Torque-Speed Curve

\[ s = \frac{n_s - n}{n_s}, \quad P_r = |I_1|^2 \frac{R_2}{s}, \quad I_1 = \frac{E_g}{Z_1 + \frac{R_2}{s}} \]

\[ T = \frac{9.55 P_r}{n_s} = \frac{9.55 |E_g|^2 R_2}{Z_1 + \frac{R_2}{s}} = \frac{9.55 |E_g|^2 R_2}{Z_1 + \frac{R_2 n_s}{n_s - n}} (n_s - n) \]

- The curve is nearly linear between no-load and full-load because \( s \) is small and \( R_2/s \) is big (\( Z_1 \) is ignored)

\[ I_1 \approx s E_g / R_2 \]

\[ T \approx \frac{9.55 |E_g|^2}{R_2 n_s^2} (n_s - n) = \frac{9.55 |E_g|^2}{R_2 n_s} s \]
Two practical squirrel-cage induction motors

Motor rating:

5 hp, 60 Hz, 1800 r/min, 440 V, 3-phase
full-load current: 7 A
locked-rotor current: 39 A

\[ r_1 = \text{stator resistance } 1.5 \, \Omega \]
\[ r_2 = \text{rotor resistance } 1.2 \, \Omega \]
\[ jx = \text{total leakage reactance } 6 \, \Omega \]
\[ jX_m = \text{magnetizing reactance } 110 \, \Omega \]
\[ R_m = \text{no-load losses resistance } 900 \, \Omega \]

(The no-load losses include the iron losses plus windage and friction losses.)

Figure 15.12
Equivalent circuit of a 5 hp squirrel-cage induction motor. Because there is no external resistor in the rotor, \( R_2 = r_2 \).

Motor rating:

5000 hp, 60 Hz, 600 r/min, 6900 V, 3-phase
full-load current: 358 A
locked-rotor current: 1616 A

\[ r_1 = \text{stator resistance } 0.083 \, \Omega \]
\[ r_2 = \text{rotor resistance } 0.080 \, \Omega \]
\[ jx = \text{total leakage reactance } 2.6 \, \Omega \]
\[ jX_m = \text{magnetizing reactance } 46 \, \Omega \]
\[ R_m = \text{no-load losses resistance } 600 \, \Omega \]

The no-load losses of 26.4 kW (per phase) consist of 15 kW for windage and friction and 11.4 kW for the iron losses.

Figure 15.13
Equivalent circuit of a 5000 hp squirrel-cage induction motor. Although this motor is 1000 times more powerful than the motor in Fig. 15.12, the circuit diagram remains the same.
The 5 hp induction motor

\[ E_g = 440/1.73 \text{ V} \]
\[ R_2 = 1.2 \text{ \Omega} \]
\[ Z_1 = 1.5 + 6j \text{ \Omega} \]
\[ n_s = 1800 \text{ r/min} \]
When the motor is stalled, i.e. locked-rotor condition, the current is 5-6 times the full-load current, making $I^2R$ losses 25-36 times higher than normal, so the rotor must never remain locked for more than a few seconds.

Small motors (15 hp and less) develop their breakdown torque at about 80% of $n_s$. 

The rated power of 5 hp is developed at $s = 0.026$. 

### Table 15A: Torque-Speed Characteristic

<table>
<thead>
<tr>
<th>$s$</th>
<th>$I_1$ [A]</th>
<th>$P_r$ [W]</th>
<th>$T$ [N·m]</th>
<th>$n$ [r/min]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0125</td>
<td>2.60</td>
<td>649</td>
<td>3.44</td>
<td>1777</td>
</tr>
<tr>
<td>0.025</td>
<td>5.09</td>
<td>1243</td>
<td>6.60</td>
<td>1755</td>
</tr>
<tr>
<td>0.026</td>
<td>5.29</td>
<td>1291</td>
<td>6.85</td>
<td>1753</td>
</tr>
<tr>
<td>0.05</td>
<td>9.70</td>
<td>2256</td>
<td>12.0</td>
<td>1710</td>
</tr>
<tr>
<td>0.1</td>
<td>17.2</td>
<td>3547</td>
<td>18.8</td>
<td>1620</td>
</tr>
<tr>
<td>0.2</td>
<td>26.4</td>
<td>4196</td>
<td>22.3</td>
<td>1440</td>
</tr>
<tr>
<td>0.4</td>
<td>33.9</td>
<td>3441</td>
<td>18.3</td>
<td>1080</td>
</tr>
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<td>0.6</td>
<td>36.6</td>
<td>2674</td>
<td>14.2</td>
<td>720</td>
</tr>
<tr>
<td>0.8</td>
<td>37.9</td>
<td>2150</td>
<td>11.4</td>
<td>360</td>
</tr>
<tr>
<td>1</td>
<td>38.6</td>
<td>1788</td>
<td>9.49</td>
<td>0</td>
</tr>
</tbody>
</table>

5 hp, 440 V, 1800 r/min, 60 Hz squirrel-cage induction motor
Torque-Speed Curve: 5000 hp motor

Big motors (>1500 hp):

- Relatively low starting (locked-rotor) torque
- Breakdown torque at about 98% of $n_s$
- Rated $n$ is close to $n_s$
**Effect of rotor resistance (5 hp induction motor)**

**Equation 1:**

\[ T = \frac{9.55 |E_g|^2 R_2}{|Z_1 + \frac{R_2 n_s}{n_s - n}|^2(n_s - n)} \]

**Equation 2:**

\[ I_1 = \frac{E_g}{Z_1 + \frac{R_2 n_s}{n_s - n}} \]

- **Rated Torque = 15 N·m**
- **Current at the rated torque**

- **Rotor Resistances:**
  - \( R_2 \)
  - \( 2.5R_2 \)
  - \( 10R_2 \)
**Effects of rotor resistance**

- When \( R_2 \uparrow \), starting (locked-rotor) torque \( T_{LR} \uparrow \), starting current \( I_{1,LR} \downarrow \), and breakdown torque \( T_b \) remains the same

\[
T_{LR} = \frac{9.55 |E_g|^2 R_2}{|Z_1 + R_2|^2 n_s} \approx \frac{9.55 |E_g|^2}{|Z_1|^2 n_s} R_2
\]

\[
I_{1,LR} = \frac{E_g}{Z_1 + R_2}
\]

\[
P_{jr} = |I_1|^2 R_2
\]

\[
T_b = \frac{9.55 |E_g|^2}{4n_s |Z_1| \cos^2 \frac{\alpha}{2}}
\]

- **Pros & Cons** with a high rotor resistance \( R_2 \)
  - It produces a high starting torque \( T_{LR} \) and a relatively low starting current \( I_{1,LR} \)
  - However, because the torque-speed curve becomes flat it produces a rapid fall-off in speed with increasing load around the rated torque, and the motor has high copper losses and low efficiency and tends to overheat

- **Solution**
  - For a squirrel-cage induction motor, design the rotor bars in a special way so that the rotor resistance \( R_2 \) is high at starting and low under normal operations
  - If the rotor resistance needs to be varied over a wide range, a wound-rotor motor needs to be used.
Asynchronous generator

Connect the 5 hp, 1800 r/min, 60Hz motor to a 440 V, 3-phase line and drive it at a speed of **1845** r/min

\[ s = \frac{n_s - n}{n_s} = \frac{1800 - 1845}{1800} = -0.025 < 0 \]

\[ R_2/s = 1.2/(-0.025) = 48 \Omega < 0 \]

The negative resistance indicates the actual power flow from the rotor to the stator

**Power flow from the rotor to the stator:**

\[ |E| = \frac{440}{1.73} = 254 \text{ V} \]

\[ |I_1| = \frac{|E|}{|48 + 1.5 + j6|} = \frac{254}{46.88} = 5.42 \text{ A} \]

\[ P_r = |I_1|^2 R_2/s = -1410 \text{ W} \text{ (in fact, rotor} \rightarrow \text{ stator)} \]

**Mech. power & torque inputs to the shaft:**

\[ P_{jr} = |I_1|^2 R_2 = 35.2 \text{ W} \]

\[ P_m = P_r + P_{jr} = 1410 + 35.2 = 1445 \text{ W} \]

\[ T = 3 \times 9.55 \times P_m/n = 22.3 \text{ N} \cdot \text{m} \]

**Total active power delivered to the line:**

\[ P_{js} = |I_1|^2 r_1 = 44.1 \text{ W}, \quad P_f = |E|^2/R_m = 71.1 \text{ W} \]

\[ P_e = P_r - P_{js} - P_f = 1410 - 44.1 - 71.7 = 1294 \text{ W} \]

\[ P_{3\phi} = 3P_e = 3882 \text{ W} \]

**Reactive power absorbed from the line:**

\[ Q_{3\phi} = (|I_1|^2 x + |E|^2/X_m) \times 3 = (176 + 586) \times 3 = 2286 \text{ var} \]

**Complex power delivered to the line:**

\[ S_{3\phi} = P_{3\phi} - Q_{3\phi} = 3882 - j2286 \text{ VA} \]

\[ \cos \theta = 86.2\% \]

**Efficiency of this asynchronous generator**

\[ \eta = \frac{P_e}{P_m} = \frac{1294}{1445} = 89.5\% \]