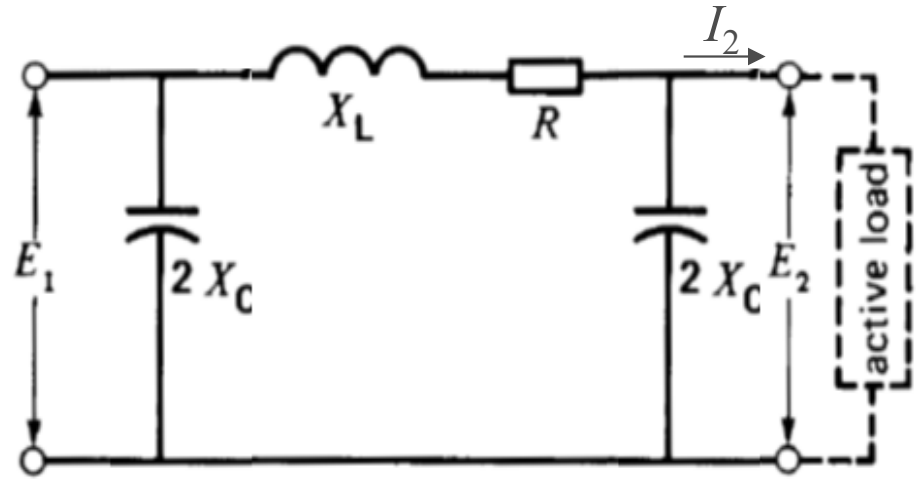
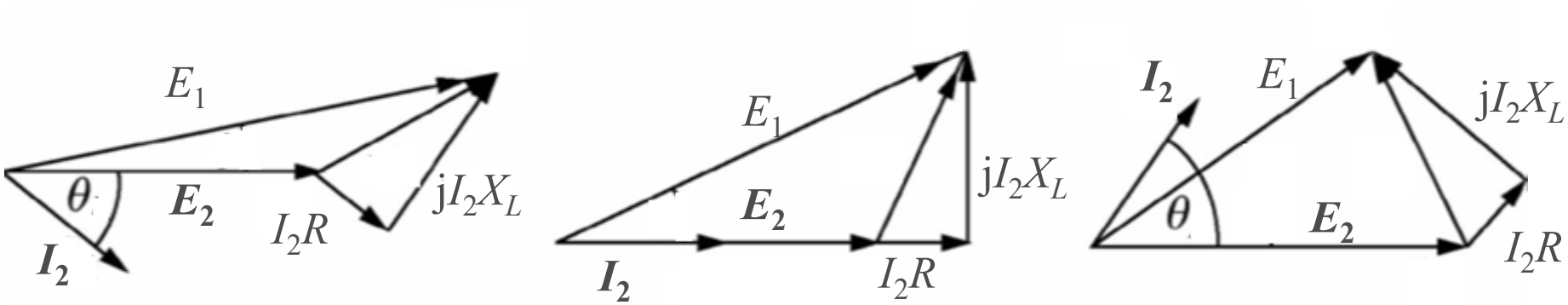
Voltage regulation

$$\text{Voltage Regulation} = \frac{|E_{2,NL}| - |E_{2,FL}|}{|E_{2,FL}|} \times 100\%$$

(if ignoring X_C) $$\approx \frac{|E_1| - |E_{2,FL}|}{|E_{2,FL}|} \times 100\%$$



- VR is a measure of line voltage drop and usually should not exceed ±5% (or ±10%)
- VR depends on the load power factor:
  - VR is often high (bad) for a low lagging power factor
  - Perhaps, VR<0 for a leading power factor (i.e. $|E_1|<|E_2|$).
  - If ignore X_C, three typical loads with lagging, unity and leading power factors



Lagging PF Unity PF Leading PF

Resistive line

- There is an upper limit on the power transferred by the line to the load

$$P = |E_R| \cdot |I| = |I|^2 (kR) = \left| \frac{E_S}{R + kR} \right|^2 kR$$

$$= \frac{|E_S|^2}{R(1/k + k + 2)} \le \frac{|E_S|^2}{4R}$$

$P = P_{max} = |E_S|^2/4R$ when $k=1$, i.e. $E_R = E_S/2$

- If $E_R \ge 0.95 E_S$ (around 5% VR) is required, the line can support a load that is only 19% of P_{max}
- The total power from the sender is $P + |I|^2 R$
- VR is a key factor that limit the power transmission capacity

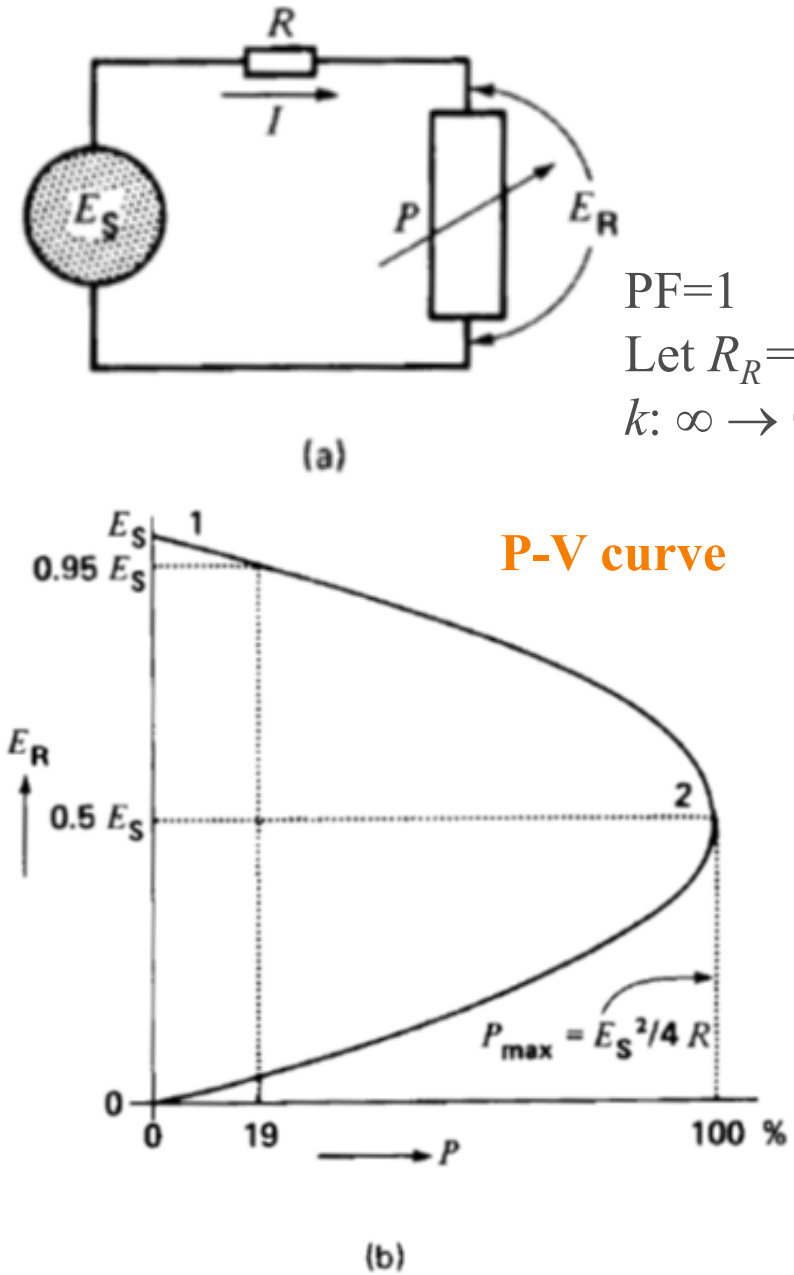


Figure 25.21
Characteristics of a resistive line.

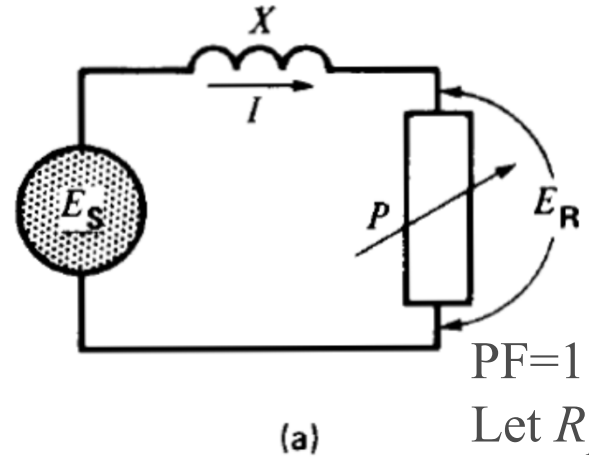
Inductive line

$$P = |I|^2 (kX) = \left| \frac{E_S}{jX + kX} \right|^2 kX$$

$$= \frac{|E_S|^2}{kX + X/k} \le \frac{|E_S|^2}{2X}$$

$P_{max}=|E_S|^2/2X$ when k=1, $|E_R|=|E_S|/\sqrt{2}=0.707|E_S|$

- A inductive line can deliver twice as much power as a resistive line (if X=R)
- If $E_R \ge 0.95E_S$ (around 5% VR) is required, the line can support a load that is 60% of P_{max}, i.e. 3x as much as power as a resistive line
- VR is a key factor that limit the power transmission capacity
- The total power from the sender is $P+j|I|^2X$



PF=1
Let $R_R=kX$
k: $\infty \rightarrow 0$

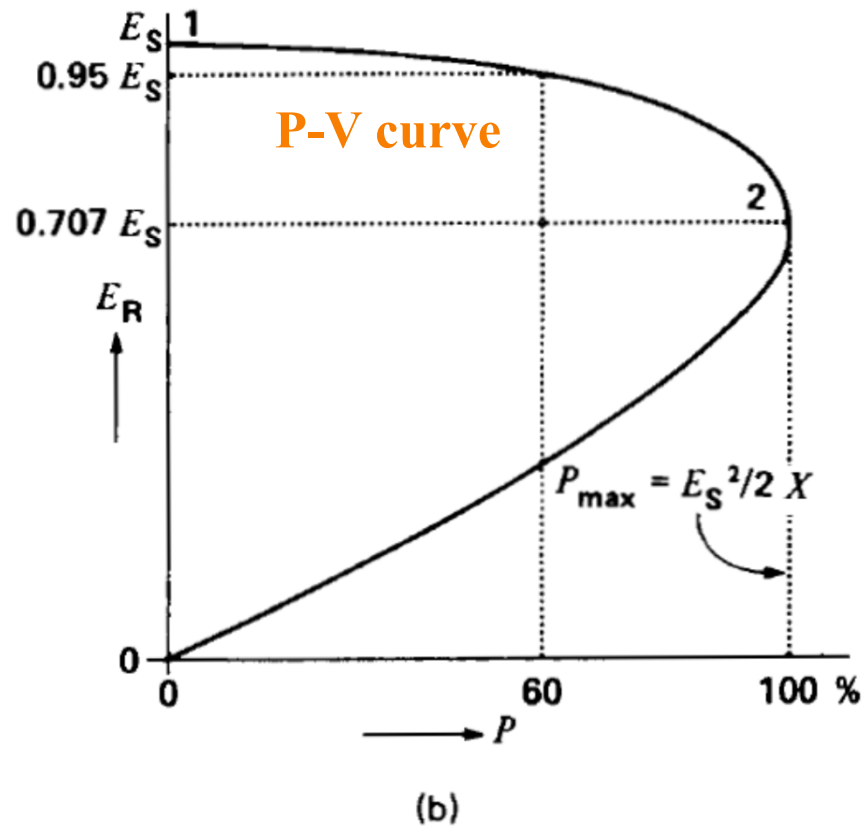
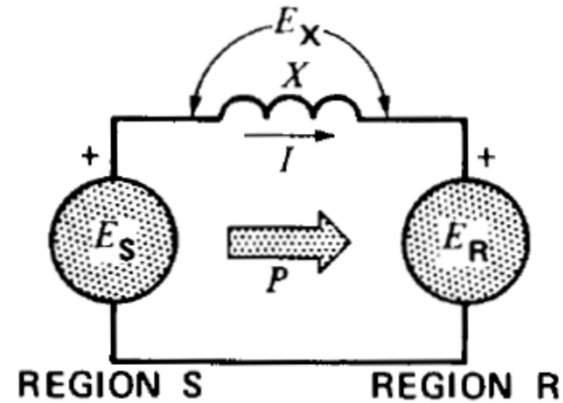


Figure 25.22
Characteristics of an inductive line.

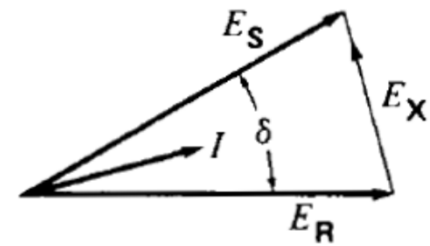
Inductive line connecting two systems



$$S = E_S I^* = E_S \left(\frac{E_S - E_R}{jX} \right)^* = |E_S| \angle \delta \left(\frac{|E_S| \angle -\delta - |E_R| \angle 0^\circ}{X \angle -90^\circ} \right)$$

$$= \frac{|E_S|^2}{X} \angle 90^\circ - \frac{|E_S||E_R|}{X} \angle (\delta + 90^\circ)$$

$$P = -\frac{|E_S||E_R|}{X} \cos(\delta + 90^\circ) = \frac{|E_S||E_R|}{X} \sin\delta \approx \frac{|E_S||E_R|}{X} \delta (\text{in rad})$$



$$Q = \frac{|E_S|^2}{X} - \frac{|E_S||E_R|}{X} \sin(\delta + 90^\circ) = \frac{|E_S|^2}{X} - \frac{|E_S||E_R|}{X} \cos\delta$$

$$= \frac{|E_S|}{X} \left(|E_S| - |E_R| \cos\delta \right) \approx \frac{|E_S|}{X} \left(|E_S| - |E_R| \right)$$

$$\boxed{P = \frac{|E_S||E_R|}{X} \sin\delta}$$

- The size of reactive flow depends on the voltage drop from the sending end to the receiving end.
- If $|E_S|=|E_R|=E$, $Q \approx 0$, i.e. almost no reactive flow

$$\boxed{P = \frac{E^2}{X} \sin\delta \le \frac{E^2}{X}}$$

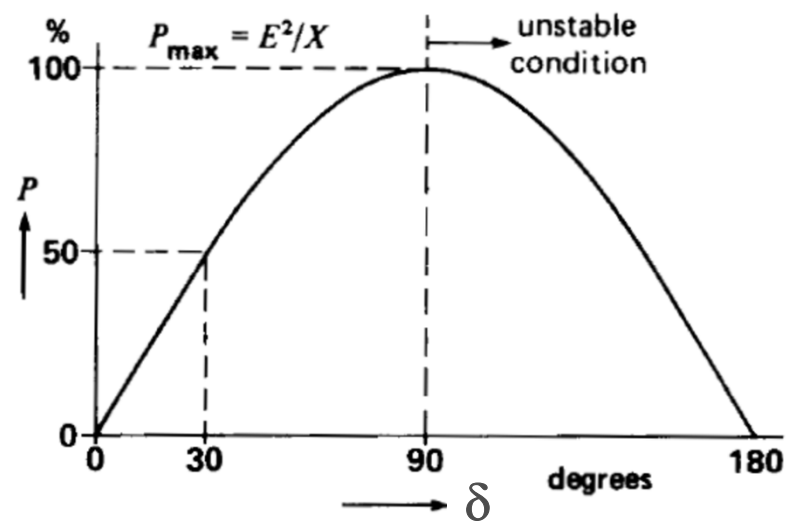


Figure 25.30a
Power versus angle characteristic.

Increasing the power transmission capacity

$$S = P + jQ = \frac{|E_S||E_R|}{X}\sin\delta + j|I|^2 X$$

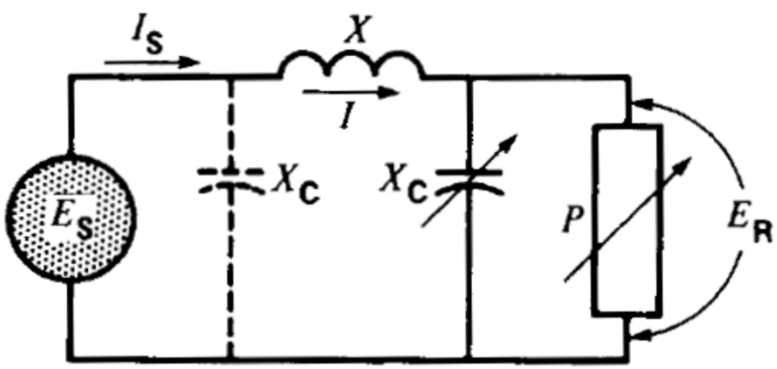
1. Use a shunt capacitor

- To have $|E_R|=|E_S|$, the line can fully be compensated by adding a shunt capacitor to the receiving end whose X_C is adjustable so that

$$|E_S|^2/X_C = 0.5|I|^2 X$$

while the other $0.5|I|^2X$ is provided by the source or another capacitor with X_C at the sending end.

- Thus, $P_{max}=|E_S|^2/X$



To further increase P_{max}, an approach is to reduce line reactance X

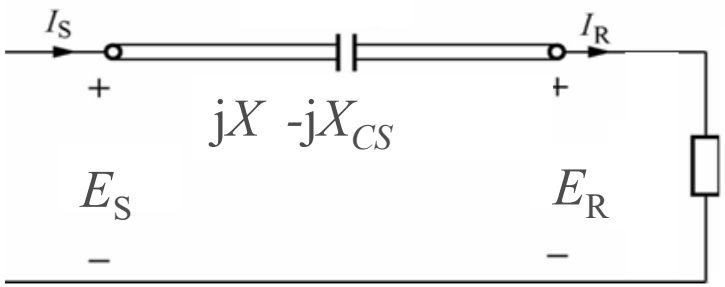
2. Use parallel lines:

$X \rightarrow X/2 \ldots X/N$, so $P_{max} \rightarrow 2P_{max} \ldots N \times P_{max}$

Also improving security against a line trip.

3. Use a series capacitor

$P_{max}=|E_S|^2/(X-X_{CS})$



- It may cause sub-synchronous resonance (SSR)

$$f_{SSR} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = f\sqrt{\frac{X_{CS}}{X}}$$

If $X_{CS}/X=1/4$ (25% compensation), $f_{SSR}=f/2$ =30Hz

Voltage Regulation for EHV lines

A 3-phase 735kV 60Hz 600km line, operated at 727kV, has inductive reactance of 0.5 Ω/km and capacitive reactance of 300kΩ/km.

- At no-load (open-circuit) conditions,

  for each phase,

  $|E_S|=727/\sqrt{3}=420\text{kV}$,

  $X_L=0.5\times600=300\Omega$, $X_C=300\text{k}/600=500\Omega$,

  $X_{C1}=X_{C2}=2X_C=1000\ \Omega$

  $E_R=E_S\times(-jX_{C2})/(jX_L-jX_{C2})$
  $=420\angle0^\circ\times1000/(1000-300)=600\angle0^\circ\ \text{kV}$

To bring $|E_R|$ back to $|E_S|$, add a shunt reactor of X_{L2} at the receiving end:

If $X_{L2}=X_{C2}$, then $-jX_{C2}//jX_{L2}=\infty$ and $|E_R|=|E_S|$.

The reactive power generated by X_{C2} is entirely absorbed by X_{L2} (cancelling each other)

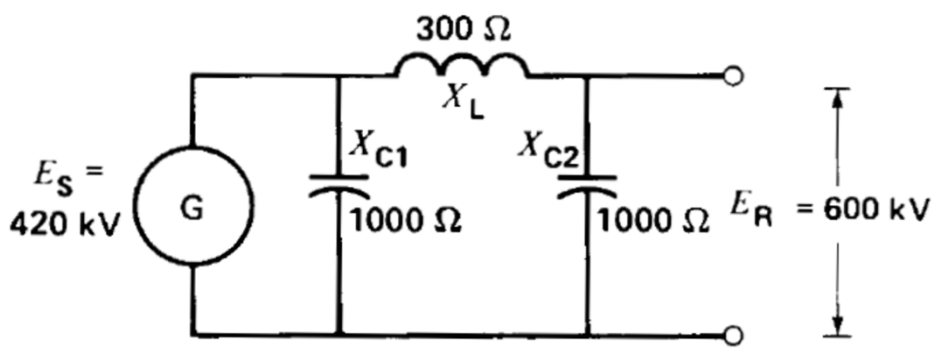


Figure 25.35
EHV transmission line at no-load.

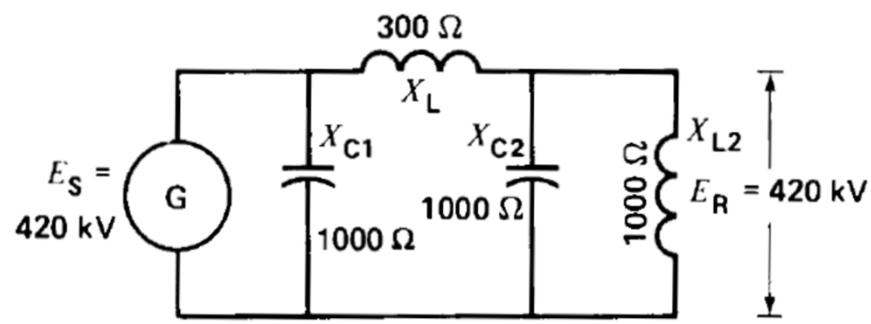


Figure 25.36
EHV reactor compensation.

Surge-impedance load (SIL)

- When connected to a gradually increasing load with PF=1, $|E_R|$ decreases from $|E_{R,NL}|$ (open-circuit) to 0 (short-circuit).
- When $|E_R|=|E_S|$, the amount of load is called the surge-impedance load (SIL) and the corresponding load impedance is called the surge impedance, which has $Z_Y \approx 400\Omega$ (line-to-neutral impedance) for aerial lines

$$\text{SIL} = E_S^2/Z_Y = E_S^2/400 \text{ (MW)} \qquad E_S\text{: 3-phase line voltage in kV}$$

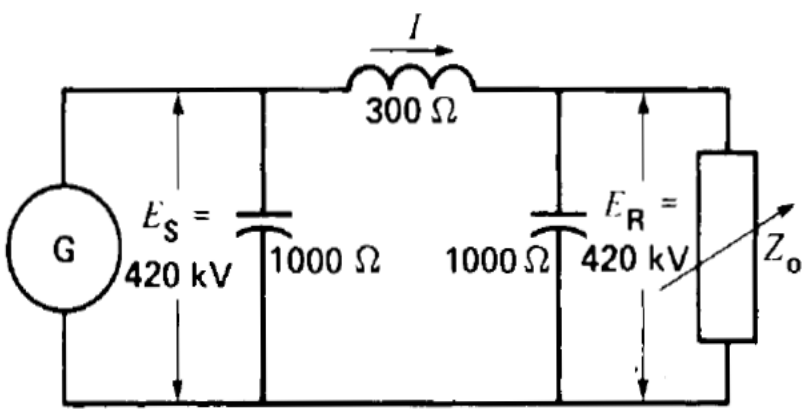
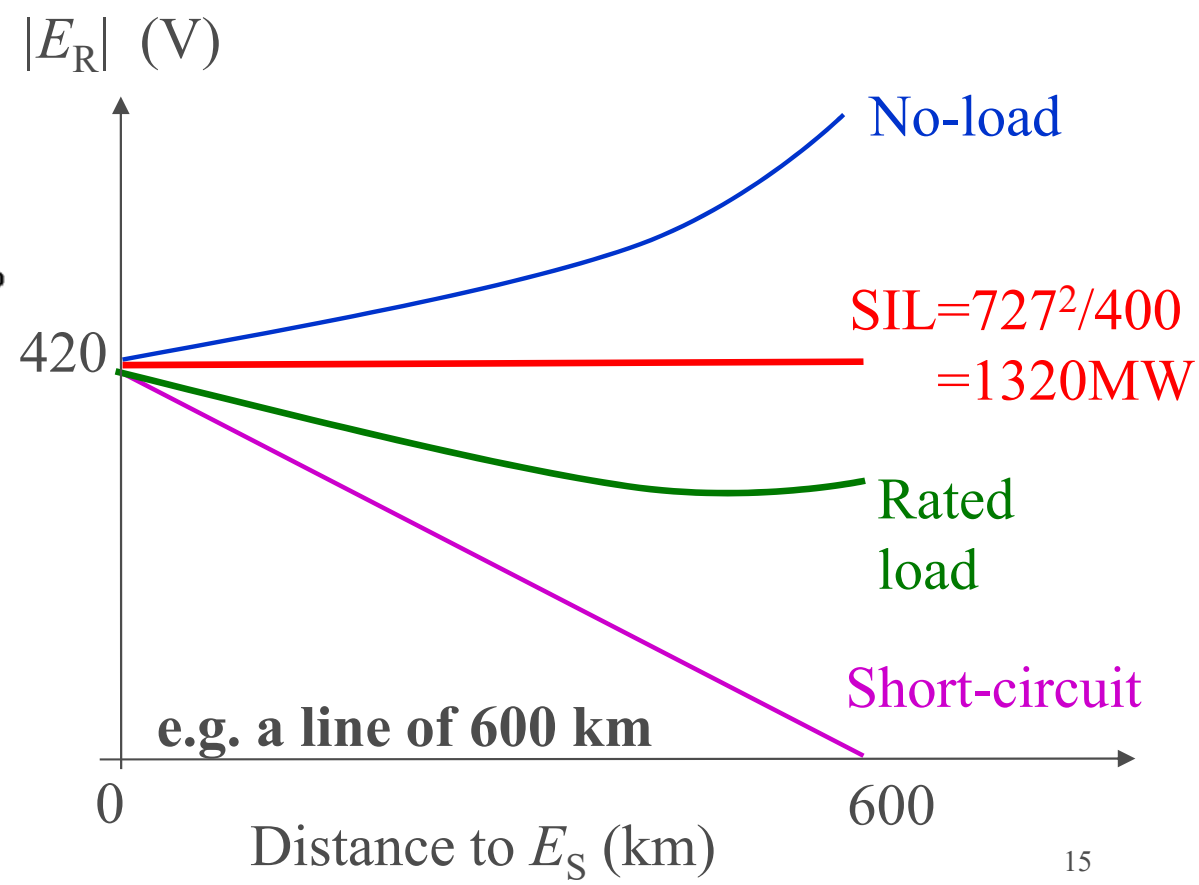


Figure 25.38
Surge impedance loading of a line.



Inter-region power exchange: Example 25-8

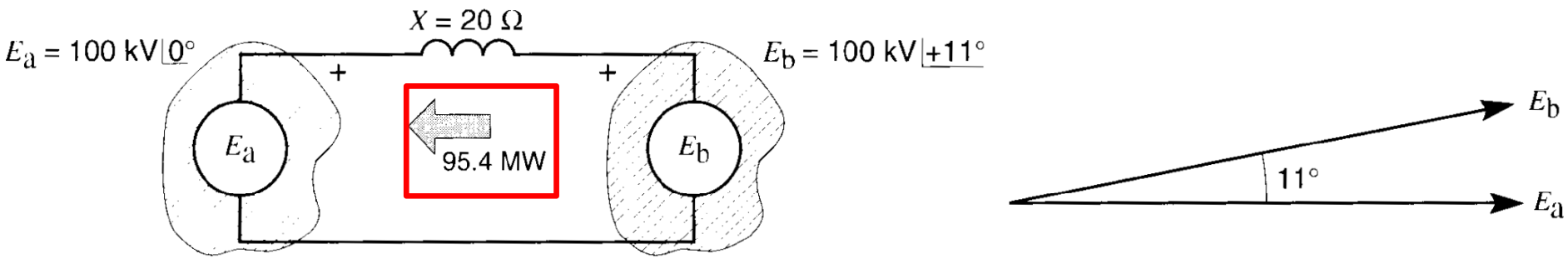


Figure 25.41a
An ordinary transmission line causes power to flow in the wrong direction.

Calculate: (1) the power transmitted by the line and (2) the required phase-shift enabling transmitting 70MW from E_a to E_b

(1) $|E_a|=|E_b|=E$, so $P_{ab}=(E^2/X)\sin(\delta_a-\delta_b)=(100^2/20)\sin(-11^o)= -95.4$MW (in fact, a←b)

(2) $P_{ab}=70=(100^2/20)\sin(\delta_a-\delta_b)$, so $\delta_a-\delta_b=8^o$, i.e. 8-(-11)=19° phase-shift of E_a

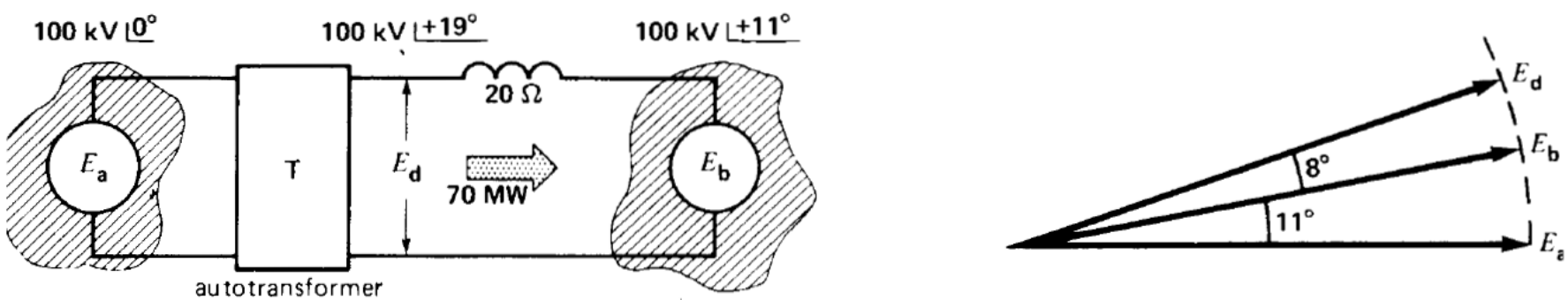


Figure 25.41b
A phase-shift autotransformer can force power to flow in the desired direction (Example 25-8).