ECE 421/521 FALL 2013 - MIDTERM EXAM 1

**Problem 1 (25 points):** A load $Z_L=50+j40\Omega$ is supported by a 300V rms, 60-Hz source through a line with impedance $Z_T=0+j10\Omega$.

a. Find the total current, the total real and reactive powers absorbed by the load, and the voltage and overall power factor on the load side

b. Find the capacitance of the shunt capacitor to improve the overall power factor on the load side to 0.9 lagging. Then, what is the new voltage on the load side?

\[
\begin{align*}
Z &= Z_T + Z_L = 50 + j50 \Omega \\
I &= \frac{V}{Z} = \frac{300}{(50 + j50)} = 3 - j3 \text{ A} \\
V_L &= I \times Z_L = (3-j3)(50+j40) = 270-j30 = 271.66 \angle -6.34^\circ \text{ V} \\
S_L &= P_L + jQ_L = V_L \times I^* = (270-j30)(3+j3) = 900+j720 \text{ VA} \\
|S_L| &= 1152.6 \text{ W} \\
PF &= \frac{P_L}{|S_L|} = 0.7809 \\
\end{align*}
\]

Another method for b: If the load and shunt capacitor has a terminal voltage $= V_L$

\[
\begin{align*}
Q_L + Q_C &= P_L \times \tan(25.84^\circ) = 436 \text{ MVar} \\
Q_C &= 436 - 270 = -284 \text{ MVar} \\
X_C &= \frac{|V_L|^2}{Q_C} = -259.7 \Omega \\
C &= -1/(\omega X_C) = -1/(377 \times 259.7) = 10.21 \mu \text{F} \\
\end{align*}
\]

\[
\begin{align*}
Z_{L_{\text{new}}} &= \frac{Z_L}{X_C} = \frac{Z_L \times jX_C}{(Z_L + jX_C)} \\
&= \frac{(50+j40) \times jX_C}{(50+j40+jX_C)} \\
&= \frac{(50+j40) \times (40/X_C + 1-j50/X_C)} \\
\text{Angle}(Z_{L_{\text{new}}}) &= \cos^{-1}(0.9) = 0.451 \text{ rad} = 25.84^\circ \\
&= \text{Angle}(Z_L) - \text{Angle}(40/X_C + 1-j50/X_C) \\
\text{Angle}(Z_L) &= 0.6747 \text{ rad} \\
\text{Angle}(40/X_C + 1-j50/X_C) &= 0.6747 - 0.451 = 0.2237 \text{ rad} \\
= -\frac{50}{(40/X_C + 1)} &= -\frac{50}{(40+X_C)} = \tan(0.2237) = 0.2275 \\
\text{Solve } X_C: \\
X_C &= -259.7 \Omega \\
C &= -1/(\omega X_C) = -1/(377 \times 259.7) = 10.21 \mu \text{F} \\
Z_{L_{\text{new}}} &= 66.42 + j32.17 \Omega \\
V_{L_{\text{new}}} &= \frac{Z_{L_{\text{new}}}}{(Z_T + Z_{L_{\text{new}}})} \times V \\
&= 279.56-j32.19 = 281.41 \angle -6.57^\circ \text{ V} \\
\end{align*}
\]
Problem 2 (25 points): A three-phase line has an impedance of \( Z_L = \text{j}2 \Omega \) and feeds two balanced three-phase loads connected in parallel. The first load is \( \Delta \)-connected and has an impedance of \( Z_1 = 24 \Omega \). The second load is \( Y \)-connected and has an impedance of \( Z_2 = 12 \Omega \). The line is energized at the sending end from a three-phase balanced supply of line-to-line voltage \( |V_L| = 415.69 \text{V} \). Take the line voltage \( V_{AB} \) on the supply side as the reference, i.e. \( V_{AB} = |V_L| \angle 0^\circ \).

a. Determine phase current \( I_A \), three-phase real power and reactive power drawn from the supply, and the power factor on the supply side.

b. Determine line voltages \( V_{ab} \) and phase voltage \( V_{an} \) at the combined load side. Draw a phasor diagram having \( V_{an}, V_{abs}, I_A \) and \( V_{AB} \).

c. Determine the total real and reactive powers absorbed by each three-phase load and the three-phase line.

\[ Z_{1Y} = \frac{24}{3} = 8 \Omega \]
\[ Z_L = \frac{Z_{1Y}}{Z_2} = 12 \times \frac{8}{(12+8)} = 4.8 \Omega \]
\[ V_{AB} = 415.69 \text{V} \]
\[ V_{An} \text{ lags } V_{AB} \text{ by } 30 \text{ degree} \]
\[ V_{An} = \frac{415.69}{\sqrt{3}} \angle -30^\circ = 240 \angle -30^\circ \text{ V} \]
\[ Z = Z_L + Z_L = 4.8 + \text{j}2\Omega \]
\[ I_a = \frac{V_{An}}{Z} = 28.02 - \text{j}36.67 = 46.15 \angle -52.62^\circ \]
\[ S_{\phi} = P_{\phi} + jQ_{\phi} = 3V_{An}I_a^* \]
\[ = 30675 + j12781 \text{VA} = 33230 \angle 22.62^\circ \text{ W} \]
\[ |S_{\phi}| = 33230 \text{ W} \]
\[ PF = \frac{P_{\phi}}{|S_{\phi}|} = 0.9231 \]

b. \[ V_{an} = I_aZ_L = 134.5 - \text{j}176.0 = 221.54 \angle -52.62^\circ \text{ V} \]
\[ V_{ab} = V_{an} \times \sqrt{3} \angle 30^\circ \]
\[ = 204.5 - \text{j}85.21 = 383.72 \angle -22.62^\circ \text{ V} \]

\[ I_a = 46.15 \angle -52.62^\circ \text{ A} \]
\[ V_{ab} = 383.72 \angle -22.62^\circ \text{ V} \]
\[ V_{an} = 221.54 \angle -52.62^\circ \text{ V} \]

\[ S_{\phi} = P_{\phi} + jQ_{\phi} = 3V_{an}I_a^* \]
\[ = 18405 \text{ VA} \]
\[ P_{\phi} = 18405 \text{ W} \]
\[ Q_{\phi} = 0 \]

\[ S_{\phi} = P_{\phi} + jQ_{\phi} = 3V_{an}I_a^* \]
\[ = 12270 \text{ VA} \]
\[ P_{\phi} = 12270 \text{ W} \]
\[ Q_{\phi} = 0 \]
Problem 3 (30 points): Short answers

a. Why did AC win over DC in the “AC vs. DC battle” in the 1890s? List at least two reasons.

   Give any two of the three:
   1. Voltage levels can easily be transformed in AC
   2. AC generators are simpler
   3. HVAC is easier to implement in order to reduce transmission loss

b. Which of these generation resources utilize steam turbines in generating electric power?
   - Coal-fired power plant
   - Combined-cycle power plant
   - Parabolic Trough
   - Solar Tower
   - Pressurized water reactor

   Check all

c. A load supplied by a voltage source has 0.7 power factor lagging. A shunt capacitor connected across the load will necessarily improve the power factor of the load side. True or false? Briefly explain why.

   False. If the capacitance is too large, it may become <1 power factor leading.

d. A two-winding transformer is first operated as a conventional two-winding transformer and then as an autotransformer to supply the same load. When it is operated as the autotransformer, it has a higher rating, a higher efficiency and a lower loss than it is operated as the conventional two-winding transformer. True or false? Briefly explain why.

   True. For the autotransformer,
   - rating (not the actual loading) will be increased,
   - loss will decrease since the currents in windings will decrease to generate the same amount of total current as the two-winding transformer does
   - then the efficiency will increase considering the loading is the same.
e. When a round-rotor generator is supporting a load as shown in the figure, $V$, $R_a$ and $X_s$ are respectively its emf, terminal voltage, resistance and synchronous reactance. Assume $R_a \approx 0$. Draw phasor diagrams to respectively illustrate two cases: $|E| = |V|$ and $|E| < |V|$. Describe the directions of real and reactive power flows for each case.

\[ |E| = |V| \quad \text{P is from the generator to the load and Q is 0.} \]
\[ |E| < |V| \quad \text{P is from the generator to the load and Q is from the load to the generator.} \]

f. (ECE521) Re-do the above question for a salient-pole generator, which has parameters $E$, $R_a \approx 0$, $X_d$ and $X_q$. Assume $X_d = 2X_q$.

\[ |E| = |V| \quad \text{P is from the generator to the load and Q is from the load to the generator} \]
\[ |E| < |V| \quad \text{P is from the generator to the load and Q is from the load to the generator.} \]
Problem 4 (30 points): For the electric power system, the three-phase power and line-line ratings are given below:

- G1: 80MVA 22kV X=10%
- T1: 90MVA 22/240kV X=15%
- T2: 90MVA 240/20kV X=20%
- M: 80MVA 20kV X=9%

The line has impedance $0+j100\Omega$ and the load at bus 4 absorbs 40MW+j70Mvar at 20kV. Both the generator and motor are round-rotor machines and the stator resistances are ignored.

a. Draw an impedance diagram showing all impedances in per-unit on a common base of 100MVA and 20kV on the generator side.

b. The motor operates at full-load 0.9 power factor leading with a terminal voltage of 20kV (bus 4). Take the voltage at bus 4 as the reference. Determine the generator’s terminal voltage (bus 1) in per-unit and kV, and determine the internal emfs of the generator and motor in per-unit and kV.

c. (ECE521) Under the above system condition in b, draw a phasor diagram including the following quantities in per-unit:
   i. The emfs of the generator and motor
   ii. The voltages at buses 1, 2, 3 and 4
   iii. The currents of the line, load and motor

d. (ECE521) If the voltage of bus 4 is maintained at 20kV, calculate the steady-state stability limit of the generator’s real power output in per-unit and in MW.

a. Once a voltage base has been selected for a point of a system, the remaining voltage bases are no longer independent; they are determined by the various transformer turns ratios

\[
\begin{align*}
V_{B1} &= 20kV \\
V_{B2} = V_{B3} &= 20 \times (240/22) = 218.1818 \text{ kV} \\
V_{B4} &= 218.1818 \times (20/240) = 18.1818 \text{ kV} \\
Z_{B2} &= (V_{B2})^2/S_B = (218.18)^2/100 = 476.025 \Omega \\
Z_T &= Z_{T}/Z_{B} = j0.2101 \text{ pu} \\
Z_L &= 20^2/(40-j70) = 2.4615+j4.3077 \Omega \\
Z_{B4} &= (V_{B4})^2/S_B = (18.18)^2/100 = 3.3051 \Omega \\
Z_L &= 0.7448+j1.3033 \text{ pu} \\
\end{align*}
\]

Impedance diagram
\( S_m = 80/100 \angle -25.84^\circ = 0.8 \angle -25.84^\circ \) pu

Take \( V_4 \) as the reference

\[
\begin{align*}
V_4 &= 20/VB_4 = 1.1 \text{pu} \\
I_m &= S_m*/V_4* = 0.8 \angle 25.84^\circ / 1.1 \\
&= 0.65 + j0.32 \text{pu} = 0.72 \angle 26.2^\circ \text{pu} \\
I_L &= V_4/Z_L = 1.1/0.7448 + j1.3033 \\
&= 0.36 - j0.64 \text{pu} = 0.73 \angle -60.6^\circ \text{pu} \\
I &= I_m + I_L = 1.01 - j0.32 \text{pu} = 1.06 \angle -17.6^\circ \text{pu}
\end{align*}
\]

\[
\begin{align*}
V_1 &= V_4 + (X_{T1} + Z_T + X_{T2}) \times I = 1.1 + j0.6807 \times (1.01 - j0.32) \\
&= 1.3178 + j0.6875 \text{pu} = 1.4864 \angle 27.55^\circ \text{pu} \\
&= 1.4864 \times \text{VB}_1 \angle 27.55^\circ = 29.6 \angle 27.55^\circ \text{kV}
\end{align*}
\]

\[
\begin{align*}
E_g &= V_1 + X_G \times I = 1.3662 + j0.8403 \\
&= 1.604 \angle 31.6^\circ \text{pu} = 32 \angle 31.6^\circ \text{kV}
\end{align*}
\]

\[
\begin{align*}
E_m &= V_4 - X_m \times I = 1.1436 - j0.0885 \\
&= 1.1470 \angle -4.42^\circ \text{pu} = 20.73 \angle -4.42^\circ \text{kV}
\end{align*}
\]
c. 

\[ V_2 = 1.34 \angle 21.41^\circ \quad V_3 = 1.22 \angle 13.24^\circ \]

d. 

\[ P_{\text{max}}(3\phi) = \frac{|E_g||V_4|}{X_G + X_{T1} + X_{T2} + Z_T} = 3 \times \frac{1.60 \times 1.1}{0.15 + 0.20 + 0.21 + 0.27} = 2.17\, \text{pu} = 2.17 \times S_{B1} = 217\, \text{MW} \]