ECE 421/599
Electric Energy Systems
3 – Generators, Transformers and the Per-Unit System

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Outline

• Synchronous Generators

• Power Transformers

• The Per-Unit System
Synchronous Generators

Salient-pole rotor

Cylindrical/round rotor

Field winding

Stator

Armature winding

Field current

3φ ac

I_a

I_b

I_c

3φ

Stator

Rotor
Types of Rotors

• Salient pole rotors
  – Concentrated windings on poles and non-uniform air gap
  – Short axial length and large diameter
  – Hydraulic turbines operated at low speeds (large number of poles)
  – Have damper windings to help damp out speed oscillations

• Round rotors
  – 70% of large synchronous generators (150~1500MVA)
  – Distributed winding and uniform air gap
  – Large axial length and small diameter to limit the centrifugal forces
  – Steam and gas turbines, operated at high speeds, typically 3600 or 1800rpm (2 or 4-pole)
  – Eddy in the solid steal rotor gives damping effects

(Source: http://emadrlc.blogspot.com)
Generator Model

- Flux linkage with coil a (leading the axis of $aa'$ by $\omega t$)
  \[ \lambda_a = N\phi \cos \omega t \]

- Induced voltage:
  \[ e_a = -\frac{d\lambda_a}{dt} = \omega N\phi \sin \omega t = E_{\text{max}} \sin \omega t \]
  \[ = E_{\text{max}} \cos(\omega t - \frac{\pi}{2}) \]

  \[ E_{\text{max}} = \omega N\phi = 2\pi fN\phi \quad \text{(occurring when } \omega t = \pi/2) \]

  \[ |E| = 4.44 fN\phi \quad \text{(rms value)} \]

  \[ f = \frac{P}{2} \frac{n}{60} \quad \text{ (n: synchronous speed in rpm; } P: \text{ the number of poles)} \]

- Assume: $i_a$ is lagging $e_a$ by $\psi$ ($i_a$ reaches the maximum when $mn$ aligns with $aa'$)

  \[ i_a = I_{\text{max}} \sin(\omega t - \psi) \quad i_b = I_{\text{max}} \sin(\omega t - \psi - \frac{2}{3}\pi) \quad i_c = I_{\text{max}} \sin(\omega t - \psi - \frac{4}{3}\pi) \]
• Magneto-motive forces (mmf’s) of three phases:

\[ F_a = K_i a = K I_{\text{max}} \sin(\omega t - \psi) = F_m \sin(\omega t - \psi) \]

\[ F_b = K_i b = K I_{\text{max}} \sin(\omega t - \frac{2}{3} \pi) = F_m \sin(\omega t - \frac{2}{3} \pi) \]

\[ F_c = K_i c = K I_{\text{max}} \sin(\omega t - \frac{4}{3} \pi) = F_m \sin(\omega t - \frac{4}{3} \pi) \]

• Total mmf \( F_s \) due to three phases of the stator

Component along \( mn \):

\[ F_1 = F_m \sin(\omega t - \psi) \cos(\omega t - \psi) + F_m \sin(\omega t - \frac{2}{3} \pi) \cos(\omega t - \frac{2}{3} \pi) + F_m \sin(\omega t - \frac{4}{3} \pi) \cos(\omega t - \frac{4}{3} \pi) \]

\[ F_1 = \frac{F_m}{2} \left[ \sin 2(\omega t - \psi) + \sin 2(\omega t - \frac{2\pi}{3}) + \sin 2(\omega t - \frac{4\pi}{3}) \right] = 0 \]

Component orthogonal to \( mn \)

\[ F_2 = F_m \sin(\omega t - \varphi) \sin(\omega t - \psi) + F_m \sin(\omega t - \frac{2\pi}{3}) \sin(\omega t - \frac{2\pi}{3}) + F_m \sin(\omega t - \frac{4\pi}{3}) \sin(\omega t - \frac{4\pi}{3}) \]

\[ F_2 = \frac{F_m}{2} \left[ 3 - \cos 2(\omega t - \psi) + \cos 2(\omega t - \frac{2\pi}{3}) + \cos 2(\omega t - \frac{4\pi}{3}) \right] = \frac{3}{2} F_m \]

\[ \sin^2 \alpha = \frac{1 - \cos 2\alpha}{2} \]

\[ F_s = \frac{3}{2} F_m \]

\( F_s \), the resultant total armature/stator mmf has constant amplitude, and is orthogonal to \( mn \) and revolving synchronously with \( F_r \)
Phasor Diagram

- No-load conditions:
  - No phase currents and \( F_s = 0 \)
  - In each phase, field mmf \( F_r \) produces no-load \( E \) (excitation voltage)

- Loading conditions:
  - Armature carries three-phase currents
  - \( F_s \neq 0 \) is orthogonal to \( m_n \) and induces emf \( E_{ar} \)
  - The sum of \( F_r \) and \( F_s \) gives mmf \( F_{sr} \) in air gap
  - The resultant air gap flux \( \phi_{sr} \) induces emf \( E_{sr} = E + E_{ar} \)
  - Define the reactance of the armature reaction
    \[
    X_{ar} = -E_{ar}/(jI_a)
    \]

\[
E = E_{sr} + jX_{ar}I_a
\]

- Terminal voltage \( V \), resistance \( R_a \) and leakage reactance \( X_l \)

\[
E = V + [R_a + j(X_l + X_{ar})]I_a = V + (R_a + jX_s)I_a
\]

\( X_s = X_l + X_{ar} \) is known as the synchronous reactance

(Here, we ignore the difference between d and q axes, so it is good for round-rotor generators)
Circuit Model (per Phase)

- When $F_r$ is ahead $F_{sr}$ by $\delta_r$, the machine is operating as a generator.
- When $F_r$ falls behind of $F_{sr}$ by $\delta_r$ ($I_a$ changes the direction) the machine acts as a motor.
- Synchronous condenser ($\delta_r=0$), used to supply or absorb VARs to regulate line voltages.
- Since $X_l \approx 0$, $\delta_r \approx \delta$.
- Generated real power:

$$P_{3\phi} \approx 3 \frac{|E||V|}{X_s} \sin \delta$$

- $\delta_r$ or $\delta$ is known as the power angle.
\[
E = V + (R_a + jX_s)I_a
\]

\[
I_a = \frac{|E| \angle \delta - |V| \angle 0}{|Z_s| \angle \gamma}
\]

- Consider different power factors
Real and Reactive Power Outputs

• Consider a generator connected to an infinite bus, which is an equivalent bus of a large power network and has constant voltage $V$ (magnitude, angle and frequency).

\[
E = V + jX_s I_a
\]

\[
S_{3\phi} = 3VI_a^*
\]

\[
P_{3\phi} = 3 \frac{|E||V|}{X_s} \sin \delta \quad T = \frac{P_{3\phi}}{\omega} = 3 \frac{|E||V|}{\omega X_s} \sin \delta
\]

\[
P_{\max(3\phi)} = 3 \frac{|E||V|}{X_s} \quad T_{\max} = 3 \frac{|E||V|}{\omega X_s}
\]

\[
Q_{3\phi} = 3 \frac{|V|}{X_s} (|E| \cos \delta - |V|) \approx 3 \frac{|V|}{X_s} (|E| - |V|)
\]
• Consider constant real power output:

\[ P_{3\phi} = \Re \left[ 3V I_a^* \right] = 3|V||I_a|\cos \theta = \text{const.} \]

\[ |I_a|\cos \theta = |ab| = \text{const.} \]

\[ |cd| = E_1 \sin \delta_1 = X_s I_a \cos \theta_1 = \text{const.} \]

\[ Q_{3\phi} \approx 3 \frac{|V|}{X_s} (|E| - |V|) \]

• Reactive power output can be controlled by means of the rotor excitation (adjusting \( E \)) while maintaining a constant real power output:
  
  - If \(|E| > |V|\), the generator delivers reactive power to the bus, and the generator is said to be overexcited.
  - If \(|E| < |V|\), the generator is absorbing reactive power from the bus.
  - Generators are main source of reactive power, so they are normally operated in the overexcited mode.
  - \( \delta \uparrow \) \(|E| \downarrow \) Minimum excitation: when \( \delta = \delta_{\text{max}} = 90^\circ \), i.e. the stability limit.
Examples 3.1

A 50-MVA, 30-kV, three-phase, 60-Hz synchronous generator has a synchronous reactance of 9 Ω per phase and a negligible resistance. The generator is delivering rated power at a 0.8 power factor lagging at the rated terminal voltage to an infinite bus.

(a) Determine the excitation voltage per phase $E$ and the power angle $\delta$.

(b) With the excitation held constant at the value found in (a), the driving torque is reduced until the generator is delivering 25 MW. Determine the armature current and the power factor.

(c) If the generator is operating at the excitation voltage of part (a), what is the steady-state maximum power the machine can deliver before losing synchronism? Also, find the armature current corresponding to this maximum power.

(a) The three-phase apparent power is

$$S_{3\phi} = 50 \angle \cos^{-1} 0.8 = 50 \angle 36.87^\circ \text{ MVA}$$

$$= 40 \text{ MW} + j30 \text{ Mvar}$$

The rated voltage per phase is

$$V = \frac{30}{\sqrt{3}} = 17.32 \angle 0^\circ \text{ kV}$$

The rated current is

$$I_a = \frac{S_{3\phi}^*}{3V^*} = \frac{(50 \angle -36.87)10^3}{3(17.32 \angle 0^\circ)} = 962.25 \angle -36.87^\circ \text{ A}$$

The excitation voltage per phase from (3.12) is

$$E = 17320.5 + (j9)(962.25 \angle -36.87) = 23558 \angle 17.1^\circ \text{ V}$$

The excitation voltage per phase (line to neutral) is 23.56 kV and the power angle is 17.1°.
(b) When the generator is delivering 25 MW from (3.21) the power angle is

\[ \delta = \sin^{-1} \left( \frac{(25)(9)}{(3)(23.56)(17.32)} \right) = 10.591^\circ \]

The armature current is

\[ I_a = \frac{(23,558 \angle 10.591^\circ - 17,320 \angle 0^\circ)}{j9} = 807.485 \angle -53.43^\circ \text{ A} \]

The power factor is given by \( \cos(53.43) = 0.596 \) lagging.

(c) The maximum power occurs at \( \delta = 90^\circ \)

\[ P_{\text{max}(3\phi)} = 3 \left( \frac{|E||V|}{X_s} \right) = 3 \left( \frac{23.56(17.32)}{9} \right) = 136 \text{ MW} \]

The armature current is

\[ I_a = \frac{(23,558 \angle 90^\circ - 17,320 \angle 0^\circ)}{j9} = 3248.85 \angle 36.32^\circ \text{ A} \]

The power factor is given by \( \cos(36.32) = 0.8057 \) leading.
Example 3.2

The generator of Example 3.1 is delivering 40 MW at a terminal voltage of 30 kV. Compute the power angle, armature current, and power factor when the field current is adjusted for the following excitations.

(a) The excitation voltage is decreased to 79.2 percent of the value found in Example 3.1.
(b) The excitation voltage is decreased to 59.27 percent of the value found in Example 3.1.
(c) Find the minimum excitation below which the generator will lose synchronism.

(a) The new excitation voltage is

\[ E = 0.792 \times 23,558 = 18,657 \text{ V} \]

From (3.21) the power angle is

\[ \delta = \sin^{-1}\left[\frac{(40)(9)}{(3)(18.657)(17.32)}\right] = 21.8^\circ \]

The armature current is

\[ I_a = \frac{(18657\angle21.8^\circ - 17320\angle0^\circ)}{j9} = 769.8\angle0^\circ \]

The power factor is given by \(\cos(0) = 1\).
(b) The new excitation voltage is

\[ E = 0.5927 \times 23,558 = 13,963 \text{ V} \]

From (3.21) the power angle is

\[ \delta = \sin^{-1} \left( \frac{(40)(9)}{(3)(13.963)(17.32)} \right) = 29.748^\circ \]

The armature current is

\[ I_a = \frac{(13,963 \angle 29.748^\circ - 17,320 \angle 0^\circ)}{j9} = 962.3 \angle 36.87^\circ \text{ A} \]

From current phase angle, the power factor is \( \cos 36.87 = 0.8 \) leading. The generator is underexcited and is actually receiving reactive power.

(c) From (3.23), the minimum excitation corresponding to \( \delta = 90^\circ \) is

\[ E = \frac{(40)(9)}{(3)(17.32)(1)} = 6.928 \text{ kV} \]

The armature current is

\[ I_a = \frac{(6,928 \angle 90^\circ - 17,320 \angle 0^\circ)}{j9} = 2073 \angle 68.2^\circ \text{ A} \]

The current phase angle shows that the power factor is \( \cos 68.2 = 0.37 \) leading. The generator is underexcited and is receiving reactive power.
## Comparison

Fix $P_{\text{max,3}\phi}=40\text{MW}$ and $|V|=30\text{kV}$

<table>
<thead>
<tr>
<th>$E$</th>
<th>$\delta$</th>
<th>$I_a$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.6kV</td>
<td>10.6°</td>
<td>807.5A</td>
<td>-54.43° (lagging)</td>
</tr>
<tr>
<td>18.7kV</td>
<td>21.8°</td>
<td>769.8A</td>
<td>0</td>
</tr>
<tr>
<td>14.0kV</td>
<td>29.7°</td>
<td>962.3A</td>
<td>36.87° (leading)</td>
</tr>
<tr>
<td>6.9KV (minimum)</td>
<td>90°</td>
<td>2073A</td>
<td>68.2° (leading)</td>
</tr>
</tbody>
</table>

- When excitation decreases,
  - armature current $|I_a|$ first decreases and then increases (V-shape function)
  - power factor angle changes from lagging to leading
• V Curve of a synchronous machine:
  
  – $|I_a| - |I_f|$ : armature current $|I_a|$ as the function of field current $|I_f|$
  
  – Example 3.3: for the generator of Example 3.1, construct the V curve for the rated power of 40MW with varying field excitation from 0.4 P.F. leading to 0.4 P.F. lagging. Assume the open-circuit characteristics in the operating region is given by $|E|=2000|I_f| (V)$

\[
P = 40\text{(MW)} \quad V = \frac{30}{\sqrt{3}} \text{(kV)}
\]

\[
|I_a| = \frac{P_3\phi}{3|V|\cos \theta} = \frac{40}{30\sqrt{3}\cos \theta} \text{(kA)}
\]

$\theta = \cos^{-1}(0.4) - \cos^{-1}(0.4)$

\[
E = V + jX_s I_a = \frac{30}{\sqrt{3}} + j9 |I_a| \angle \theta \text{(kV)}
\]

\[
|I_f| = |E| \times \frac{1000}{2000} \text{(A)}
\]

FIGURE 3.7
V curve for generator of Example 3.3.
Salient-Pole Synchronous Generators

- The model developed in previous slides assumes uniform magnetic reluctance in the air gap (round rotors).

\[ E = V + jX_{ar}I_a + (R_a + jX_{r})I_a \]
\[ = V + R_a I_a + j(X_{ar} + X_r)I_a = V + (R_a + jX_s)I_a \]

- For a salient-pole rotor, the reluctance in the rotor polar axis (d axis) is less than that in the inter-polar axis (q axis)
  - No such a constant \( X_s \) being independent of the rotor position
  - Represent the machine by two circuits respectively for d and q axes
  - Define d-axis and q-axis reactances \( (X_d > X_q) \) to replace a single synchronous reactance \( X_s \)
  - Resolve \( I_a \) into d-axis and q-axis components: \( I_d \) and \( I_q \)

Note: When the direction of \( e_a \) (axis of coil) is parallel with q axis of the rotor, \( e_a \) reaches the maximum, so in the phasor diagram, q axis is the axis of \( E \)
Consider a simple case ignoring $R_a$ and $X_l$

$$|E| = |V| \cos \delta + X_d I_d$$

$$I_d = \frac{|E| - |V| \cos \delta}{X_d}$$

$$|V| \sin \delta = X_q I_q$$

$$I_q = \frac{|V| \sin \delta}{X_q}$$

$$P_{3\phi} = 3 |V| |I_a| \cos \theta$$

$$|I_a| \cos \theta = ab + de = I_q \cos \delta + I_d \sin \delta$$

$$P_{3\phi} = 3 |V| (I_q \cos \delta + I_d \sin \delta) = 3 \frac{|E| |V|}{X_d} \sin \delta + 3 |V|^2 \frac{X_d - X_q}{2 X_d X_q} \sin 2\delta$$

Approximately, the second item can be ignored:

$$P_{3\phi} \approx 3 \frac{|E| |V|}{X_d} \sin \delta$$
Power Transformer

• Ideal transformers
  – Winding resistance is negligible (no power loss due to winding)
  – No leakage flux
  – Permeability of the core is infinite such that net mmf to establish flux in core is zero. (mmf balance)
  – No core loss (no power loss due to core)

\[ F = I \cdot N = \Phi \cdot R = \frac{\Phi}{P} \]

• Real transformers:
  – Windings have resistance
  – Windings do not link the same flux
  – Permeability of the core is finite
  – Core losses (hysteresis losses and eddy current losses due to time varying flux)

(Source: wikipedia.org)
Ideal Transformer

• Primary winding (assuming sinusoidal flux)

\[ \phi = \Phi_{\text{max}} \cos \omega t \]

\[ e_1 = N_1 \frac{d\phi}{dt} = -\omega N_1 \Phi_{\text{max}} \sin \omega t \]

\[ = E_{1\text{max}} \cos (\omega t + 90^\circ) \]

\[ E_{1\text{max}} = 2\pi f N_1 \Phi_{\text{max}} \]

\[ E_1 = 4.44 f N_1 \Phi_{\text{max}} \angle 90^\circ \]

• Secondary winding
  – No flux leakage:

\[ e_2 = N_2 \frac{d\phi}{dt} = E_{2\text{max}} \cos (\omega t + 90^\circ) \quad E_2 = 4.44 f N_2 \Phi_{\text{max}} \angle 90^\circ \]

  – Zero reluctance in core (exact mmf balance between the primary and secondary)

\[ I_1 N_1 = I_2 N_2 \]

\[ \frac{E_1}{E_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} \]

Note: \( e_1 \) is the load voltage of the primary side and \( e_2 \) is the source voltage of the secondary side
Real Transformers

- Modeling the current under no-load conditions (due to finite core permeability and core losses)

\[ I_2 = I'_2 = 0 \quad I_1 = I_0 \neq 0 \quad I_0 = I_m + I_c \]

- \( I_m \) is the magnetizing current to set up the core flux
  - In phase with flux and lagging \( E_1 \) by \( 90^\circ \), modeled by \( E_1/(jX_{m1}) \)
- \( I_c \) supplies the eddy-current and hysteresis losses in the core
  - A power component, so it is in phase with \( E_1 \), modeled by \( E_1/(R_{c1}) \)

- Modeling flux leakages
  - Primary and secondary flux leakage reactances: \( X_1 \) and \( X_2 \)

- Modeling winding resistances
  - Primary and secondary winding resistances: \( R_1 \) and \( R_2 \)
Exact Equivalent Circuit

\[ E_2 = V_2 + Z_2 I_2 \]

\[ E_1 = \frac{N_1}{N_2} E_2 \]

\[ = \frac{N_1}{N_2} V_2 + \frac{N_1}{N_2} Z_2 I_2 \]

\[ = \frac{N_1}{N_2} V_2 + \left( \frac{N_1}{N_2} \right)^2 Z_2 I'_2 \]

\[ = V'_2 + Z'_2 I'_2 \]

\[ Z'_2 = R'_2 + jX'_2 \]

\[ = \left( \frac{N_1}{N_2} \right)^2 R_2 + j \left( \frac{N_1}{N_2} \right)^2 X_2 \]

Note: \( V_1 \approx E_1 \), so \( Z_1 \) may be moved to the right side of the shunt branch and combined with \( Z'_2 \)
Approximate Equivalent Circuits

• Utilize $V_1 \approx E_1$

$R_1$ and $X_1$ can be combined with $R'_2$ and $X'_2$ to obtain equivalent $R_{e1}$ and $X_{e1}$

Approximate equivalent circuit referred to the primary side

$$V_1 = V'_2 + (R_{e1} + jX_{e1}) I'_2$$

$$R_{e1} = R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2$$

$$X_{e1} = X_1 + \left(\frac{N_1}{N_2}\right)^2 X_2$$

$$I'_2 = \frac{S_L^*}{3V_2^*}$$

Approximate equivalent circuit referred to the secondary side

$$V'_1 = V_2 + (R_{e2} + jX_{e2}) I_2$$
Simplified Circuits Referred to One Side

• Power transformers are generally designed with very high permeability core and very small core loss.
• If ignoring the shunt branch:
Determination of Equivalent Circuit Parameters

• Open-circuit (no-load) test
  – Neglect \((R_1 + jX_1)I_0\) (since \(|R_1 + jX_1| \ll |R_{c1}/jX_{m1}|\))
  – Measure input voltage \(V_1\), current \(I_0\), power \(P_0\) (core/iron loss)

\[
R_{c1} = \frac{V_1^2}{P_0}, \quad I_c = \frac{V_1}{R_{c1}}, \quad I_m = \sqrt{I_0^2 - I_c^2}, \quad X_{m1} = \frac{V_1}{I_m}
\]
• Short-circuit test
  – Apply a low voltage $V_{SC}$ to create rated current $I_{SC}$
  – Neglect the shunt branch due to the low core flux

$$Z_{e1} = \frac{V_{sc}}{I_{sc}}$$

$$R_{e1} = \frac{P_{sc}}{(I_{sc})^2}$$

$$X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2}$$
Transformer Performance

- Efficiency: 95% ~ 99%
- Given $|V_{2,rated}|$, $|S|=3|V_{2,rated}|I_{2,rated}$ (full-load rated VA) and PF
- Actual load is $|I_2|=n|I_{2,rated}|$ where $n$ is the fraction of the full load power

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{P_{out}(n)}{P_{out}(n) + P_{var}(n) + P_{const}}$$

$$= \frac{n \times |S| \times PF}{n \times |S| \times PF + n^2 \times P_{cu} + P_c} = \frac{|S| \times PF}{|S| \times PF + n \times P_{cu} + P_c / n}$$

\[ P_{cu} = 3R_{e2}|I_{2,rated}|^2 \quad \text{full-load copper loss (current dependent)} \]

\[ P_c: \text{ core/iron loss at rated voltage (mainly voltage dependent, almost constant)} \]

- Maximum efficiency (constant PF) occurs when

$$\frac{d\eta}{d|I_2|} = 0 \iff \frac{d\eta}{dn} = 0$$

$$n = \sqrt{\frac{P_c}{P_{cu}}}$$

Learn Example 3.4
Voltage Regulation

• % change in terminal voltage from no-load to full load (rated)

\[ VR = \frac{|V_{nl}| - |V_{\text{rated}}|}{|V_{\text{rated}}|} \times 100 \]

- Generator

\[ VR = \frac{|E| - |V_{\text{rated}}|}{|V_{\text{rated}}|} \times 100 \]

- Transformer

\[ VR = \frac{|V_{2,nl}| - |V_{2,\text{rated}}|}{|V_{2,\text{rated}}|} \times 100 \]

Utilizing the equivalent circuit referred to the primary/secondary side

\[ V_1 = V_2' + (R_{e1} + jX_{e1}) I_2' \]

\[ VR = \frac{|V_1| - |V_2'|}{|V_2'|} \times 100 \]

\[ V_1' = V_2 + (R_{e2} + jX_{e2}) I_2 \]

\[ VR = \frac{|V_1'| - |V_2|}{|V_2|} \times 100 \]
Three-Phase Transformer Connections

- A bank of three single-phase transformers connected in Y or Δ arrangements
- Four possible combinations: Y-Y, Δ-Δ, Y-Δ and Δ-Y

- **Y-connection**: lower insulation costs, with neutral for grounding, 3\(^{rd}\) harmonics problem (\(3^{rd}\) harmonic voltages/currents are all in phase, i.e. \(v_{an3}=v_{bn3}=v_{cn3}=V_m \cos 3 \omega t\))
- **Δ-connection**: more insulation costs, no neutral, providing a path for 3\(^{rd}\) harmonics (all triple harmonics are trapped in the Δ loop), able to operate with only two phases (V-connection)
- **Y-Y** and **Δ-Δ**: HV/LV ratio is same for line and phase voltages; Y-Y is rarely used due to the 3\(^{rd}\) harmonics problem.
- **Y-Δ**: commonly used as voltage step-down transformers
- **Δ-Y**: commonly used as voltage step-up transformers
Y-Δ and Δ -Y Connections

- Y-Δ and Y-Δ connections result in a 30° phase shift between the primary and secondary line-to-line voltages.
- According to the American Standards Association (ASA), the windings are arranged such that the HV side line voltage leads the LV side line voltage by 30°.
  - E.g. Y-Δ (HV-LV) Connection with the ratio of turns $a= N_H/N_X > 1$

**HV Side (indicated by “H”) in Y connection:**

$$\frac{V_{H,P}}{V_{X,P}} = \frac{V_{An}}{V_{ab}} = \frac{N_H}{N_X} = a$$

$$\frac{V_{H,L}}{V_{H,P}} = \frac{V_{AB}}{V_{An}} = \sqrt{3} \angle 30^\circ$$

$$V_{X,L} = V_{X,P}$$

**LV Side (indicated by “X”) in Δ connection:**

$$V_{ca} = V_{ab} = V_{X,P} = V_{X,L}$$

$$\frac{V_{H,L}}{V_{X,L}} = \frac{V_{AB}}{V_{ab}} = \sqrt{3} a \angle 30^\circ$$

(Complex ratio)
Per-Phase Model for Y-△ or △ -Y Connection

• Neglect the shunt branch
• Replace the △ connection by an equivalent Y connection
• Work with only one phase (equivalent impedances are line-to-neutral values $Z_{eY} = Z_{e\Delta}/3$)

E.g. for Y-△ Connection, $V_1$ is the phase voltage of the Y side and $V_2$ is the line-to-line voltage of the △ side
Autotransformers

- A conventional two-winding transformer can be changed into an autotransformer by connecting its two coils in series.
- The connection may use a sliding contact to providing variable output voltage.
- An autotransformer has kVA rating increased but loses insulation between primary and secondary windings.

(Source: EPRI Power System Dynamic Tutorial)
Autotransformer Model

\[
\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} = a
\]

- Apparent power:

\[
S_{\text{auto}} = (V_1 + V_2)I_1 = (1 + \frac{N_2}{N_1})V_1I_1 = (1 + \frac{1}{a})S_{2-w} = S_{2-w} + S_{\text{conducted}}
\]

Power rating advantage

Transformed power (thru EM induction)

Conducted power \((V_2I_1)\)

Equivalent Circuit (if the equivalent impedance is referred to the HV side)
Example 3.5

A two-winding transformer is rated at 60 kVA, 240/1200 V, 60 Hz. When operated as a conventional two-winding transformer at rated load, 0.8 power factor, its efficiency is 0.96. This transformer is to be used as a 1440/1200-V step-down autotransformer in a power distribution system.

(a) Assuming ideal transformer, find the transformer kVA rating when used as an autotransformer.

(b) Find the efficiency with the kVA loading of part (a) and 0.8 power factor.

The two-winding transformer rated currents are:

\[ I_1 = \frac{60,000}{240} = 250 \text{ A} \]

\[ I_2 = \frac{60,000}{1200} = 50 \text{ A} \]

The autotransformer connection is as shown in Figure 3.21.

![Figure 3.21](image)

**FIGURE 3.21**
Auto transformer connection for Example 3.5.

(a) The autotransformer secondary current is

\[ I_L = 250 + 50 = 300 \text{ A} \]

With windings carrying rated currents, the autotransformer rating is

\[ S = (1200)(300)(10^{-3}) = 360 \text{ kVA} = 1440 \times 250 \times 10^{-3} \]

Therefore, the power advantage of the autotransformer is

\[ \frac{S_{\text{auto}}}{S_{2-w}} = \frac{360}{60} = 6 \]

(b) When operated as a two-winding transformer at full-load, 0.8 power factor, the losses are found from the efficiency formula

\[ \frac{(60)(0.8)}{(60)(0.8) + P_{\text{loss}}} = 0.96 \]

Solving the above equation, the total transformer loss is

\[ P_{\text{loss}} = \frac{48(1 - 0.96)}{0.96} = 2.0 \text{ kW} \]

Since the windings are subjected to the same rated voltages and currents as the two-winding transformer, the autotransformer copper loss and the core loss at the rated values are the same as the two-winding transformer. Therefore, the autotransformer efficiency at rated load, 0.8 power factor, is

\[ \eta = \frac{(360)(0.8)}{(360)(0.8) + 2} \times 100 = 99.31 \text{ percent} \]
True or False?

A two-winding transformer is first operated as a conventional two-winding transformer and then as an autotransformer to supply the same load. When it is operated as the autotransformer, it has

– a higher rating,
– a higher efficiency and
– a lower loss

than it is operated as the conventional two-winding transformer.

• Answer: True. For the autotransformer, rating (not the actual loading) will be increased, loss will decrease since currents in two windings will decrease to generate output current $I_L$ equal to output current $I_2$ of the two-winding transformer. Then the efficiency will increase considering the loading is the same.
Three-Winding Transformers

- Primary (P), secondary (S) and tertiary (T) windings
- Typical applications:
  - The same source supplying two independent loads at different voltages
  - Interconnection of two transmission systems of different voltages
  - In a substation, the tertiary winding is used to provide voltage for auxiliary power purposes or to supply a local distribution system
  - Reactive power compensation by switched reactors or capacitors connected to the tertiary winding
Three-Winding Transformer Model

• $Z_p$, $Z_s$ and $Z_t$ are impedances of three windings referred to the P side

• Define

\[
Z_{ps} = Z_p + Z_s
\]
\[
Z_{pt} = Z_p + Z_t
\]
\[
Z_{st} = Z_s + Z_t
\]

\[
Z_p = \left( \frac{Z_{ps} + Z_{pt} - Z_{st}}{2} \right)
\]
\[
Z_s = \left( \frac{Z_{ps} + Z_{st} - Z_{pt}}{2} \right)
\]
\[
Z_t = \left( \frac{Z_{pt} + Z_{st} - Z_{ps}}{2} \right)
\]

• Estimate impedances by short-circuit tests:

$Z_{ps}$ = measured P impedance, with S short-circuited and T open

$Z_{pt}$ = measured P impedance, with T short-circuited and S open

$Z'_{st}$ = measured S impedance, with T short-circuited and P open

• Referring $Z'_{st}$ to the P side:

\[
Z_{st} = \left( \frac{N_p}{N_s} \right)^2 Z'_{st}
\]
Voltage Control of Transformers

• Voltage magnitude → Reactive power flow
• Voltage phase angle → Real power flow

• Two commonly used methods
  – Tap changing Transformers
  – Regulating transformers
Tap Changing Transformers

• Practically all power transformers in transmission systems and many distribution transformers have taps in one or more windings for changing turns ratios

• Off-load tap changing transformers
  – They are adjusted only when the transformer current flow has been completely interrupted
  – Typically, 4 taps with 2.5% of rated voltage each tap to allow an operating range of -5% ~ +5% of rated voltage

• Under-load tap changing (ULTC) transformers
  – More flexible when the transformer is in-service
  – May have built-in voltage sensing circuitry for automatic tap changing to keep voltage constant
  – Typically, off-load tap changers at the HV side for -5% ~ +5%; ULTC at the LV side with 32 taps of 0.625% each for an operating range of -10% ~ +10%
ULTC Model

• How to adjust \( t_S \) and \( t_R \) to maintain \( V'_1 \) and \( V'_2 \) at desired levels?

\[
V_R = V_S - (R + jX) I
\]

Since phase shift \( \delta \) is small

\[
|V_S| \approx |V_S| \cos \delta
\]

\[
= |V_R| + ab + de
\]

\[
= |V_R| + |I| R \cos \theta + |I| X \sin \theta
\]

\[
= |V_R| + \frac{R}{|V_R|} |I| |V_R| \cos \theta + \frac{X}{|V_R|} |I| |V_R| \sin \theta
\]

\[
= |V_R| + \frac{RP_{\phi} + XQ_{\phi}}{|V_R|}
\]

\[
t_S |V_1'| = t_R |V_2'| + \frac{RP_{\phi} + XQ_{\phi}}{t_R |V_2'|}
\]

\[
t_s = \frac{1}{|V_1'|} \left( t_R |V_2'| + \frac{RP_{\phi} + XQ_{\phi}}{t_R |V_2'|} \right)
\]
Limitations of ULTC in Voltage Control

- Normally, when the turns ratio is adjusted, Mvar flow across the transformer also changes.
- However since a transformer itself absorbs Mvar to build its internal magnetic field, when its secondary voltage is raised via a tap change, its Mvar usage increases and its primary voltage often drops. The greater the tap change ($V_2 \uparrow$ and $I_2 \uparrow$) and the weaker the primary side (high impedance), the greater the primary voltage drop.
- If the primary side is weak, the tap change may not necessarily increase the secondary voltage. Therefore, spare Mvar must be available for a tap change to be successful.

An example:

±10% / 33-position ULTC

(Source: EPRI Dynamic Tutorial)
Regulating Transformers – Voltage Magnitude Control

\[ V'_{an} = V_{an} \pm \Delta V_{an} \]

Using \( V_{an} \) to induce a voltage (\( \parallel \)) added to \( V_{an} \).
Regulating Transformers – Phase Angle Control

• Phase shifting transformer (PST), phase angle regulator (PAR) or Quadrature booster

\[ V'_{an} = V_{an} \pm \Delta V_{bc} \]

Using \( V_{bc} \) to induce a voltage (\( \perp \)) added to \( V_{an} \)

400MVA 220/155kV phase-shifting transformer
(source: wikipedia.org)
Application of PST

The Exciting Voltage is $90^\circ$ Out-of-Phase with the Series Voltage. You can Advance or Retard the Voltage Phase Angle Difference by Varying the Series Tap Position.

Figure 10-29. Varying Construction of a PST

(Source: EPRI Dynamic Tutorial)
Figure 10-31. Phasor Diagram for Regulating Transformers

(Source: EPRI Dynamic Tutorial)
The Per-Unit System

- Quantity in **per-unit** = **Actual** quantity / **Base** value of quantity
- Why per-unit system?

- Neglecting different voltage levels of transformers, lines and generators
- Powers, voltages, currents and impedances are expressed as decimal fractions of respective base quantities

(Source: EPRI Dynamic Tutorial)
• Four base quantities are required to completely define a per-unit system

\[ S_{pu} = \frac{S}{S_B} \quad V_{pu} = \frac{V}{V_B} \quad I_{pu} = \frac{I}{I_B} \quad Z_{pu} = \frac{Z}{Z_B} \]

• Usually, two independent bases are selected, and the other two are calculated

  – \( S_B \) or \( \text{MVA}_B \)  Three-phase base volt-ampere
  – \( V_B \) or \( \text{kV}_B \)  Line-to-line base voltage

\[ I_B = \frac{S_B}{\sqrt{3}V_B} \quad Z_B = \frac{V_B / \sqrt{3}}{I_B} = \frac{(V_B)^2}{S_B} = \frac{(\text{kV}_B)^2}{	ext{MVA}_B} \]

• The phase and line quantities expressed in per-unit are the same

\[ S_{pu} = V_{pu} I_{pu}^* \quad V_{pu} = Z_{pu} I_{pu}^* \]

• Relationship between the load power and per-unit impedance

\[ S_{3\phi} = 3V_P I_P^* \quad Z_P = \frac{V_P}{I_P} = \frac{3 V_P^2}{S_{3\phi}^*} = \frac{V_{L-L}^2}{S_{3\phi}^*} \]

\[ Z_{pu} = \frac{Z_P}{Z_B} = \frac{V_{L-L}^2}{S_{3\phi}^*} \left( \frac{S_B}{V_B} \right)^2 = \left( \frac{V_{L-L}}{V_B} \right)^2 \left( \frac{S_{3\phi}^*}{S_{3\phi}} \right) = \frac{V_{pu}^2}{S_{pu}} \]
ULTC Per-Unit Model

• How to adjust \( t_S \) and \( t_R \) to maintain \( V'_1 \) and \( V'_2 \) at desired levels?

\[
t_S = \frac{1}{|V'_1|} \left( t_R |V'_2| + \frac{RP_\phi + XQ_\phi}{t_R |V'_2|} \right)
\]

• If all quantities are in per-unit, \( t_S \approx 1 \text{pu} \) and \( t_R \approx 1 \text{pu} \)

• Assuming \( t_S t_R = 1 \)

\[
t_S = \sqrt{ \frac{|V'_2|}{|V'_1|} } \frac{RP_\phi + XQ_\phi}{1 - \frac{RP_\phi + XQ_\phi}{|V'_1||V'_2|}} \quad t_R = 1/t_S
\]

• Example 3.6: \( t_S = 1.08 \text{pu} \) and \( t_R = 0.926 \text{pu} \)
Change of Base

\[
Z_{pu}^{\text{old}} = \frac{Z}{Z_B^{\text{old}}} = Z \frac{S_B^{\text{old}}}{(V_B^{\text{old}})^2}
\]

\[
Z_{pu}^{\text{new}} = \frac{Z}{Z_B^{\text{new}}} = Z \frac{S_B^{\text{new}}}{(V_B^{\text{new}})^2}
\]

Eliminate Z

\[
Z_{pu}^{\text{new}} = Z_{pu}^{\text{old}} \frac{S_B^{\text{new}}}{S_B^{\text{old}}} \left( \frac{V_B^{\text{old}}}{V_B^{\text{new}}} \right)^2
\]

(Simplified equation if \( V_B \) is the same)

- The nameplates of transformers and generators usually give impedance as \( X\% \) based on its rated voltage and rated MVA. Also, it is usually operated under the same (or very close to) rated voltage level while the base MVA may be quite different. Then, the impedance under the new, system MVA base can be calculated using the above simplified equation.
Advantages of the Per-Unit System

• Gives a clear idea of relative magnitudes of various quantities, e.g. voltage, current, power and impedance

• The per-unit impedance of equipment of the same general type based on their own ratings fall in a narrow range regardless of the rating of the equipment. Whereas their impedance in ohms vary greatly with the rating

• The per-unit values of impedance, voltage and current of a transformer are the same regardless of whether they are referred to the primary or the secondary side

• Idea for computerized analysis and simulation of complex power system problems

• The circuit laws are valid in per-unit systems. For three-phase systems, factors of $\sqrt{3}$ and 3 are eliminated by properly selecting base quantities
Example 3.7

The one-line diagram of a three-phase power system is shown in Figure 3.29. Select a common base of 100 MVA and 22 kV on the generator side. Draw an impedance diagram with all impedances including the load impedance marked in per-unit. The manufacturer’s data for each device is given as follow:

\[
\begin{align*}
G: & \quad X = 0.18 \left( \frac{100}{90} \right) = 0.20 \text{ pu} \\
T_1: & \quad X = 0.10 \left( \frac{100}{50} \right) = 0.20 \text{ pu} \\
T_2: & \quad X = 0.06 \left( \frac{100}{40} \right) = 0.15 \text{ pu} \\
T_3: & \quad X = 0.064 \left( \frac{100}{40} \right) = 0.16 \text{ pu} \\
T_4: & \quad X = 0.08 \left( \frac{100}{40} \right) = 0.20 \text{ pu} \\
M: & \quad X = 0.185 \left( \frac{100}{66.5} \right) \left( \frac{10.45}{11} \right)^2 = 0.25 \text{ pu}
\end{align*}
\]

\[
\begin{align*}
\text{Line 1:} & \quad Z_{B2} = \frac{(220)^2}{100} = 484 \ \Omega \quad X = \left( \frac{48.4}{484} \right) = 0.10 \text{ pu} \\
\text{Line 2:} & \quad Z_{B5} = \frac{(110)^2}{100} = 121 \ \Omega \quad X = \left( \frac{65.43}{121} \right) = 0.54 \text{ pu}
\end{align*}
\]

The three-phase load at bus 4 absorbs 57 MVA, 0.6 power factor lagging at 10.45 kV. Line 1 and line 2 have reactances of 48.4 and 65.43 Ω, respectively.

\[
\begin{align*}
V_{B3} &= V_{B2} = 22 \left( \frac{220}{22} \right) = 220 \ \text{kV} \\
V_{B4} &= 220 \left( \frac{11}{220} \right) = 11 \ \text{kV} \\
V_{B5} &= V_{B6} = 22 \left( \frac{110}{22} \right) = 110 \ \text{kV}
\end{align*}
\]
Example 3.8

The motor of Example 3.7 operates at full-load 0.8 power factor leading at a terminal voltage of 10.45 kV.

(a) Determine the voltage at the generator bus bar (bus 1).
(b) Determine the generator and the motor internal emfs.

(a) The per-unit voltage at bus 4, taken as reference is

\[ V_4 = \frac{10.45}{11} = 0.95^\circ 0^\circ \text{ pu} \]

The motor apparent power at 0.8 power factor leading is given by

\[ S_m = \frac{66.5}{100} \angle -36.87^\circ \text{ pu} \]

Therefore, current drawn by the motor is

\[ I_m = \frac{S_m}{V_4} = \frac{0.665/36.87}{0.95/0^\circ} = 0.56 + j0.42 \text{ pu} \]

and current drawn by the load is

\[ I_L = \frac{V_4}{Z_L} = \frac{0.95/0^\circ}{0.95 + j1.2667} = 0.36 - j0.48 \text{ pu} \]

Total current drawn from bus 4 is

\[ I = I_m + I_L = (0.56 + j0.42) + (0.36 - j0.48) = 0.92 - j0.06 \text{ pu} \]

The equivalent reactance of the parallel branches is

\[ X_{\parallel} = \frac{0.45 \times 0.9}{0.45 + 0.9} = 0.3 \text{ pu} \]

The generator terminal voltage is

\[ V_1 = V_4 + Z_q I = 0.95/0^\circ + j0.3(0.92 - j0.06) = 0.968 + j0.276 \]

\[ = 1.0^\circ 15.91^\circ \text{ pu} \]

\[ = 22^\circ 15.91^\circ \text{ kV} \]

(b) The generator internal emf is

\[ E_g = V_1 + Z_q I = 0.968 + j0.276 + j0.20(0.92 - j0.06) = 1.0826/25.14^\circ \text{ pu} \]

\[ = 23.82/25.14^\circ \text{ kV} \]

and the motor internal emf is

\[ E_m = V_4 - Z_m I_m = 0.95 + j0 - j0.25(0.56 + j0.42) = 1.064/\angle -7.56^\circ \text{ pu} \]

\[ = 11.71/\angle -7.56^\circ \text{ kV} \]