Voltage regulation

Voltage Regulation = \frac{|E_{2,NL}| - |E_{2,FL}|}{|E_{2,FL}|} \times 100\%

(if ignoring \(X_C\)) \approx \frac{|E_1| - |E_{2,FL}|}{|E_{2,FL}|} \times 100\%

- VR is a measure of line voltage drop and usually should not exceed ±5% (or ±10%)
- VR depends on the load power factor:
  - VR is often high (bad) for a low lagging power factor
  - Perhaps, VR<0 for a leading power factor (i.e. \(|E_1|<|E_2|\)).
  - If ignore \(X_C\), three typical loads with lagging, unity and leading power factors

<table>
<thead>
<tr>
<th>Voltage Regulation</th>
<th>Lagging PF</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E_1)</td>
<td>(E_2)</td>
</tr>
<tr>
<td>(I_2)</td>
<td>(I_2R)</td>
</tr>
<tr>
<td>(\theta)</td>
<td></td>
</tr>
<tr>
<td>(jI_2X_L)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Voltage Regulation</th>
<th>Unity PF</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E_1)</td>
<td>(E_2)</td>
</tr>
<tr>
<td>(jI_2X_L)</td>
<td>(jI_2X_L)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Voltage Regulation</th>
<th>Leading PF</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E_1)</td>
<td>(E_2)</td>
</tr>
<tr>
<td>(I_2)</td>
<td>(I_2R)</td>
</tr>
<tr>
<td>(\theta)</td>
<td></td>
</tr>
</tbody>
</table>
Resistive line

• There is an upper limit on the power transferred by the line to the load

\[
P = |E_R| \cdot |I| = |I|^2 (kR) = \left| \frac{E_S}{R + kR} \right|^2 kR
\]

\[
= \frac{|E_S|^2}{R(1/k + k + 2)} \leq \frac{|E_S|^2}{4R}
\]

\[P = P_{\text{max}} = |E_S|^2/4R \text{ when } k=1, \text{i.e. } E_R = E_S/2\]

• If \(E_R \geq 0.95E_S\) (around 5% VR) is required, the line can support a load that is only 19% of \(P_{\text{max}}\)

• The total power from the sender is \(P + |I|^2 R\)

• VR is a key factor that limit the power transmission capacity
**Inductive line**

\[ P = |I|^2 (kX) = \frac{E_s}{jX + kX} \]

\[ = \frac{|E_s|^2}{kX + X/k} \leq \frac{|E_s|^2}{2X} \]

\[ P_{\text{max}} = |E_s|^2/2X \text{ when } k=1, \ |E_R| = |E_s|/\sqrt{2} = 0.707|E_s| \]

- A inductive line can deliver twice as much power as a resistive line (if \(X=R\))
- If \(E_R \geq 0.95E_s\) (around 5% VR) is required, the line can support a load that is 60% of \(P_{\text{max}}\), i.e. 3x as much as power as a resistive line
- VR is a key factor that limit the power transmission capacity
- The total power from the sender is \(P + j|I|^2X\)

**Figure 25.22**
Characteristics of an inductive line.
Inductive line connecting two systems

\[ S = E_S I^* = E_S \left( \frac{E_S - E_R}{jX} \right)^* = |E_S| \angle \delta \left( \frac{|E_S| \angle - \delta - |E_R| \angle 0^\circ}{X \angle -90^\circ} \right) \]

\[ = \frac{|E_S|^2}{X} \angle 90^\circ - \frac{|E_S||E_R|}{X} \angle (\delta + 90^\circ) \]

\[ P = \frac{|E_S||E_R|}{X} \cos(\delta + 90^\circ) = \frac{|E_S||E_R|}{X} \sin \delta \approx \frac{|E_S||E_R|}{X} \delta \text{ (in rad)} \]

\[ Q = \frac{|E_S|^2}{X} - \frac{|E_S||E_R|}{X} \sin(\delta + 90^\circ) = \frac{|E_S|^2}{X} - \frac{|E_S||E_R|}{X} \cos \delta \]

\[ = \frac{|E_S|}{X} (|E_S| - |E_R| \cos \delta) \approx \frac{|E_S|}{X} (|E_S| - |E_R|) \]

\[ P = \frac{|E_S||E_R|}{X} \sin \delta \]

- The size of reactive flow depends on the voltage drop from the sending end to the receiving end.
- If \(|E_S| = |E_R| = E\), \(Q \approx 0\), i.e. almost no reactive flow

\[ P = \frac{E^2}{X} \sin \delta \leq \frac{E^2}{X} \]

**Figure 25.30a**
Power versus angle characteristic.
Increasing the power transmission capacity

\[ S = P + jQ = \frac{|E_S||E_R|}{X} \sin \delta + j|I|^2X \]

1. Use a shunt capacitor
   - To have \( |E_R| = |E_S| \), the line can fully be compensated by adding a shunt capacitor to the receiving end whose \( X_C \) is adjustable so that
     \[ |E_S|^2/X_C = 0.5|I|^2X \]
   - Thus, \( P_{\text{max}} = |E_S|^2/X \)

2. Use parallel lines:
   - \( X \rightarrow X/2 \ldots X/N \), so \( P_{\text{max}} \rightarrow 2P_{\text{max}} \ldots N \times P_{\text{max}} \)
   - Also improving security against a line trip.

3. Use a series capacitor
   - \( P_{\text{max}} = |E_S|^2/(X-X_{CS}) \)
   - It may cause sub-synchronous resonance (SSR)
     \[ f_{\text{SSR}} = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} = f \sqrt{\frac{X_{CS}}{X}} \]
   - If \( X_{CS}/X = 1/4 \) (25% compensation), \( f_{\text{SSR}} = f/2 = 30 \text{Hz} \)
Voltage Regulation for EHV lines

A 3-phase 735kV 60Hz 600km line, operated at 727kV, has inductive reactance of 0.5 Ω/km and capacitive reactance of 300kΩ/km.

- At no-load (open-circuit) conditions, for each phase,
  \[ |E_S| = \frac{727}{\sqrt{3}} = 420 \text{kV}, \]
  \[ X_L = 0.5 \times 600 = 300 \Omega, \quad X_C = \frac{300k}{600} = 500 \Omega, \]
  \[ X_{C1} = X_{C2} = 2X_C = 1000 \Omega \]
  \[ E_R = E_S \times \frac{-jX_{C2}}{jX_L - jX_{C2}} = 420 \angle 0^\circ \times 1000 / (1000 - 300) = 600 \angle 0^\circ \text{kV} \]

To bring \( |E_R| \) back to \( |E_S| \), add a shunt reactor of \( X_{L2} \) at the receiving end:

- If \( X_{L2} = X_{C2} \), then \( -jX_{C2} / jX_{L2} = \infty \) and \( |E_R| = |E_S| \).

The reactive power generated by \( X_{C2} \) is entirely absorbed by \( X_{L2} \) (cancelling each other)

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**Figure 25.35**
EHV transmission line at no-load.

**Figure 25.36**
EHV reactor compensation.
Surge-impedance load (SIL)

• When connected to a gradually increasing load with PF=1, $|E_R|$ decreases from $|E_{R,\text{NL}}|$ (open-circuit) to 0 (short-circuit).

• When $|E_R|=|E_S|$, the amount of load is called the surge-impedance load (SIL) and the corresponding load impedance is called the surge impedance, which has $Z_Y \approx 400\Omega$ (line-to-neutral impedance) for aerial lines

\[
\text{SIL} = \frac{E_S^2}{Z_Y} = \frac{E_S^2}{400} \text{ (MW)}
\]

$E_S$: 3-phase line voltage in kV

\[
\text{e.g. a line of 600 km}
\]

\[
\text{SIL} = \frac{727^2}{400} = 1320\text{MW}
\]

Figure 25.38
Surge impedance loading of a line.
Inter-region power exchange: Example 25-8

Calculate: (1) the power transmitted by the line and (2) the required phase-shift enabling transmitting 70MW from $E_a$ to $E_b$

(1) $|E_a|=|E_b|=E$, so $P_{ab}=(E^2/X)\sin(\delta_a-\delta_b)=(100^2/20) \sin(-11^\circ)=-95.4\text{MW}$ (in fact, $a\leftarrow b$)

(2) $P_{ab}=70=(100^2/20) \sin(\delta_a-\delta_b)$, so $\delta_a-\delta_b=8^\circ$, i.e. $8-(-11)=19^\circ$ phase-shift of $E_a$

Figure 25.41a
An ordinary transmission line causes power to flow in the wrong direction.

Figure 25.41b
A phase-shift autotransformer can force power to flow in the desired direction (Example 25-8).