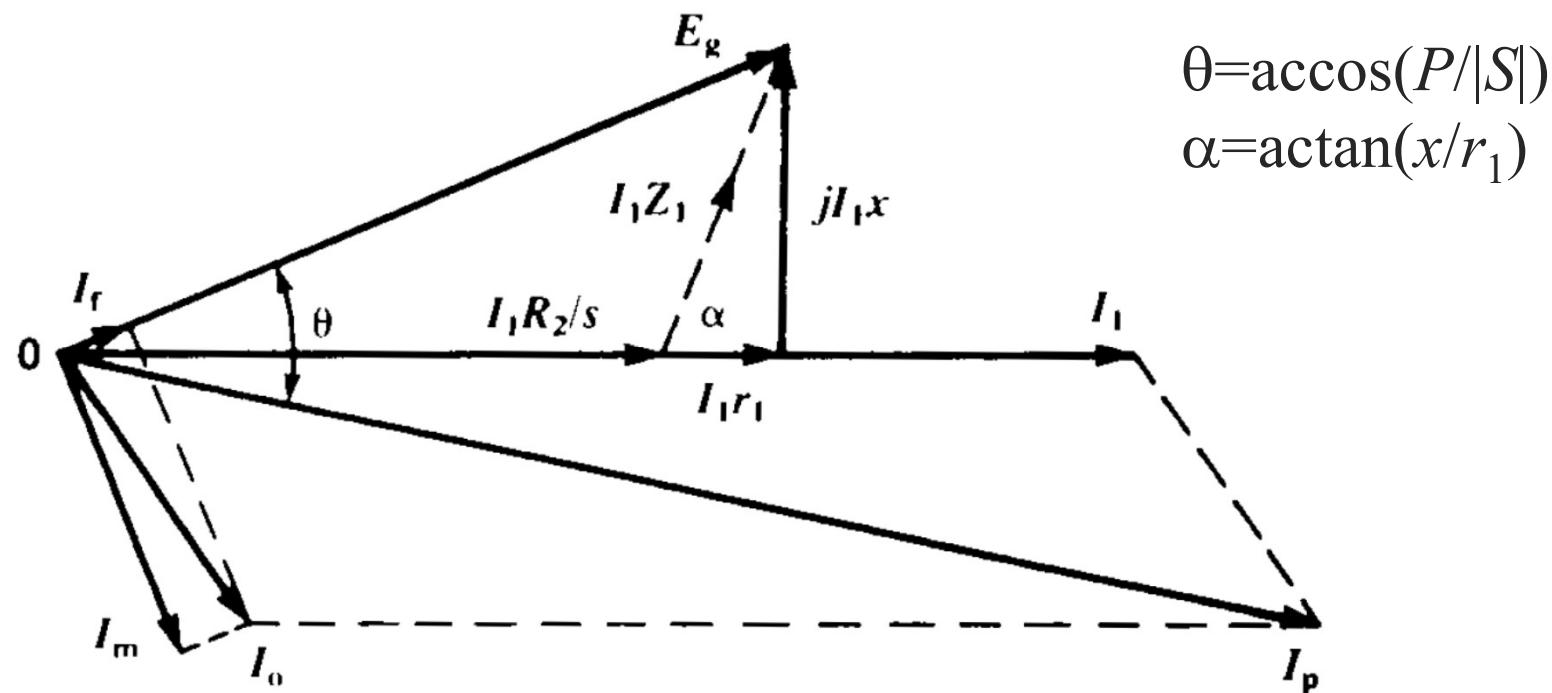
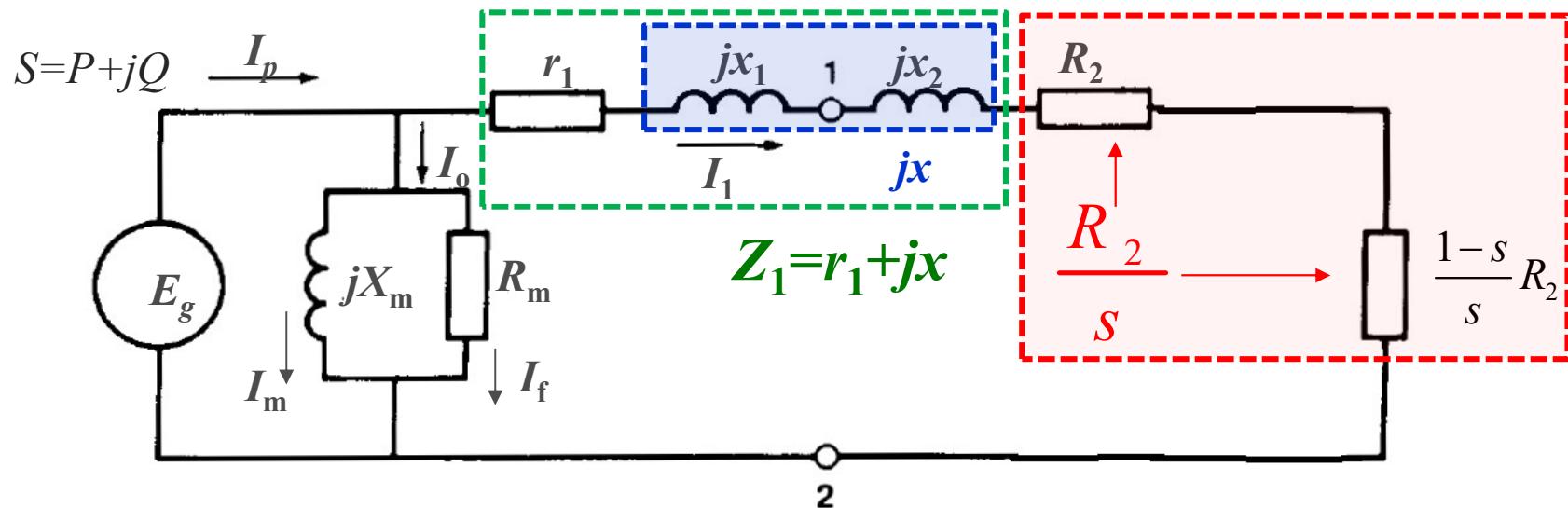
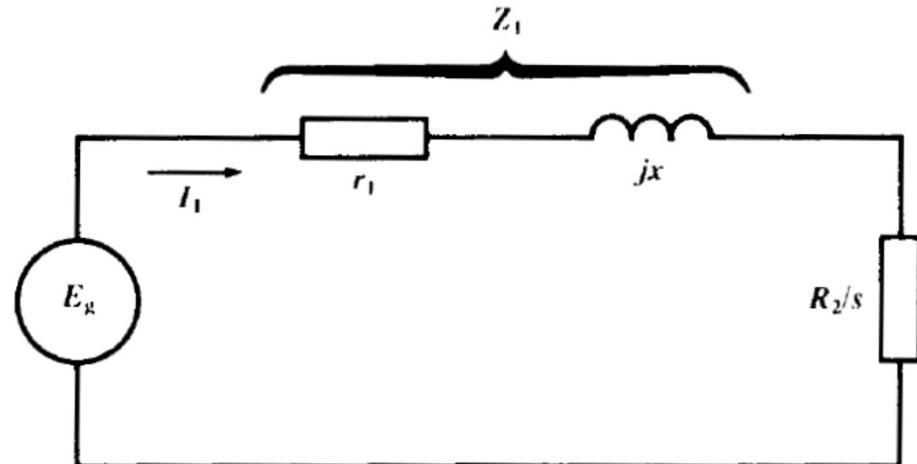
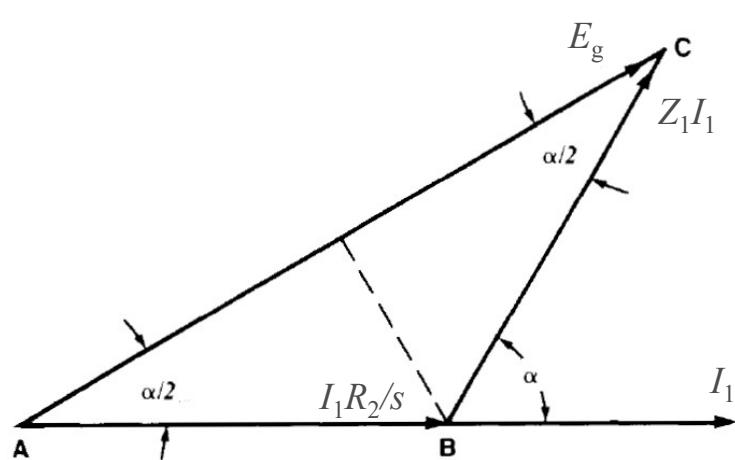


# Phasor diagram of the induction motor



## Breakdown (maximum) Torque

- When  $|Z_1|=|R_2/s|$ ,  $P_r$  and torque  $T$  both reach their maximum values



$$\frac{|E_g|}{2} = \frac{|I_1| R_2}{s} \cos \frac{\alpha}{2} = |I_1 Z_1| \cos \frac{\alpha}{2}$$



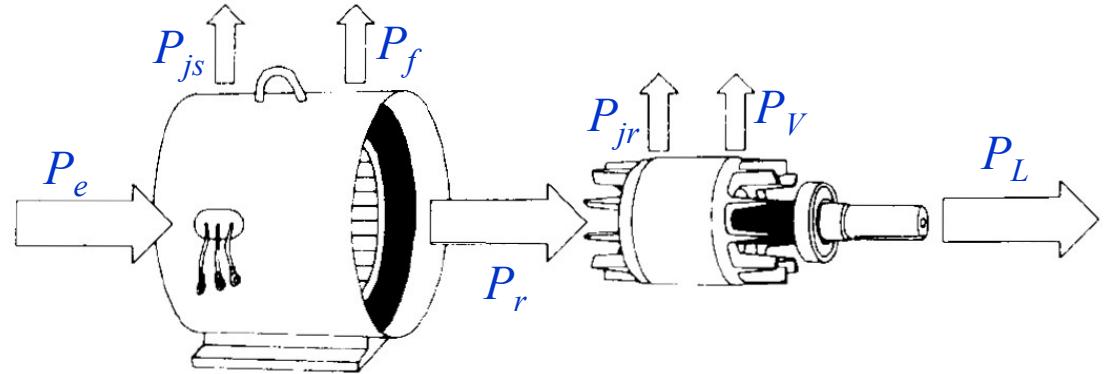
$$|I_{1b}| = \frac{|E_g|}{2 |Z_1| \cos \frac{\alpha}{2}} \quad s_b = \frac{R_2}{|Z_1|}$$

$$T_{\max} \triangleq T_b = \frac{9.55 P_r|_{I_1=I_{1b}, s=s_b}}{n_s} = \frac{9.55 |I_{1b}|^2 R_2 / s_b}{n_s}$$

$$T_b = \frac{9.55 |E_g|^2}{4 n_s |Z_1| \cos^2 \frac{\alpha}{2}}$$

## Example 13-5

A 3-phase induction motor having a synchronous speed of 1200 r/min draws 80 kW from a 3-phase feeder. The copper losses and iron losses in the stator amount to 5 kW. If the motor runs at 1152 r/min, calculate



- a. The active power transmitted to the rotor

$$P_r = P_e - P_{js} - P_f = 80 - 5 = 75 \text{ kW}$$

- b. The rotor  $I^2R$  losses, i.e.  $P_{jr}$

$$s = (n_s - n) / n_s = (1200 - 1152) / 1200 = 0.04, \quad P_{jr} = s P_r = 0.04 \times 75 = 3 \text{ kW}$$

- c. The mechanical power developed

$$P_m = P_r - P_{jr} = 75 - 3 = 72 \text{ kW}$$

- d. The mechanical power delivered to the load, knowing that the windage and friction losses equal to 2 kW

$$P_L = P_m - P_V = 72 - 2 = 70 \text{ kW}$$

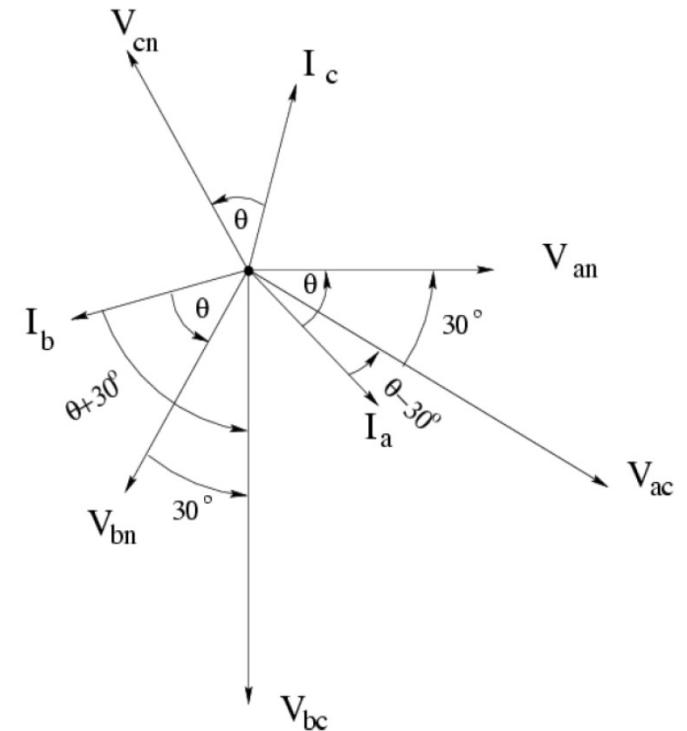
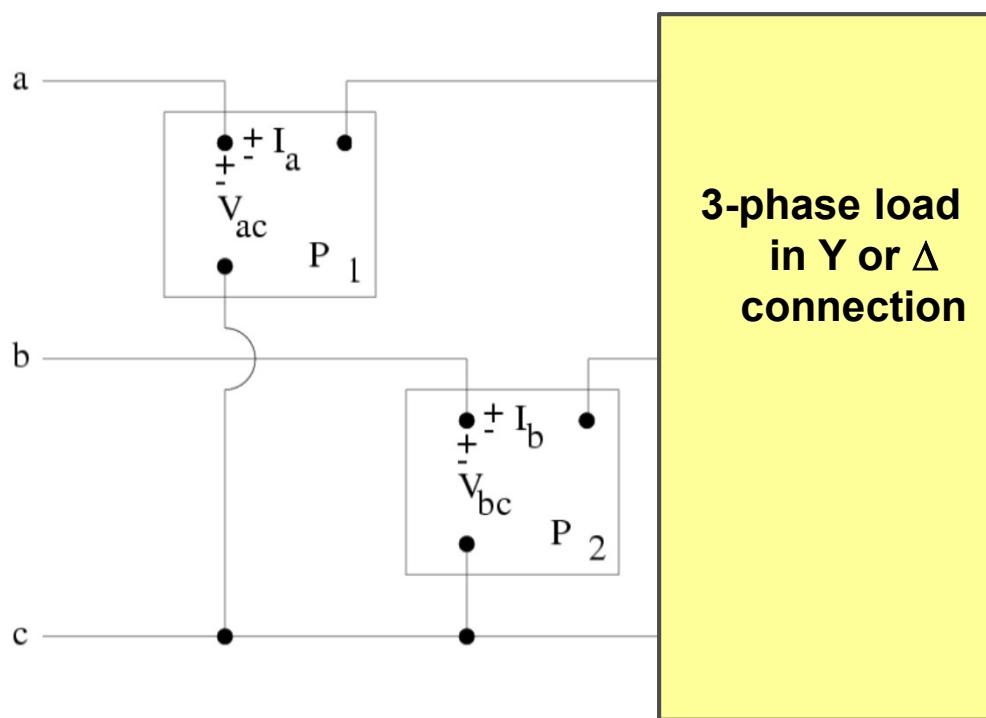
- e. The efficiency of the motor

$$\eta = P_L / P_e = 70 / 80 = 87.5\%$$

- f. The torque developed by the motor

$$T = 9.55 P_r / n_s = 9.55 \times 75000 / 1200 = 597 \text{ N}\cdot\text{m}$$

## Two-wattmeter method to measure 3-phase active power



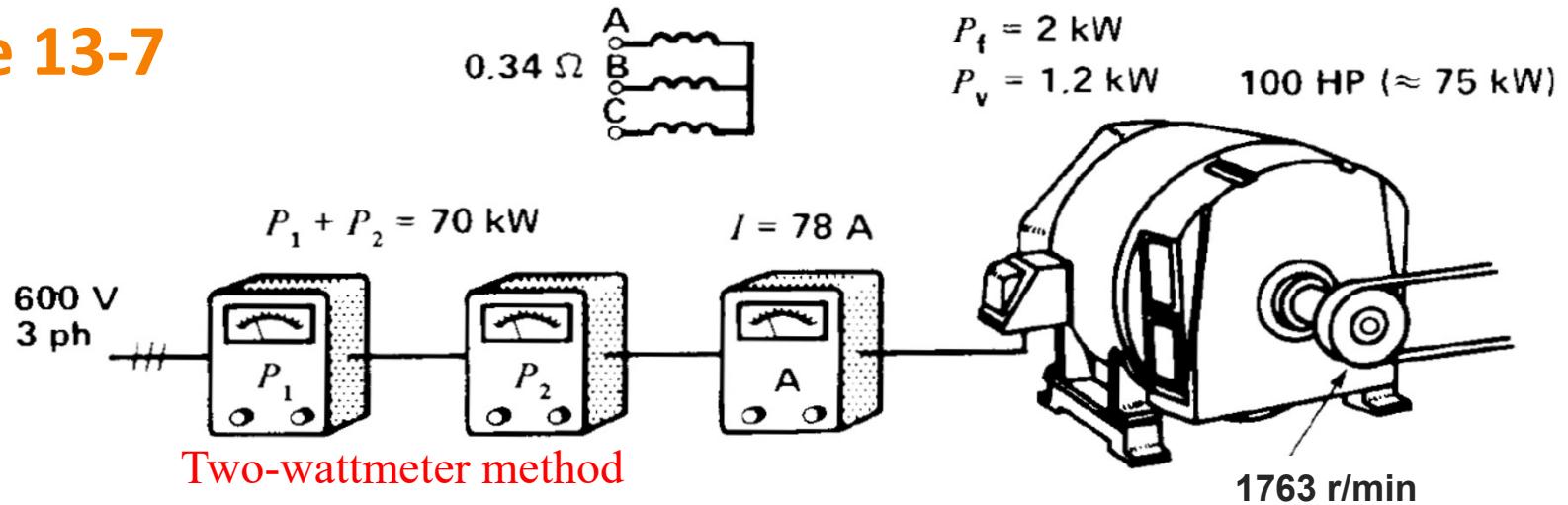
$$P_1 = |V_{ac}| |I_a| \cos(\theta - 30^\circ) = E_L I_L \cos(\theta - 30^\circ)$$

$$P_2 = |V_{bc}| |I_b| \cos(\theta + 30^\circ) = E_L I_L \cos(\theta + 30^\circ)$$

$$P_1 + P_2 + j\sqrt{3}(P_1 - P_2) = P_{3\phi} + jQ_{3\phi} = S_{3\phi}$$

$$\begin{aligned} \text{Proof: } & E_L I_L [\cos(\theta - 30^\circ) + \cos(\theta + 30^\circ)] + j\sqrt{3} E_L I_L [\cos(\theta - 30^\circ) - \cos(\theta + 30^\circ)] \\ &= E_L I_L 2 \cos \theta \cos 30^\circ + j\sqrt{3} E_L I_L 2 \sin \theta \sin 30^\circ \\ &= \sqrt{3} E_L I_L \cos \theta + j\sqrt{3} E_L I_L \sin \theta = S_{3\phi} \end{aligned}$$

## Example 13-7



A 3-phase induction motor having a nominal rating of 100 hp ( $\sim 75$  kW) and a synchronous speed of 1800 r/min is connected to a 3-phase 600 V source.

Resistance between two stator terminals =  $0.34\Omega$ . Calculate

a. Power supplied to the motor

$$P_e = P_1 + P_2 = 70 \text{ kW}$$

b. Rotor  $I^2R$  losses  $P_{jr}$

$$r_1 = 0.34/2 = 0.17\Omega$$

$$P_{js} = 3I_1^2r_1 = 3 \times 78^2 \times 0.17 = 3.1 \text{ kW}$$

$$P_r = P_e - P_{js} - P_f = 70 - 3.1 - 2 = 64.9 \text{ kW}$$

$$s = (n_s - n)/n_s = (1800 - 1763)/1800 = 0.0205$$

$$P_{jr} = sP_r = 0.0205 \times 64.9 = 1.33 \text{ kW}$$

c. Mechanical power supplied to the load

$$P_m = P_r - P_{jr} = 64.9 - 1.33 = 63.5 \text{ kW}$$

$$\begin{aligned} P_L &= P_m - P_V = 63.5 - 1.2 = 62.3 \text{ kW} \\ &= 62.3 \times 1.34 = 83.5 \text{ hp} \end{aligned}$$

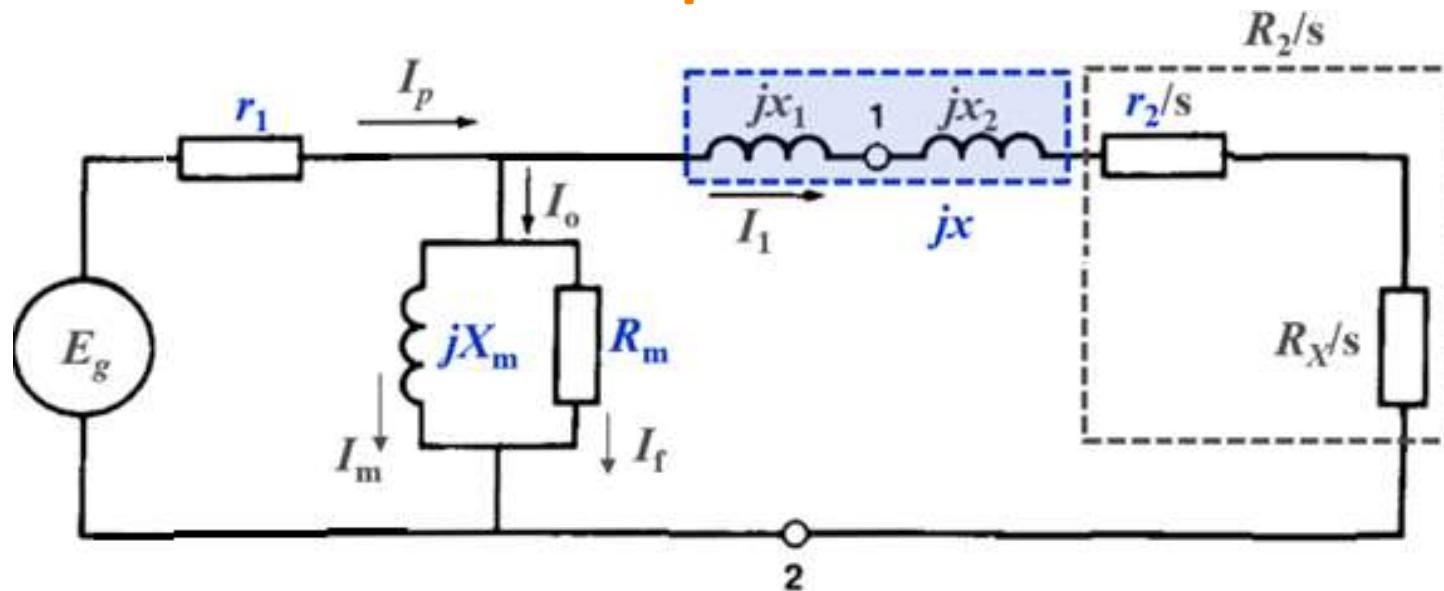
d. Efficiency

$$\eta = P_L/P_e = 62.3/70 = 89\%$$

e. Torque developed at 1763 r/m

$$\begin{aligned} T_m &= 9.55 P_r/n_s = 9.55 \times 649000/1800 \\ &= 344 \text{ N}\cdot\text{m} \end{aligned}$$

## Tests to determine the equivalent circuit



- Estimate  $r_1$ ,  $r_2$ ,  $X_m$ ,  $R_m$  and  $x$  (note:  $r_2 + R_X = R_2$  where  $R_X$  is the external resistance)
  1. No-load test
  2. Locked-rotor test
- Learn Example 15-1

## No-load test

At no-load, slip  $s \approx 0 \rightarrow$

$R_2/s$  is high,  $I_1 \ll I_o$

Steps:

1. Measure stator resistance  $R_{LL}$  between any two terminals (assuming a Y connection)

$$r_1 = R_{LL}/2$$

2. Run the motor at no-load using rated line-to-line voltage  $E_{NL}$ . Measure no-load current  $I_{NL}$  and 3-phase active power  $P_{NL}$

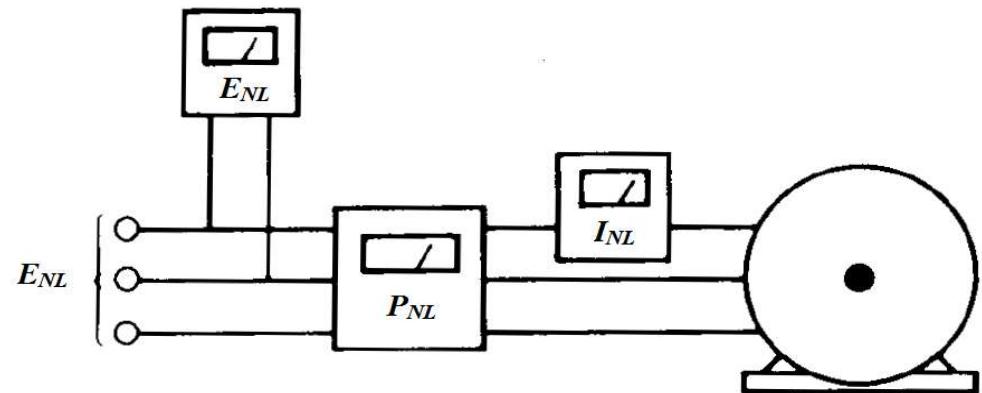
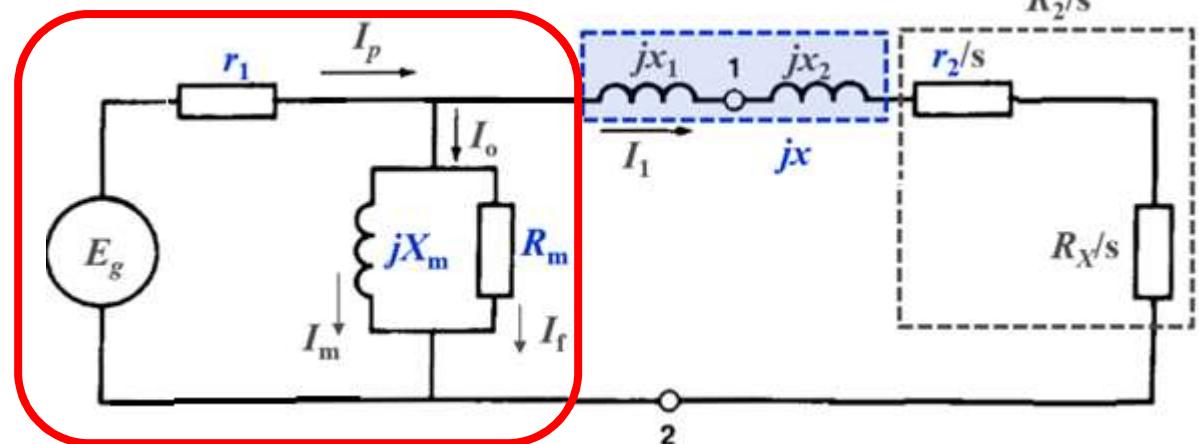
$$S_{NL} = \sqrt{3}E_{NL}I_{NL}$$

$$P_{NL} \approx P_f + 3I_{NL}^2 r_1 \Leftrightarrow P_f \approx P_{NL} - 3I_{NL}^2 r_1 \quad (\text{Ignoring } P_V)$$

$$Q_{NL} = \sqrt{S_{NL}^2 - P_{NL}^2}$$

$$R_m = \frac{(E_{NL}/\sqrt{3})^2}{P_f/3} = \frac{E_{NL}^2}{P_{NL} - 3I_{NL}^2 r_1}$$

$$X_m = \frac{(E_{NL}/\sqrt{3})^2}{(Q_{NL})/3} = \frac{E_{NL}^2}{Q_{NL}}$$



**Figure 15.17**

A no-load test permits the calculation of  $X_m$  and  $R_m$  of the magnetizing branch.

## Locked-rotor test

When the rotor is locked,  
 $s=1$ ,  $I_p \gg I_o \rightarrow r_2 = r_2/s \approx R_2/s$ ,  
neglect the magnetizing branch

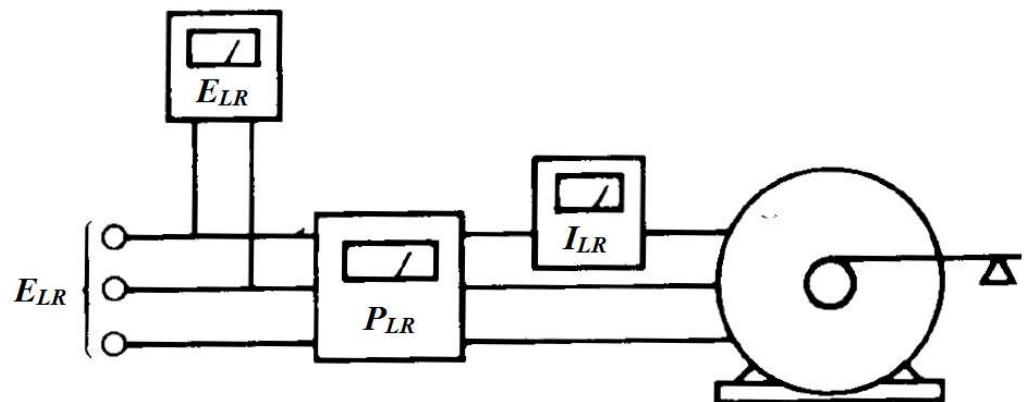
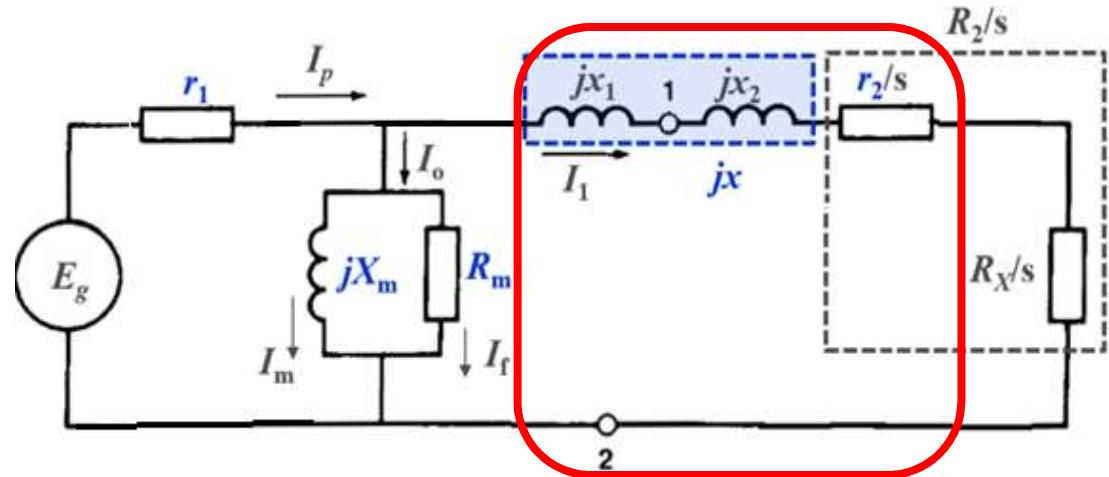
Steps:

1. Apply reduced 3-phase voltage  $E_{LR}$  to the stator so that the stator current  $I_p \approx$  the rated value
2. Measure line-to-line voltage  $E_{LR}$ , current  $I_{LR}$  and 3-phase active power  $P_{LR}$

$$S_{LR} = \sqrt{3}E_{LR}I_{LR}$$

$$Q_{LR} = \sqrt{S_{LR}^2 - P_{LR}^2}$$

$$x = \frac{Q_{LR}}{3I_{LR}^2}$$



**Figure 15.18**

A locked-rotor test permits the calculation of the total leakage reactance  $x$  and the total resistance  $(r_1 + r_2)$ . From these results we can determine the equivalent circuit of the induction motor.

$$P_{LR} \approx 3I_{LR}^2 r_1 + 3I_{LR}^2 r_2 \Rightarrow r_2 \approx P_{LR} / (3I_{LR}^2) - r_1$$