Example 13-3

- A 0.5 hp, 6-pole induction motor is excited by a 3-phase, 60 Hz source. Calculate the slip, and the frequency of the rotor current under the following conditions:
  a. At standstill
  b. Motor turning at 500r/min in the same direction as the revolving field
  c. Motor turning at 500r/min in the opposite direction to the revolving field
  d. Motor turning at 2000r/min in the same direction as the revolving field

Solution:

$n_s = 120f/p = 120 \times 60/6 =$ **1200** r/min

a. $s = (n_s - n)/n_s = (1200-0)/1200 = 1$ pu

$f_2 = sf = 60\text{Hz}$

b. $s = (1200-500)/1200 = 0.583$ pu

$f_2 = sf = 0.583 \times 60\text{Hz} = 35$ Hz

c. $s = [1200-(-500)]/1200 = 1.417$ pu

$f_2 = sf = 1.417 \times 60\text{Hz} = 85$ Hz

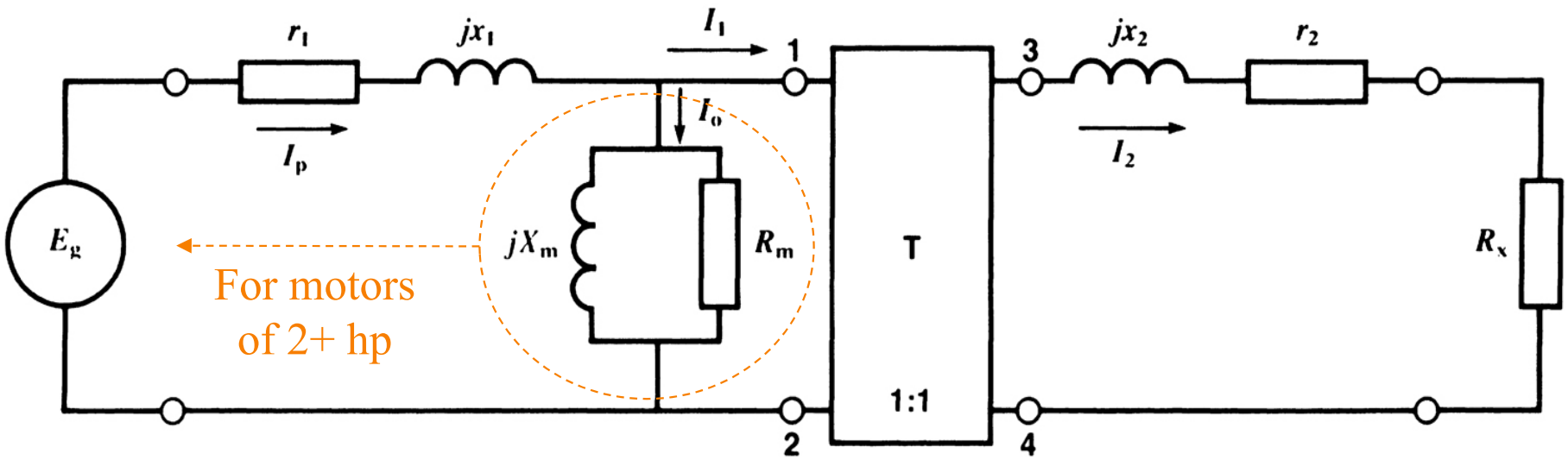
d. $s = [1200-2000)]/1200 = -0.667$ pu

$f_2 = sf = -0.667 \times 60\text{Hz} = -40$ Hz

Slip	Rotor speed & freq	Operating mode
$s=1$	$n=0$ $f_2=f$	Transformer (standstill)
$0<s<1$	$0<n<n_s$ $0<f_2<f$	Motor
$s>1$	$n<0$ $f_2>f$	Brake
$s<0$	$n>n_s$ $f_2<0$	Generator with reversed phase sequence A-C-B

Equivalent circuits: wound-rotor induction motor

1. **When the motor is at standstill (i.e. *n*=0, *s*=1)**, it acts exactly like a conventional transformer. Assume a Y-connection for both the stator and rotor, and a turns ratio of **1:1**. Consider the per-phase equivalent circuit:



E_g: source voltage, line to neutral,

x_1, r_1: stator leakage reactance and winding resistance

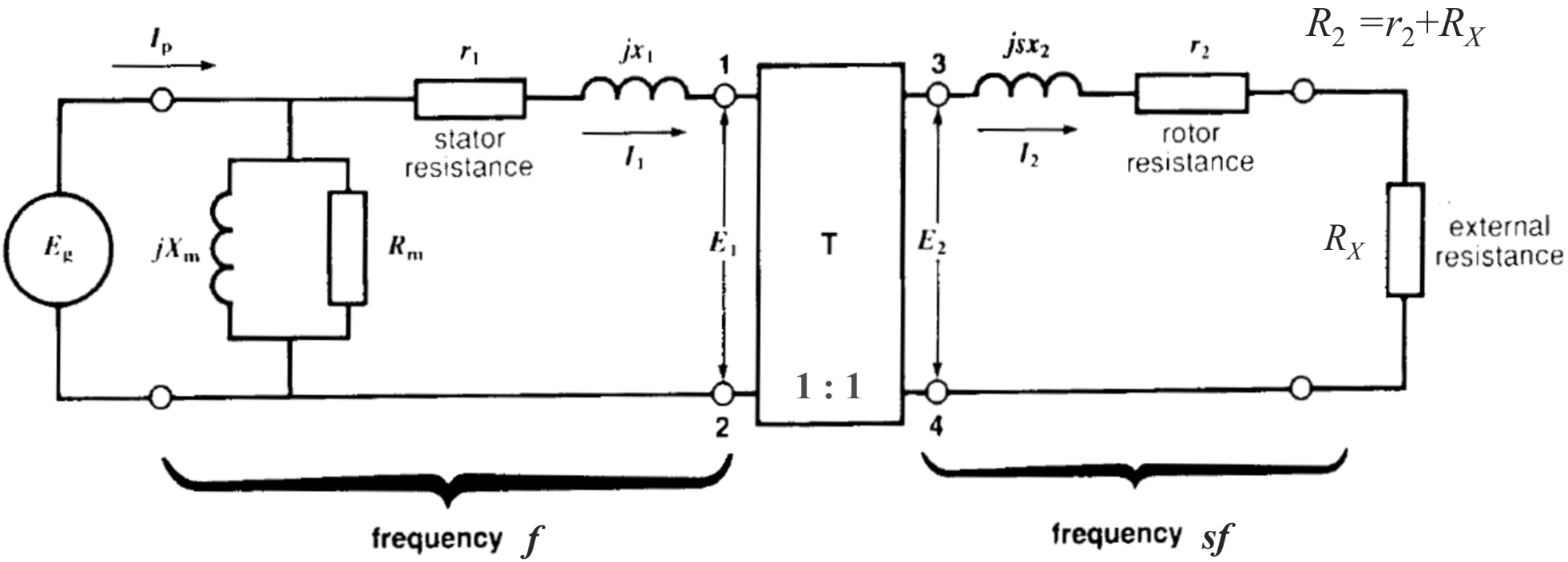
x_2, r_2: rotor leakage reactance and winding resistance

X_m, R_m: magnetizing reactance and resistance modeling losses of iron, windage and friction

R_x: external resistance, connecting one slip-ring to the neutral of the rotor

Note: The magnetizing branch cannot be ignored since I_o may reach 40% of I_p due to the air gap between the stator and rotor (>> the air gap between two windings of a transformer). For motors bigger than 2hp, it is often moved to the terminals of the source.

2. When the motor runs at a slip s, i.e. $n=(1-s)n_s$



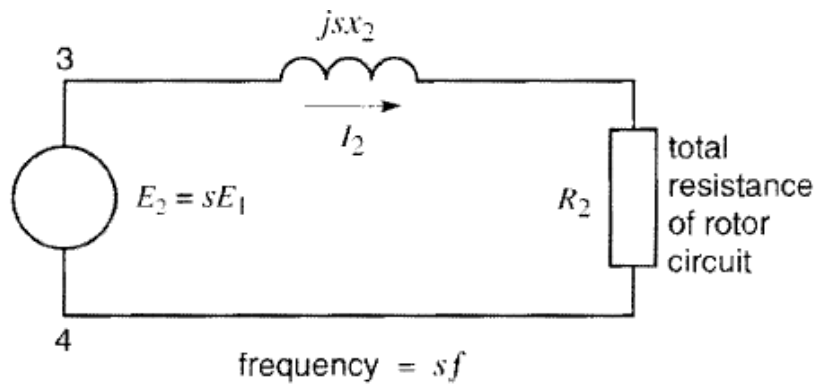
$|E_2|=|sE_1|$, $|I_2|=|I_1|$, $jx_2 \rightarrow jsx_2$ r_2 and R_X do not change

$$I_2 = \frac{E_2}{R_2 + jsx_2} = \frac{|sE_1|\angle 0}{|R_2 + jsx_2|\angle\beta} = \frac{|sE_1|\angle -\beta}{\sqrt{R_2^2 + (sx_2)^2}} \qquad \beta = \arctan\frac{sx_2}{R_2}$$

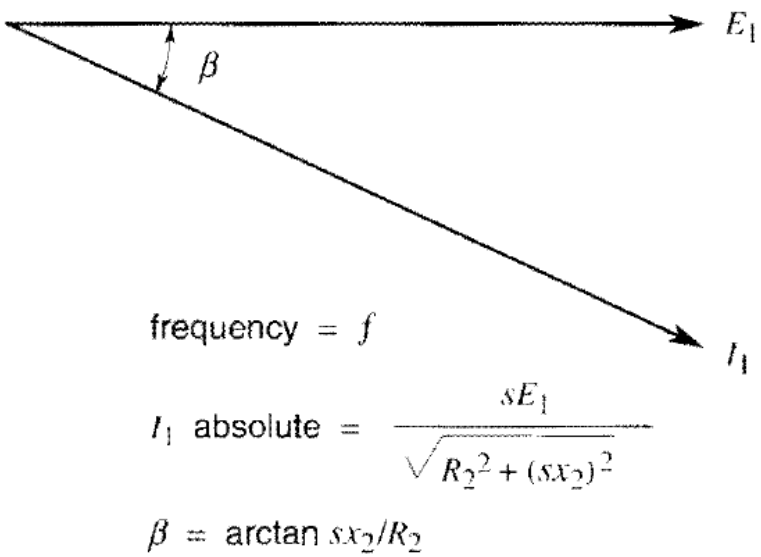
Note: the phasors on the primary side (E_1 and I_1) and the secondary side (E_2 and I_2) cannot be drawn in one phasor diagram because they have different frequencies

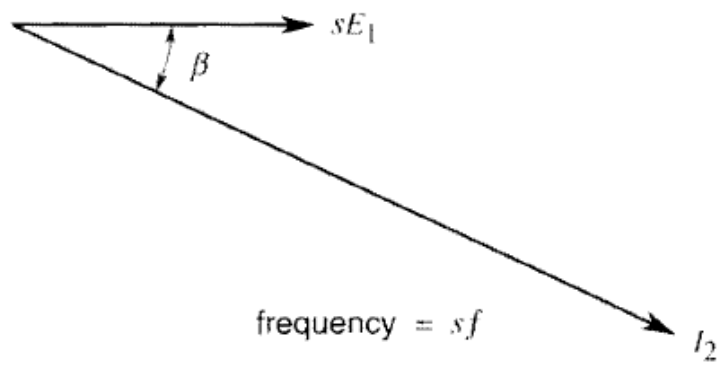
Phasor diagrams on the rotor and stator

- $|I_1|=|I_2|$
- $|sE_1|=|E_2|$
- E_1 leads I_1 and E_2 leads I_2 both by β even though they have different frequencies



(a)





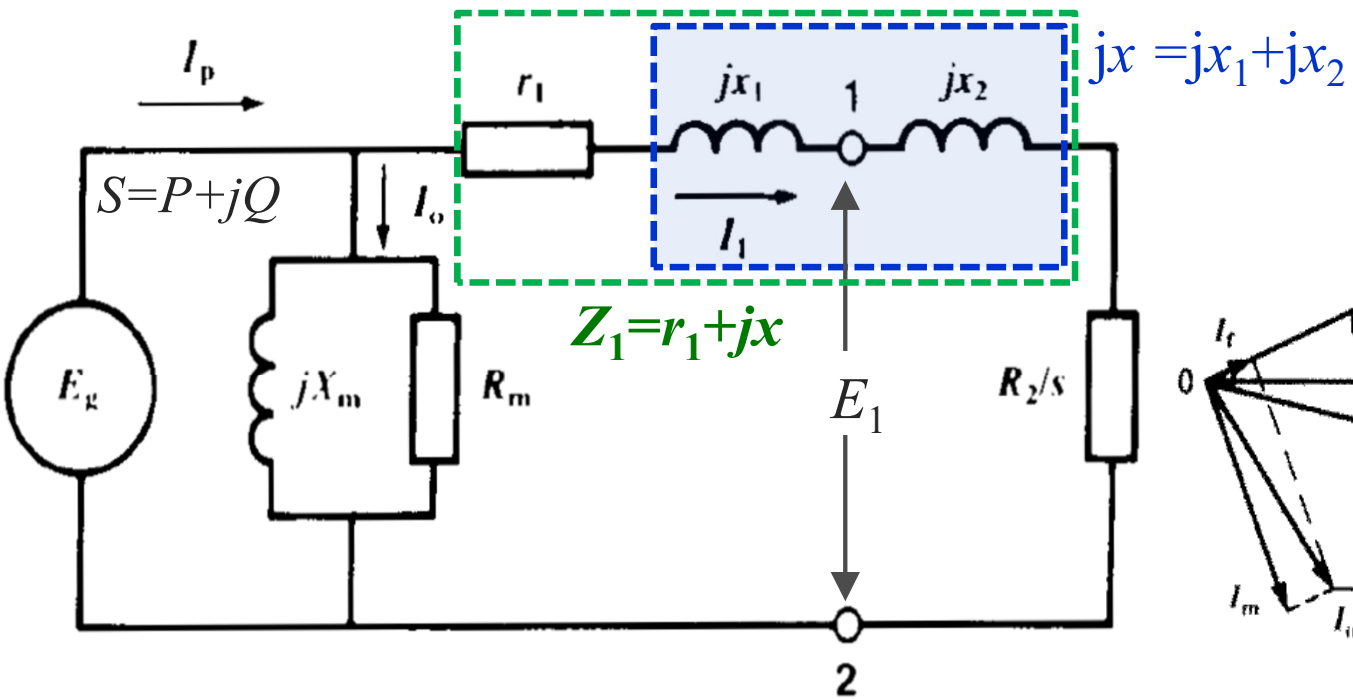
(b)

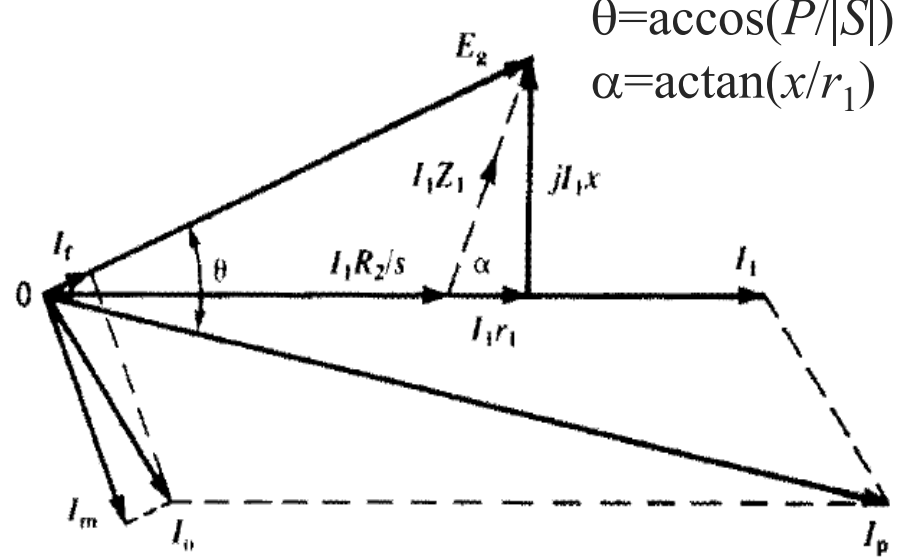
Figure 15.4
a. Equivalent circuit of the rotor; E_2 and I_2 have a frequency sf.
b. Phasor diagram showing the current lagging behind the voltage by angle β.

Figure 15.5
The voltage and current in the stator are separated by the same phase angle β, even though the frequency is different.

Simplified equivalent circuit: referred to the stator side

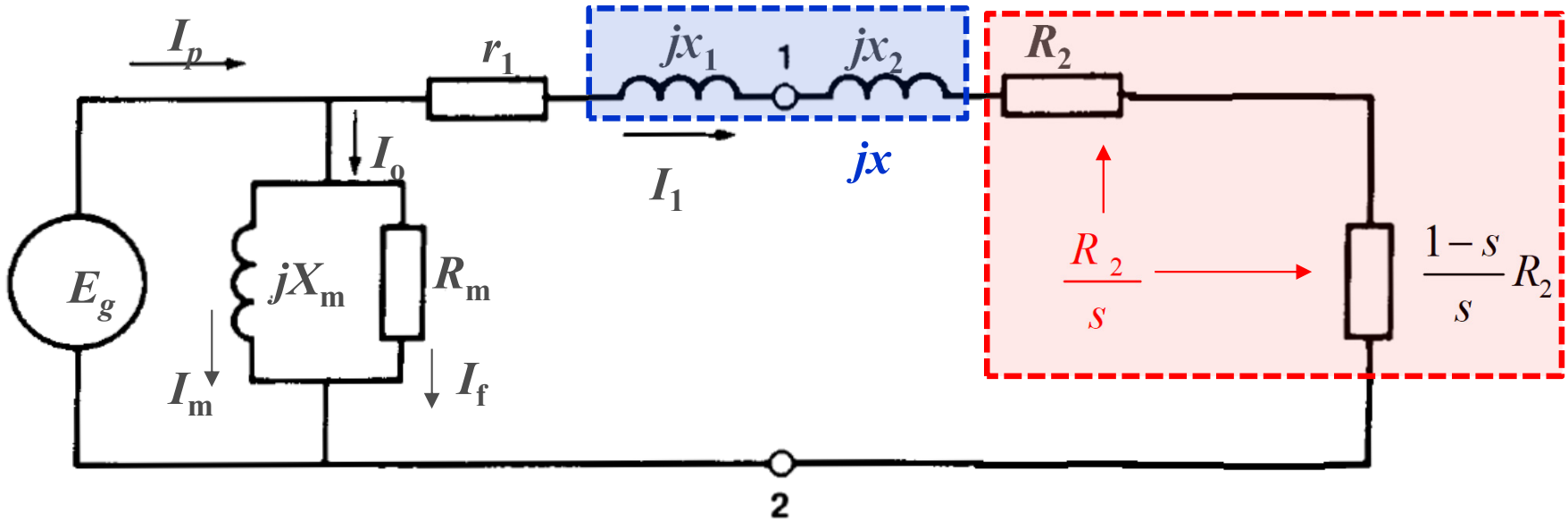
$$I_1 = I_2 = \frac{sE_1}{R_2 + \mathrm{j}sx_2} \Rightarrow I_1 = \frac{E_1}{\frac{R_2}{s} + \mathrm{j}x_2} = \frac{E_1}{Z_2} \qquad Z_2 \overset{\text{def}}{=} \frac{R_2}{s} + \mathrm{j}x_2 = \frac{E_1}{I_1}$$





Note:

- Phasor equation $I_1=I_2$ is assumed just for simplicity of calculation; actually, they have different frequencies
- The value of R_2/s will vary from R_2 to ∞ as the motor goes from start-up (s=1) to n_s (s=0)



- Total power absorbed by the motor:

$$S=\frac{|E_g|^2}{R_m}+j\frac{|E_g|^2}{X_m}+|I_1|^2(r_1+jx+\frac{R_2}{s})$$

$$P=\frac{|E_g|^2}{R_m}+|I_1|^2 r_1+|I_1|^2\frac{R_2}{s}$$

- Stator iron losses

$$P_f=|E_g|^2/R_m$$

- Stator copper losses

$$P_{js}=|I_1|^2 r_1$$

- Active power supplied to the rotor:

$$P_r=|I_1|^2 R_2/s$$

- Total I^2R losses in the rotor circuit

$$P_{jr}=|I_1|^2 R_2=sP_r$$

- Mechanical power developed by the motor

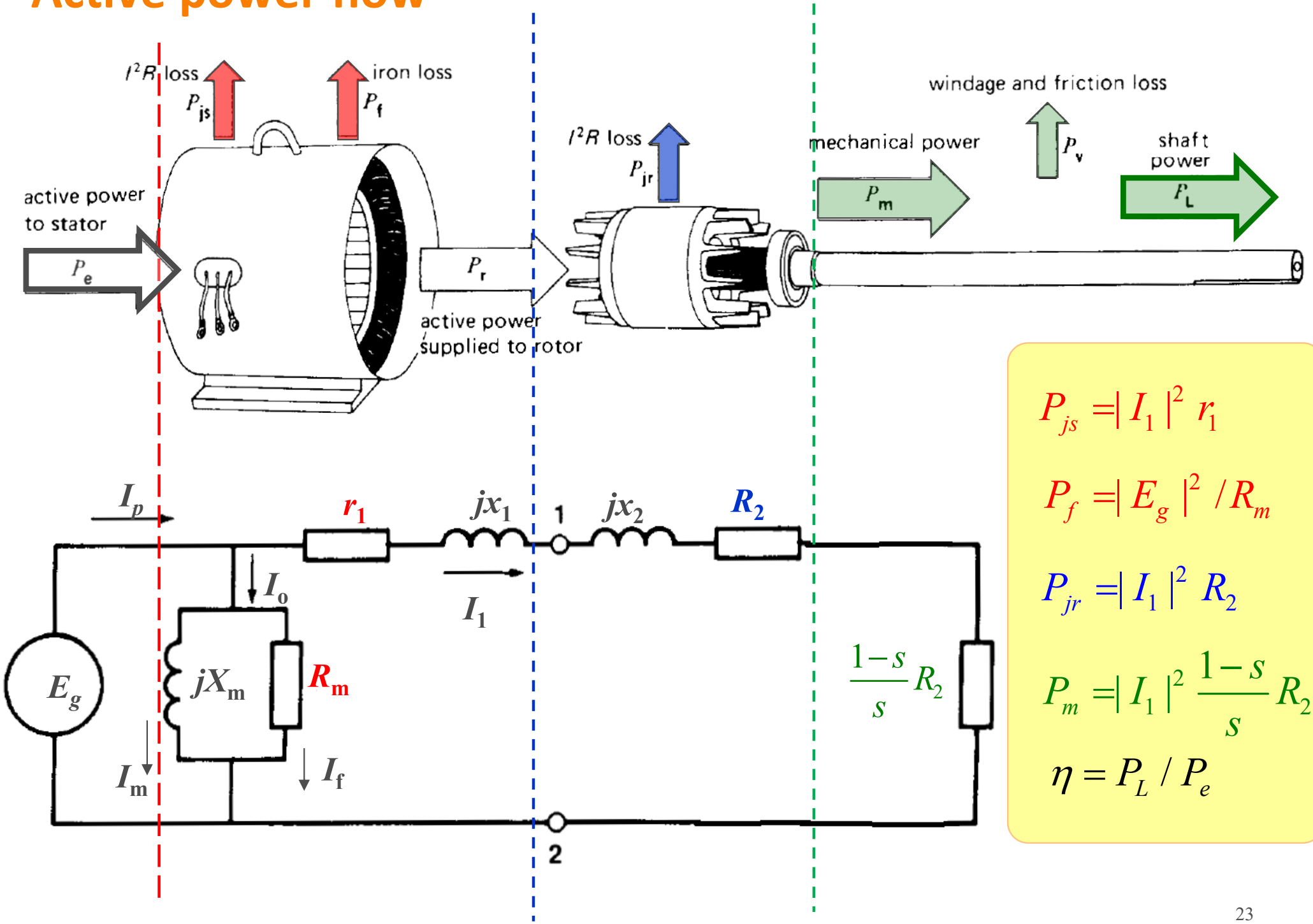
$$P_m=P_r-P_{jr}=P_r(1-s)=|I_1|^2\frac{1-s}{s}R_2$$

- Torque developed by the motor

$$T=\frac{9.55P_m}{n}=\frac{9.55P_r(1-s)}{n_s(1-s)}=\frac{9.55P_r}{n_s}$$

Note: The torque only depends on P_r

Active power flow



$$P_{js} = |I_1|^2 r_1$$

$$P_f = |E_g|^2 / R_m$$

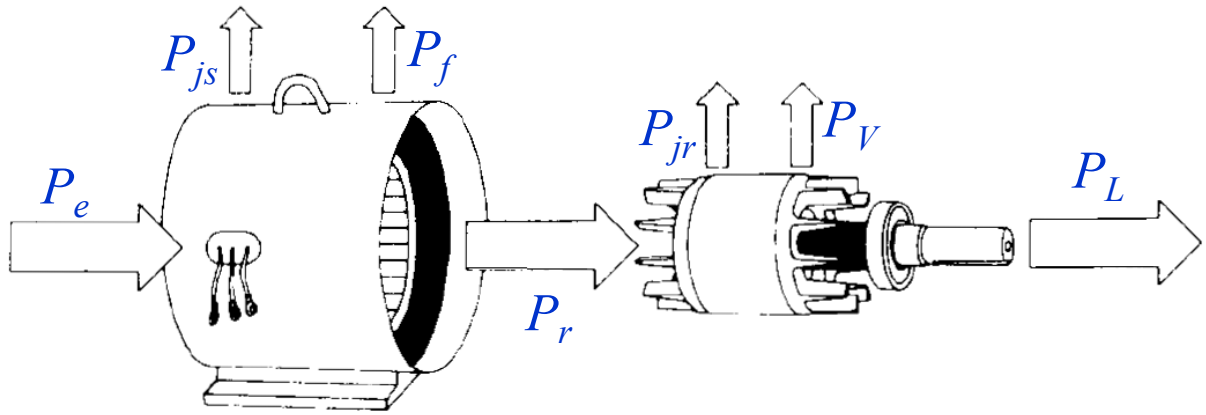
$$P_{jr} = |I_1|^2 R_2$$

$$P_m = |I_1|^2 \frac{1-s}{s} R_2$$

$$\eta = P_L / P_e$$

Example 13-5

A 3-phase induction motor having a synchronous speed of 1200 r/min draws 80 kW from a 3-phase feeder. The copper losses and iron losses in the stator amount to 5 kW. If the motor runs at 1152 r/min, calculate



a. The active power transmitted to the rotor

$P_r = P_e - P_{js} - P_f = 80 - 5 = 75$ kW

b. The rotor I^2R losses, i.e. P_{jr}

$s = (n_s - n)/n_s = (1200-1152)/1200 = 0.04$, $P_{jr} = sP_r = 0.04 \times 75 = 3$ kW

c. The mechanical power developed

$P_m = P_r - P_{jr} = 75 - 3 = 72$ kW

d. The mechanical power delivered to the load, knowing that the windage and friction losses equal to 2 kW

$P_L = P_m - P_V = 72 - 2 = 70$ kW

e. The efficiency of the motor

$\eta = P_L/P_e = 70/80 = 87.5\%$

f. The torque developed by the motor

$T = 9.55\ P_r/n_s = 9.55 \times 75000/1200 = 597$ N·m