ECE 325 – Electric Energy System Components
2- Fundamentals of Electrical Circuits

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Content

• Fundamentals of electrical circuits (Ch. 2.0-2.15, 2.32-2.39)

• Active power, reactive power and apparent power (Ch. 7)

• Three-phase AC systems (Ch. 8)
Notations: Current and Alternating Current

- Arbitrarily determine a positive direction, e.g. 1→2
  - If a current of 2A flows from 1 to 2, $I=+2A$
  - If a current of 2A flows from 2 to 1, $I=-2A$
Notations: Voltage

1. **Double-subscript notation:**
   \[ E_{21} = +100\text{V} \] (the voltage between 2 and 1 is 100V, and 2 is positive w.r.t 1)
   \[ E_{12} = -100\text{V} \]

2. **Sign notation:**
   Arbitrarily mark a terminal with (+); \( E > 0 \) if and only if that marked terminal is positive w.r.t the other.
   E.g. if \( E_{21} = +100\text{V} \), \( E = E_{21} = +100\text{V} \).

Both the double-subscript notation and sign notation apply to alternating voltage
Notations: Alternating Voltage
Notations: Sources and Loads

- **Definition**: given the instantaneous, **actual** polarity of voltage and **actual** direction of current
  - **Actual Source**: whenever current flows out of the terminal (+)
  - **Actual Load**: whenever current flows into the terminal (+)
- How about these?
  - Resistor, battery cell, electric motor, capacitor and inductor
1-Phase AC System with Sinusoidal Voltage and Current

\[ i(t) = I_m \cos(\omega t + \theta_i) \]

\[ e(t) = E_m \cos(\omega t + \theta_e) \]

- \( e, i \): instantaneous voltage (V) and current (A)
- \( E_m, I_m \): peak values of the sinusoidal voltage (V) and current (A)
- \( \omega = 2\pi f \) (rad/s): angular frequency, which is assumed constant here
- \( \theta_e, \theta_i \): constant phase angles (rad. or deg.) of voltage and current
- \( E_m/\sqrt{2}, I_m/\sqrt{2} \): RMS (root-mean-square, effective) values

\[ \int_{t-T}^{t} [i(t)]^2 R dt = \frac{I_m^2 RT}{2} = I_{dc}^2 RT \]

\[ \rightarrow I_{dc} = \frac{I_m}{\sqrt{2}} \triangleq \text{RMS value} \]

Equal heating effects
Phasor Representation

\[ e(t) = E_m \cos(\omega t + \theta_e) = \sqrt{2} |E| \cos(\omega t + \theta_e) \]
\[ i(t) = I_m \cos(\omega t + \theta_i) = \sqrt{2} |I| \cos(\omega t + \theta_i) \]

\[ E = \frac{E_m}{\sqrt{2}} \angle \theta_e = |E| \angle \theta_e = |E| e^{j\theta_e} \]
\[ I = \frac{I_m}{\sqrt{2}} \angle \theta_i = |I| \angle \theta_i = |I| e^{j\theta_i} \]

- \( E \) and \( I \) are called RMS phasors of \( e(t) \) and \( i(t) \);
- \( E \) leads \( I \) by \( \theta = \theta_e - \theta_i \) or in other words, \( I \) leads \( E \) by \( 2\pi - \theta \)

- Phasor:
  - mapping a time-domain sinusoidal waveform (infinitely long in time) to a single complex number
  - carries the amplitude and phase angle information of a sinusoidal signal of a common frequency \( (\omega) \) w.r.t. a chosen reference signal.
Impedance

- Impedance is a complex number (in \( \Omega \)) defined as

\[
Z = \frac{E_{12}}{I} = \frac{E}{I} = \left| \frac{E}{I} \right| \angle \theta_e = \left| Z \right| \angle (\theta_e - \theta_i) = R + jX
\]

\[\theta_e - \theta_i = \theta \] (Impedance angle)

- Purely resistive: \( Z = |Z| = R \)

- Purely inductive: \( Z = |Z| \angle 90^\circ = jX = jX_L = j\omega L \)

- Purely capacitive: \( Z = |Z| \angle -90^\circ = jX = -jX_C = -j \frac{1}{\omega C} \)
Example 2-5

• Draw the phasor diagram of the voltage and current at a frequency of 60Hz. Calculate the time interval $\Delta t$ between the positive peaks of $E$ and $I$

Solution:

$\omega=2\pi f=377$ (rad/s) = 21600 (deg/s)

$|E|=\frac{339}{\sqrt{2}}=240$ (V)

$|I|=\frac{14.1}{\sqrt{2}}=10$ (A)

Choose an arbitrary reference to draw phasors $E$ and $I$

$\Delta t=\frac{\theta}{\omega}=\frac{30}{21600}=0.00139$ (s)