Example 13-3

• A 0.5 hp, 6-pole induction motor is excited by a 3-phase, 60 Hz source. Calculate the slip, and the frequency of the rotor current under the following conditions:
  a. At standstill
  b. Motor turning at 500r/min in the same direction as the revolving field
  c. Motor turning at 500r/min in the opposite direction to the revolving field
  d. Motor turning at 2000r/min in the same direction as the revolving field

Solution:

\[ n_s = \frac{120f}{p} = 120 \times \frac{60}{6} = 1200 \text{ r/min} \]

a. \( s = \frac{(n_s - n)}{n_s} = \frac{(1200 - 0)}{1200} = 1 \text{ pu} \)
   \[ f_2 = sf = 60\text{Hz} \]

b. \( s = \frac{(1200 - 500)}{1200} = 0.583 \text{ pu} \)
   \[ f_2 = sf = 0.583 \times 60\text{Hz} = 35 \text{ Hz} \]

c. \( s = \frac{[1200 - (-500)]}{1200} = 1.417 \text{ pu} \)
   \[ f_2 = sf = 1.417 \times 60\text{Hz} = 85 \text{ Hz} \]

d. \( s = \frac{[1200 - 2000]}{1200} = -0.667 \text{ pu} \)
   \[ f_2 = sf = -0.667 \times 60\text{Hz} = -40 \text{ Hz} \]
1. When the motor is at standstill (i.e. \( n=0, s=1 \)), it acts exactly like a conventional transformer. Assume a Y-connection for both the stator and rotor, and a turns ratio of 1:1. Consider the per-phase equivalent circuit:

\[
\begin{align*}
E_g: & \quad \text{source voltage, line to neutral,} \\
x_1, r_1: & \quad \text{stator leakage reactance and winding resistance} \\
x_2, r_2: & \quad \text{rotor leakage reactance and winding resistance} \\
X_m, R_m: & \quad \text{magnetizing reactance and resistance modeling losses of iron, windage and friction} \\
R_x: & \quad \text{external resistance, connecting one slip-ring to the neutral of the rotor}
\end{align*}
\]

Note: The magnetizing branch cannot be ignored since \( I_o \) may reach 40\% of \( I_p \) due to the air gap between the stator and rotor (>> the air gap between two windings of a transformer). For motors bigger than 2hp, it is often moved to the terminals of the source.
2. When the motor runs at a slip $s$, i.e. $n=(1-s)n_s$

\[ E_2 = |sE_1|, \quad I_2 = |I_1|, \quad jx_2 \rightarrow jsx_2 \quad r_2 \text{ and } R_X \text{ do not change} \]

\[ I_2 = \frac{E_2}{R_2 + jsx_2} = \frac{|sE_1| \angle 0}{|R_2 + jsx_2| \angle \beta} = \frac{|sE_1| \angle -\beta}{\sqrt{R_2^2 + (sx_2)^2}} \quad \beta = \arctan \frac{sx_2}{R_2} \]

**Note:** the phasors on the primary side ($E_1$ and $I_1$) and the secondary side ($E_2$ and $I_2$) cannot be drawn in one phasor diagram because they have different frequencies
Phasor diagrams on the rotor and stator

- $|I_1| = |I_2|
- |sE_1| = |E_2|
- $E_1$ leads $I_1$ and $E_2$ leads $I_2$ both by $\beta$ even though they have different frequencies

**Figure 15.5**
The voltage and current in the stator are separated by the same phase angle $\beta$, even though the frequency is different.

**Figure 15.4**
a. Equivalent circuit of the rotor; $E_2$ and $I_2$ have a frequency $sf$.
b. Phasor diagram showing the current lagging behind the voltage by angle $\beta$. 

\[
\begin{align*}
n_1 \text{ absolute} &= \frac{sE_1}{\sqrt{R_2^2 + (sx_2)^2}} \\
\beta &= \arctan sx_2/R_2
\end{align*}
\]
Simplified equivalent circuit: referred to the stator side

\[ I_1 = I_2 = \frac{sE_1}{R_2 + jsx_2} \]

\[ Z_2 = \frac{R_2}{s} + jx_2 = \frac{E_1}{I_1} \]

Note:
- Phasor equation \( I_1 = I_2 \) is assumed just for simplicity of calculation; actually, they have different frequencies.
- The value of \( R_2/s \) will vary from \( R_2 \) to \( \infty \) as the motor goes from start-up \( (s=1) \) to \( n_s \) \( (s=0) \)
Active power supplied to the rotor:

\[ P_r = |I_1|^2 \frac{R_2}{s} \]

Total \( I^2R \) losses in the rotor circuit

\[ P_{jr} = |I_1|^2 R_2 = sP_r \]

Mechanical power developed by the motor

\[ P_m = P_r - P_{jr} = P_r (1 - s) = |I_1|^2 \frac{1 - s}{s} R_2 \]

Torque developed by the motor

\[ T = \frac{9.55P_m}{n} = \frac{9.55P_r (1 - s)}{n_s (1 - s)} = \frac{9.55P_r}{n_s} \]

Note: The torque only depends on \( P_r \)
Active power flow

$$P_{js} = \lvert I_1 \rvert^2 r_1$$

$$P_f = \lvert E_g \rvert^2 / R_m$$

$$P_{jr} = \lvert I_1 \rvert^2 R_2$$

$$P_m = \lvert I_1 \rvert^2 \frac{1-s}{s} R_2$$

$$\eta = P_L / P_e$$
**Example 13-5**

A 3-phase induction motor having a synchronous speed of 1200 r/min draws 80 kW from a 3-phase feeder. The copper losses and iron losses in the stator amount to 5 kW. If the motor runs at 1152 r/min, calculate

a. The active power transmitted to the rotor

\[ P_r = P_e - P_{js} - P_f = 80 - 5 = 75 \text{ kW} \]

b. The rotor \( I^2R \) losses, i.e. \( P_{jr} \)

\[ s = \frac{(n_s - n)}{n_s} = \frac{(1200 - 1152)}{1200} = 0.04, \quad P_{jr} = s P_r = 0.04 \times 75 = 3 \text{ kW} \]

c. The mechanical power developed

\[ P_m = P_r - P_{jr} = 75 - 3 = 72 \text{ kW} \]

d. The mechanical power delivered to the load, knowing that the windage and friction losses equal to 2 kW

\[ P_L = P_m - P_V = 72 - 2 = 70 \text{ kW} \]

e. The efficiency of the motor

\[ \eta = \frac{P_L}{P_e} = \frac{70}{80} = 87.5\% \]

f. The torque developed by the motor

\[ T = 9.55 \frac{P_r}{n_s} = 9.55 \times 75000 / 1200 = 597 \text{ N} \cdot \text{m} \]