ECE 325 – Electric Energy System Components
2- Fundamentals of Electrical Circuits

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Content

• Fundamentals of electrical circuits (Ch. 2.0-2.15, 2.32-2.39)

• Active power, reactive power and apparent power (Ch. 7)

• Three-phase AC systems (Ch. 8)
Notations: Current and Alternating Current

- **Arbitrarily** determine a positive direction, e.g. 1→2
  - If a current of 2A flows from 1 to 2, \( I = +2A \)
  - If a current of 2A flows from 2 to 1, \( I = -2A \)
Notations: Voltage

1. Double-subscript notation:
   \[ E_{21} = +100\text{V} \] (the voltage between 2 and 1 is 100V, and 2 is positive w.r.t 1)
   \[ E_{12} = -100\text{V} \]

2. Sign notation:
   Arbitrarily mark a terminal with (+); \( E > 0 \) if and only if that marked terminal is positive w.r.t the other.
   E.g. if \( E_{21} = +100\text{V} \), \( E = E_{21} = +100\text{V} \).

Both the double-subscript notation and sign notation apply to alternating voltage
Notations: Alternating Voltage
Notations: Sources and Loads

• **Definition**: given the *instantaneous, actual* polarity of voltage and *actual* direction of current
  – **Actual Source**: whenever current flows out of the terminal (+)
  – **Actual Load**: whenever current flows into the terminal (+)

• How about these?
  – Resistor, battery cell, electric motor, capacitor and inductor
\[ v(t) \] (Resistor)

\[ i(t) \]

\[ v(t) \] (Inductor)

\[ i(t) \] (Capacitor)
1-Phase AC System with Sinusoidal Voltage and Current

\[ i(t) = I_m \cos(\omega t + \theta_i) \]

\[ e(t) = E_m \cos(\omega t + \theta_e) \]

- \( e, i \): instantaneous voltage (V) and current (A)
- \( E_m, I_m \): peak values of the sinusoidal voltage (V) and current (A)
- \( \omega = 2\pi f \) (rad/s): angular frequency, which is assumed constant here
- \( \theta_e, \theta_i \): constant phase angles (rad. or deg.) of voltage and current
- \( E_m/\sqrt{2}, I_m/\sqrt{2} \): RMS (root-mean-square, effective) values

\[ \int_{t-T}^{t} [i(t)]^2 R dt = \frac{I_m^2 RT}{2} = I_{dc}^2 RT \]

\[ \rightarrow I_{dc} = \frac{I_m}{\sqrt{2}} \triangleq \text{RMS value} \]

Equal heating effects
Phasor Representation

\[ e(t) = E_m \cos(\omega t + \theta_e) = \sqrt{2} |E| \cos(\omega t + \theta_e) \]
\[ i(t) = I_m \cos(\omega t + \theta_i) = \sqrt{2} |I| \cos(\omega t + \theta_i) \]

\[ E = \frac{E_m}{\sqrt{2}} \angle \theta_e = |E| \angle \theta_e = |E| e^{j \theta_e} \]
\[ I = \frac{I_m}{\sqrt{2}} \angle \theta_i = |I| \angle \theta_i = |I| e^{j \theta_i} \]

- \( E \) and \( I \) are called RMS phasors of \( e(t) \) and \( i(t) \);
  \( E \) leads \( I \) by \( \theta = \theta_e - \theta_i \) or in other words, \( I \) leads \( E \) by \( 2\pi - \theta \)
- Phasor:
  - mapping a time-domain sinusoidal waveform (infinitely long in time) to a single complex number
  - carries the amplitude and phase angle information of a sinusoidal signal of a common frequency (\( \omega \)) w.r.t. a chosen reference signal.
Impedance

- Impedance is a complex number (in $\Omega$) defined as
  \[ Z \stackrel{\text{def}}{=} \frac{E_{12}}{I} = \frac{E}{I} = \frac{|E| \angle \theta_e}{|I| \angle \theta_i} = |Z| \angle (\theta_e - \theta_i) = R + jX \]
  \[ \theta_e - \theta_i = \theta \] (Impedance angle)

- Purely resistive: \( Z = |Z| = R \)

- Purely inductive: \( Z = |Z| \angle 90^\circ = jX = jX_L = j\omega L \)

- Purely capacitive: \( Z = |Z| \angle -90^\circ = jX = -jX_C = -j \frac{1}{\omega C} \)
Example 2-5

- Draw the phasor diagram of the voltage and current at a frequency of 60Hz. Calculate the time interval $\Delta t$ between the positive peaks of $E$ and $I$

Solution:

$\omega=2\pi f=377 \text{ (rad/s)}=21600 \text{ (deg/s)}$

$|E|=\frac{339}{\sqrt{2}}=240 \text{ (V)}$

$|I|=\frac{14.1}{\sqrt{2}}=10 \text{ (A)}$

Choose an arbitrary reference to draw phasors $E$ and $I$

$\Delta t=\frac{\theta}{\omega}=\frac{30}{21600}=0.00139 \text{ (s)}$

$$e(t) = 339 \cos(21600t - 90^\circ)$$

$$i(t) = 14.1 \cos(21600t - 120^\circ)$$
Kirchhoff’s Voltage Law (KVL)

• The algebraic sum of the voltages around a closed loop (CW/CCW) is zero
  \( (\Sigma \text{ voltage rises} = \Sigma \text{ voltage drops}) \)
  – Applied to both instantaneous voltages or voltage phasors

• Loop 24312 (BCDA, CW):
  \[ E_{24} + E_{43} + E_{31} + E_{12} = 0 \quad \text{or} \quad e_{24} + e_{43} + e_{31} + e_{12} = 0 \]

• Loop 2342 (ECF, CCW):
  \[ E_{23} + E_{34} + E_{42} = 0 \quad \text{or} \quad e_{23} + e_{34} + e_{42} = 0 \]
Kirchhoff’s Current Law (KCL)

• The algebraic sum of the currents arriving at a node is equal to 0. (Σ currents in = Σ currents out)

• Node A:

\[ I_1 + I_3 = I_2 + I_4 + I_5 \]

or \[ i_1 + i_3 = i_2 + i_4 + i_5 \]

\[ I_1 + I_3 + (-I_2) + (-I_4) + (-I_5) = 0 \]

or \[ i_1 + i_3 + (-i_2) + (-i_4) + (-i_5) = 0 \]
Kirchhoff’s Laws and AC Circuits

• KVL
  – Loop 24312, CW:
    \[ E_{24} + E_{43} + E_{31} + E_{12} = 0 \]
    \[ I_2Z_2 - I_3Z_3 + E_b - I_1Z_1 = 0 \]
  – Loop 2342, CCW:
    \[ E_{23} + E_{34} + E_{42} = 0 \]
    \[ I_4Z_4 + I_3Z_3 - I_2Z_2 = 0 \]
  – Loop 242, CW:
    \[ E_{24} + E_{42} = 0 \]
    \[ E_a - I_2Z_2 = 0 \]

• KCL
  – Node 2:
    \[ I_5 - I_2 - I_4 - I_1 = 0 \]
  – Node 3:
    \[ I_4 + I_1 - I_3 = 0 \]
Examples 2-14 to 2-16
**Instantaneous Power**

\[ i(t) = I_m \cos(\omega t + \theta_i) \]

\[ e(t) = E_m \cos(\omega t + \theta_e) \]

Impedance angle: \( \theta = \theta_e - \theta_i \)

- >0 for inductive load and
- <0 for capacitive load

Using trigonometric identity

\[ \cos A \cos B = \frac{1}{2} \left[ \cos(A - B) + \cos(A + B) \right] \]

\[ p(t) = e(t)i(t) = E_m I_m \cos(\phi + \theta_e) \cos(\phi + \theta_i) \]

\[ = \frac{1}{2} E_m I_m \left[ \cos(\theta_e - \theta_i) + \cos(2\phi + \theta_e + \theta_i) \right] \]

\[ = \frac{1}{2} E_m I_m \left\{ \cos(\theta_e - \theta_i) + \cos[2(\phi + \theta_e) - (\theta_e - \theta_i)] \right\} \]

\[ = \frac{1}{2} E_m I_m \left\{ \cos \theta + \cos[2(\phi + \theta_e) - \theta] \right\} \]

\[ = \frac{1}{2} E_m I_m \left[ \cos \theta + \cos \theta \cos 2(\phi + \theta_e) + \sin \theta \sin 2(\phi + \theta_e) \right] \]

\[ p(\phi) = \frac{1}{2} E_m I_m \left[ \cos \theta + \cos \theta \cos 2(\phi + \theta_e) + \sin \theta \sin 2(\phi + \theta_e) \right] \]

\[ = p_R + p_X \]

- \( p_R = \frac{1}{2} E_m I_m \cos \theta [1 + \cos 2(\phi + \theta_e)] \)
- \( p_X = \frac{1}{2} E_m I_m \sin \theta \sin 2(\phi + \theta_e) \)

| | \( E_m \)/\sqrt{2} |
|---|-----------------
| | \( I_m \)/\sqrt{2} |