ECE 421/599
Electric Energy Systems
5 – Line Model and Performance

Instructor:
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Line Models

• Short Line Model
  – 80km (50 miles) or less, 69kV or lower
  – Ignoring capacitance

• Medium Line Model
  – 80km (50 miles) ~ 250km (150 miles)
  – Lumped line parameters

• Long Line Model
  – 250km (150 miles) or longer
  – Distributed line parameters

• Example: L=1mH/km, C=0.01μF/km and r=0.01Ω/km

<table>
<thead>
<tr>
<th>$\omega = 2\pi \times 60 = 377$Hz</th>
<th>$l=80$km</th>
<th>$l=250$km</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R = r \times l \ (\Omega)$</td>
<td>0.8 = 0.027$X_L$</td>
<td>2.5 = 0.027$X_L$</td>
</tr>
<tr>
<td>$X_L = \omega L \times l \ (\Omega)$</td>
<td>30.2</td>
<td>94.3</td>
</tr>
<tr>
<td>$X_C = \frac{1}{(\omega C) \times l} \ (\Omega)$</td>
<td>3315.6 = 109.8$X_L$</td>
<td>1061.0 = 11.3$X_L$</td>
</tr>
</tbody>
</table>
Short Line Model

- Capacitance is ignored.

\[
Z = (r + j \omega L)l = R + jX
\]

\[
\begin{bmatrix}
V_S \\
I_S
\end{bmatrix} = \begin{bmatrix}
A & B \\
C & D
\end{bmatrix} \begin{bmatrix}
V_R \\
I_R
\end{bmatrix}
\]

\[
\begin{bmatrix}
V_S \\
I_S
\end{bmatrix} = \begin{bmatrix}
V_R + ZI_R \\
I_R
\end{bmatrix} = \begin{bmatrix}
1 & Z \\
0 & 1
\end{bmatrix} \begin{bmatrix}
V_R \\
I_R
\end{bmatrix}
\]

\[A = 1 \quad B = Z \quad C = 0 \quad D = 1\]

\[
S_{S(3\phi)} = 3V_SI_S^*
\]

\[
S_{L(3\phi)} = S_{S(3\phi)} - S_{R(3\phi)} \quad \eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}}
\]
- Voltage Regulation (VR):

Percent \( VR = \frac{|V_{R(NL)}| - |V_{R(FL)}|}{|V_{R(FL)}|} \times 100\% \)

- It is a measure of line voltage drop
- Depends on the load power factor: VR is poorer at low lagging power factor
- Perhaps, \( VR < 0 \) for a leading power factor (i.e. \(|V_S| < |V_R|\)). See Example 5.1
Medium Line Model

• Model the total shunt admittance of the line by

\[ Y = (g + j\omega C)\ell \approx j\omega C\ell \]

– \( g \), the shunt conductance per unit length, represents the leakage current over the insulators is negligible under normal condition.
– \( C \) is the line to neutral capacitance per unit length

• Nominal \( \pi \) model:
– Half of \( C \) is considered to be lumped at each end of the line

\[ Z = R + jX \]
• Find $V_S, I_S \leftrightarrow V_R, I_R$

$I_L = I_R + \frac{Y}{2}V_R$

$V_S = V_R + ZI_L$

$= V_R + Z(I_R + \frac{Y}{2}V_R)$

$V_S = (1 + \frac{ZY}{2})V_R + ZI_R$

$I_S = I_L + \frac{Y}{2}V_S$

$= (I_R + \frac{Y}{2}V_R) + \frac{Y}{2} \left[ (1 + \frac{ZY}{2})V_R + ZI_R \right]$

$I_S = Y(1 + \frac{ZY}{4})V_R + (1 + \frac{ZY}{2})I_R$

\[
\begin{bmatrix}
V_S \\
I_S
\end{bmatrix} =
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
V_R \\
I_R
\end{bmatrix} =
\begin{bmatrix}
1 + \frac{ZY}{2} & Z \\
Y(1 + \frac{ZY}{4}) & 1 + \frac{ZY}{2}
\end{bmatrix}
\begin{bmatrix}
V_R \\
I_R
\end{bmatrix}
\]

$A = 1 + \frac{ZY}{2}$  \hspace{1cm}  $B = Z$  \hspace{1cm}  $C = Y(1 + \frac{ZY}{4})$  \hspace{1cm}  $D = 1 + \frac{ZY}{2}$

\[
\text{det} \begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = AD - BC = 1
\]

\[
\begin{bmatrix}
V_R \\
I_R
\end{bmatrix} =
\begin{bmatrix}
D & -B \\
-C & A
\end{bmatrix}
\begin{bmatrix}
V_S \\
I_S
\end{bmatrix}
\]

Linear, passive, bilateral two-port network (no source)
Long Line Model

- Series impedance per unit length
  \[ z = r + j\omega L \]

- Shunt admittance per unit length
  \[ y = g + j\omega C \]

- Consider a small segment of \( \Delta x \) at distance \( x \) from the receiving end

\[
\begin{align*}
V(x + \Delta x) &= V(x) + z\Delta x I(x) \\
I(x + \Delta x) &= I(x) + y\Delta x V(x + \Delta x)
\end{align*}
\]

\[
\begin{align*}
d\frac{V(x)}{dx} &= zI(x) \\
d\frac{I(x)}{dx} &= yV(x)
\end{align*}
\]

\[
\begin{align*}
\frac{V(x + \Delta x) - V(x)}{\Delta x} &= zI(x) \\
\frac{I(x + \Delta x) - I(x)}{\Delta x} &= yV(x + \Delta x)
\end{align*}
\]

\[
\begin{align*}
\frac{d^2V(x)}{dx^2} &= z \frac{dI(x)}{dx} = zyV(x)
\end{align*}
\]
\[
\frac{d^2 V(x)}{dx^2} = zy V(x) \quad \Rightarrow \quad \gamma^2 = zy
\]

\[
\gamma = \alpha + j \beta = \sqrt{z}y = \sqrt{(r + j \omega L)(g + j \omega C)}
\]

If line losses are neglected, i.e. \( r=0 \) and \( g=0 \)

\[
\gamma = \alpha + j \beta = \sqrt{-\omega^2 LC} = j \omega \sqrt{LC}
\]

\[\alpha = 0, \quad \beta = \omega \sqrt{LC}\]

\[
V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}
\]

\[
I(x) = \frac{1}{z} \frac{dV(x)}{dx} = \frac{\gamma}{z} (A_1 e^{\gamma x} - A_2 e^{-\gamma x}) = \sqrt{\frac{\gamma}{z}} (A_1 e^{\gamma x} - A_2 e^{-\gamma x})
\]

\[
= \frac{1}{Z_C} (A_1 e^{\gamma x} - A_2 e^{-\gamma x})
\]

\[Z_C = \sqrt{\frac{z}{y}} \quad \text{- Characteristic impedance}\]
\[ V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x} \]

\[ I(x) = \frac{1}{Z_C} \left( A_1 e^{\gamma x} - A_2 e^{-\gamma x} \right) \]

\begin{itemize}
  \item Find \( A_1 \) and \( A_2 \): at the receiving end, \( x=0 \), \( V(x)=V_R \) and \( I(x)=I_R \)
  \end{itemize}

\[ V(0) = V_R = A_1 + A_2 \]

\[ I(0) = I_R = \frac{1}{Z_C} (A_1 - A_2) \]

\[ A_1 = \frac{V_R + Z_C I_R}{2} \]

\[ A_2 = \frac{V_R - Z_C I_R}{2} \]

\[ |A_1| > |A_2| \text{ or } |A_1| < |A_2|? \]

\[ V(x) = \frac{V_R + Z_C I_R}{2} e^{\gamma x} + \frac{V_R - Z_C I_R}{2} e^{-\gamma x} = \frac{e^{\gamma x} + e^{-\gamma x}}{2} V_R + Z_C \frac{e^{\gamma x} - e^{-\gamma x}}{2} I_R \]

\[ I(x) = \frac{V_R + I_R}{Z_c} e^{\gamma x} - \frac{V_R - I_R}{Z_c} e^{-\gamma x} = \frac{1}{Z_c} \frac{e^{\gamma x} - e^{-\gamma x}}{2} V_R + \frac{e^{\gamma x} + e^{-\gamma x}}{2} I_R \]

\[ Z_C = \sqrt{\frac{z}{y}} \text{ - Characteristic impedance} \]

\[ \cosh x = \frac{e^x + e^{-x}}{2} \quad \sinh x = \frac{e^x - e^{-x}}{2} \]
\[ V(x) = \cosh \gamma x V_R + Z_c \sinh \gamma x I_R \]

\[ I(x) = \frac{1}{Z_c} \sinh \gamma x V_R + \cosh \gamma x I_R \]
At the sending end, \( x = l, \ V(l) = V_S, \ I(l) = I_S \)

\[
V_S = \cosh \gamma l V_R + Z_c \sinh \gamma l I_R
\]

\[
I_s = \frac{1}{Z_c} \sinh \gamma l V_R + \cosh \gamma l I_R
\]

\[
\begin{bmatrix}
V_S \\
I_s
\end{bmatrix} =
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
V_R \\
I_R
\end{bmatrix}
\]

\[
A = \cosh \gamma l = 1 + \frac{Z'Y'}{2}
\]

\[
B = Z_c \sinh \gamma l = Z'
\]

\[
C = \frac{1}{Z_c} \sinh \gamma l = Y'(1 + \frac{Z'Y'}{4})
\]

\[
D = \cosh \gamma l = 1 + \frac{Z'Y'}{2}
\]

A = D, \( AD - BC = 1 \)

Linear, passive, bilateral two-port network (no source)

Compared to the medium line \( \pi \) model:

\[
V_S = (1 + \frac{Z Y}{2}) V_R + Z I_R
\]

\[
I_s = Y(1 + \frac{Z Y}{4}) V_R + (1 + \frac{Z Y}{2}) I_R
\]

\[
A = 1 + \frac{Z Y}{2} \quad B = Z
\]

\[
C = Y(1 + \frac{Z Y}{4}) \quad D = 1 + \frac{Z Y}{2}
\]

\[
Z' = Z_c \sinh \gamma l = \sqrt{\frac{Z}{Y}} \sqrt{zy \ell \frac{\sinh \gamma l}{\gamma l}} = Z \frac{\sinh \gamma l}{\gamma l}
\]

\[
Y' = \cosh \gamma l - 1 = \cosh \gamma l - 1 = \sqrt{\frac{y}{z}} \sqrt{zy \ell \frac{\tanh \gamma l}{\gamma l}} = \frac{Y \tanh \frac{\gamma l}{2}}{2}
\]
Equivalent $\pi$ Model for Long Length Lines

\[ Z = zl = (r + j\omega L)l \]
\[ Y = yl = (g + j\omega C)l \]
\[ \gamma = \sqrt{z}y = \sqrt{(r + j\omega L)(g + j\omega C)} \]

\[ \frac{Z'}{Z} = \frac{\sinh(\gamma l)}{\gamma l} \]
\[ \frac{Y'}{Y} = \frac{\tanh(\gamma l/2)}{\gamma l/2} \]
Example 4.1 in Bergen and Vittal’s Book

- A 60-Hz 138kV 3-phase transmission line is 225 mi long. The distributed line parameters are \( r = 0.169 \Omega/\text{mi} \), \( L = 2.093 \text{mH/mi} \), \( C = 0.01427 \mu\text{F/mi} \), \( g = 0 \). The transmission line delivers 40MW at 132kV with 95% power factor lagging.
  - Find the sending-end voltage and current.
  - Find the transmission line efficiency

Solution:

\[ \omega = 2\pi \times 60 = 377 \text{rad/s} \]

\[ z = r + j \omega L = 0.169 + j377 \times 2.093 \times 10^{-3} = 0.169 + j0.789 = 0.807 \angle 77.9^\circ \Omega/\text{mi} \]

\[ y = j \omega C = j377 \times 0.01427 \times 10^{-6} = j5.38 \times 10^{-6} = 5.38 \times 10^{-6} \angle 90^\circ \text{S/mi} \]

\[ Z_C = \sqrt{z/y} = 387.3 \angle -6.05^\circ \Omega/\text{mi} \approx \text{Real number} \]

\[ \gamma = \alpha + j \beta = \sqrt{zy} = 0.136 \times 10^{-6} + j1.29 \times 10^{-6} \]

\[ \gamma l = 225 \sqrt{zy} = 0.4688 \angle 83.95^\circ = 0.0494 + j0.466 \approx \text{Imaginary number} \]

\[ 2 \sinh \gamma l = e^{\gamma l} - e^{-\gamma l} = e^{0.0494} - e^{-0.0494} = 1.051 \angle 0.466 \text{ rad} -0.952 \angle -0.466 \text{ rad} \]

\[ \sinh \gamma l = 0.452 \angle 84.4^\circ \]. Similarly, \( \cosh \gamma l = 0.8950 \angle 1.42^\circ \)

Let \( \angle V_R = 0 \). \( V_R = 132 \times 10^3 / \sqrt{3} = 76.2 \text{kV} \)

\[ P_{\text{load}} = 0.95 |V_R||I_R| = 40 / 3 = 13.33 \text{MW} \quad \theta = \cos^{-1}(0.95) = 18.195^\circ \]

\[ I_R = 184.1 \angle -18.195^\circ \text{A} \]

\[ V_s = \cosh \gamma l V_R + Z_C \sinh \gamma l I_R = 89.28 \angle 19.39^\circ \text{kV} \]

\[ I_s = \frac{1}{Z_C} \sinh \gamma l V_R + \cosh \gamma l I_R = 162.42 \angle 14.76^\circ \text{A} \]

\[ \eta = \frac{P_{\text{load}}}{\text{Re}(V_s I_s^*)} = \frac{13.33}{14.45} = 92\% \]

\[ |A_1| = 72.1 \text{kV} \]

\[ |A_2| = 1.57 \text{kV} \]

\[ 13 \]
Voltage and Current Waves

\[ V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x} = A_1 e^{\alpha x} e^{i\beta x} + A_2 e^{-\alpha x} e^{-i\beta x} \]

• Instantaneous voltage as a function of \( t \) and \( x \)

\[ v(t, x) = \sqrt{2} \text{ Re} \left[ A_1 e^{\alpha x} e^{j(\omega t + \beta x)} \right] + \sqrt{2} \text{ Re} \left[ A_2 e^{-\alpha x} e^{j(\omega t - \beta x)} \right] \]

\[ = v_1(t, x) + v_2(t, x) \]

\[ v_1(t, x) = \sqrt{2} \ | A_1 | e^{\alpha x} \cos(\omega t + \beta x + \angle A_1) \]

Incident wave: amplitude ↑ when \( x \) ↑

\[ v_2(t, x) = \sqrt{2} \ | A_2 | e^{-\alpha x} \cos(\omega t - \beta x + \angle A_2) \]

Reflected wave: amplitude ↓ when \( x \) ↑
Velocity and Wavelength of Propagation

- Consider

\[ v_2(t, x) = \sqrt{2} \mid A_2 \mid \cos(\omega t - \beta x) \]

coming from the receiving end (x=0)

- For a point on the traveling wave: \( \omega t - \beta x = \text{constant} \)
- Its moving speed (velocity of propagation) and the wavelength

\[ v = \frac{dx}{dt} = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} \quad \lambda = \frac{v}{f} = \frac{2\pi}{\beta} \]

- If \( \alpha=0 \), \( \beta = \omega \sqrt{LC} \)

\[ Z_c = \sqrt{\frac{L}{C}} \]

\( \text{Surge impedance} \)

- \( GMR_L \approx GMR_C \)

For 3 bundled conductors: \( GMR_C / GMR_L = \sqrt[3]{r / r'} = e^{3 \times 4} = 1.09 \)

\[ v \approx \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}} = 3 \times 10^8 \text{ m/s} \quad \lambda \approx \frac{1}{60 \sqrt{\mu_0 \varepsilon_0}} = 5000 \text{ km} \]

\[ Z_c = \frac{1}{2\pi} \sqrt{\frac{\mu_0}{\varepsilon_0} \ln \frac{GMD}{GMR_c}} \approx 60 \ln \frac{GMD}{GMR_c} \]
**Lossless Lines**

\[ V(x) = \cosh \gamma x V_R + Z_c \sinh \gamma x I_R \]

\[ I(x) = \frac{1}{Z_c} \sinh \gamma x V_R + \cosh \gamma x I_R \]

\[ \gamma = j \beta \]

\[ \cosh \gamma x = \cosh j \beta x = \frac{e^{j \beta x} + e^{-j \beta x}}{2} = \cos \beta x \]

\[ \sinh \gamma x = \sinh j \beta x = \frac{e^{j \beta x} - e^{-j \beta x}}{2} = j \sin \beta x \]

\[ V(x) = \cos \beta x V_R + j Z_C \sin \beta x I_R \]

\[ I(x) = j \frac{1}{Z_C} \sin \beta x V_R + \cos \beta x I_R \]

- **Open circuit at the receiving end:**
  \[ I_R = 0 \quad V_{R(nl)} = V_S / \cos \beta \ell \geq V_S \]

- **Short circuit at the receiving end:**
  \[ V_R = 0 \quad V_S = j Z_C \sin \beta \ell I_R \]
  \[ I_S = \cos \beta \ell I_R \]

\[ A = D = \cos \beta \ell, \quad B = j Z C \sin \beta \ell, \quad C = j \frac{1}{Z_C} \sin \beta \ell \]

\[ V_{R(nl)} \approx V_S \]

For short lines \( \beta \ell \approx 0 \)

\[ I_S \approx I_R \rightarrow \infty \]
Surge Impedance Loading

- When $Z_L = Z_C$

\[
I_R = \frac{V_R}{Z_C}
\]

- For a lossless line, $Z_C$ is purely resistive.

Surge impedance loading (SIL) is the loading when $Z_L = Z_C$ at rated voltage

\[
SIL = 3V_R I^*_R = \frac{3 |V_R|^2}{Z_C} = \frac{3 |V_{\text{Lrated}}/\sqrt{3}|^2}{Z_C} = \frac{(kV_{\text{Lrated}})^2}{Z_C} \quad \text{MW}
\]

\[
V(x) = \cos \beta x V_R + j Z_C \sin \beta x I_R = (\cos \beta x + j \sin \beta x)V_R = V_R \angle \beta x
\]

\[
I(x) = j \frac{1}{Z_C} \sin \beta x V_R + \cos \beta x I_R = (\cos \beta x + j \sin \beta x)I_R = I_R \angle \beta x
\]
Observations from SIL

\[ V(x) = V_R \angle \beta x \quad I(x) = I_R \angle \beta x \]

- \(|V(x)| = |V_S| = |V_R|, \quad |I(x)| = |I_S| = |I_R|\)
- PF=1 for any x

- \(Q_S = Q_R = 0\): Q losses due to line inductance are exactly offset by Q supplied by shunt capacitance, i.e.

\[ \omega L |I_R|^2 = \omega C |V_R|^2 \]

- SIL is a useful measure of transmission line capacity:
  - For load >>SIL, shunt capacitors may be needed to minimize voltage drop along the line
  - For load <<SIL, shunt inductors may be needed to avoid over-voltage issues at the receiving end
Voltage profile for length up to 1/8 wavelength, $Z_c = 290.5 \text{ ohms}$

**FIGURE 5.11**
Voltage profile for length up to 1/8 wavelength.
Table 6.1 Typical overhead transmission line parameters

<table>
<thead>
<tr>
<th>Nominal Voltage</th>
<th>230 kV</th>
<th>345 kV</th>
<th>500 kV</th>
<th>765 kV</th>
<th>1,100 kV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ (Ω/km)</td>
<td>0.050</td>
<td>0.037</td>
<td>0.028</td>
<td>0.012</td>
<td>0.005</td>
</tr>
<tr>
<td>$x_L$ (Ω/km)</td>
<td>0.488</td>
<td>0.367</td>
<td>0.325</td>
<td>0.329</td>
<td>0.292</td>
</tr>
<tr>
<td>$b_C$ (μs/km)</td>
<td>3.371</td>
<td>4.518</td>
<td>5.200</td>
<td>4.978</td>
<td>5.544</td>
</tr>
<tr>
<td>$\alpha$ (nepers/km)</td>
<td>0.000067</td>
<td>0.000066</td>
<td>0.000057</td>
<td>0.000025</td>
<td>0.000012</td>
</tr>
<tr>
<td>$\beta$ (rad/km)</td>
<td>0.00128</td>
<td>0.00129</td>
<td>0.00130</td>
<td>0.00128</td>
<td>0.00127</td>
</tr>
<tr>
<td>$Z_C$ (Ω)</td>
<td>380</td>
<td>285</td>
<td>250</td>
<td>257</td>
<td>230</td>
</tr>
<tr>
<td>SIL (MW)</td>
<td>140</td>
<td>420</td>
<td>1000</td>
<td>2280</td>
<td>5260</td>
</tr>
</tbody>
</table>

Charging MVA/km $= V_0^2 b_C$

0.18  0.54  1.30  2.92  6.71

Notes:  
1. Rated frequency is assumed to be 60 Hz.  
2. Bundled conductors used for all lines listed, except for the 230 kV line.  
3. $R$, $x_L$, and $b_C$ are per-phase values.  
4. SIL and charging MVA are three-phase values.

(Source: Kundur’s book)
Complex Power Flow Through Transmission Lines

\[
\begin{bmatrix}
V_S \\
I_S \\
V_R \\
I_R
\end{bmatrix} =
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
V_R \\
I_R
\end{bmatrix} = \begin{bmatrix} A = D, \quad AD - BC = 1 \\
C = (AD - 1) / B \end{bmatrix}
\]

- Define \( V_R = |V_R| \angle 0 \) \( V_S = |V_S| \angle \delta \) \( A = |A| \angle \theta_A \) \( B = |B| \angle \theta_B \)

\[
I_R = \frac{V_S - AV_R}{B} = \frac{|V_S| \angle \delta - |A||V_R| \angle \theta_A}{|B| \angle \theta_B} = \frac{|V_S| \angle (\delta - \theta_B) - |A||V_R| \angle (\theta_A - \theta_B)}{|B|}
\]

\[
S_{R(3\phi)} = P_{R(3\phi)} + jQ_{R(3\phi)} = 3V_R I_R^* = \frac{3|V_S||V_R|}{|B|} \angle (\theta_B - \delta) - \frac{3|A||V_R|^2}{|B|} \angle (\theta_B - \theta_A)
\]

\[
\theta_A \approx 0, \theta_B \approx 90^\circ
\]

\[
I_S = \frac{DV_S - V_R}{B} = \frac{|A||V_S| \angle \theta_A + \delta - |V_R| \angle 0}{|B| \angle \theta_B} = \frac{|A||V_S| \angle (\theta_A - \theta_B + \delta)}{|B|} - \frac{|V_R| \angle - \theta_B}{|B|}
\]

\[
S_{S(3\phi)} = P_{S(3\phi)} + jQ_{S(3\phi)} = 3V_S I_S^* = \frac{A||V_{S(L-L)}|^2 \angle (\theta_B - \theta_A)}{|B|} - \frac{|V_{S(L-L)}||V_{R(L-L)}| \angle (\theta_B + \delta)}{|B|}
\]
For a lossless line, $B = jX'$, $\theta_A = 0$, $\theta_B = 90^\circ$, and $A = \cos \beta \ell$

\[
\begin{align*}
P_{S(3\phi)} &= P_{R(3\phi)} = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{X'} \sin \delta = P_{3\phi} \\
Q_{S(3\phi)} &= \frac{|V_{S(L-L)}||V_{R(L-L)}|}{X'} \cos \delta - \frac{|V_{R(L-L)}|^2}{X'} \cos \beta \ell \\
Q_{R(3\phi)} &= \frac{|V_{S(L-L)}||V_{R(L-L)}|}{X'} \cos \delta + \frac{|V_{S(L-L)}|^2}{X'} \cos \beta \ell \\
\end{align*}
\]

\[
\begin{align*}
P_L(3\phi) &= P_{S(3\phi)} - P_{R(3\phi)} \\
Q_L(3\phi) &= Q_{S(3\phi)} - Q_{R(3\phi)} \\
\end{align*}
\]
Sending & Receiving End Power Circle Diagram

\[ C_S = \frac{|A||V_{S(L-L)}|^2}{|B|} \angle \theta_B - \theta_A \]
\[ C_R = \frac{|A||V_{R(L-L)}|^2}{|B|} \angle \theta_B - \theta_A + \pi \]
\[ R = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{|B|} \]

- Can two circles intersect? (\(P_R=P_S\) and \(Q_R=Q_S\))

\[ |C_S| + |C_R| \leq 2R \quad \frac{|A|}{|B|} \left( |V_{S(L-L)}|^2 + |V_{R(L-L)}|^2 \right) \leq \frac{2|V_{S(L-L)}||V_{R(L-L)}|}{|B|} \]

\[ |A| \leq \frac{2|V_{S(L-L)}||V_{R(L-L)}|}{|V_{S(L-L)}|^2 + |V_{R(L-L)}|^2} \leq 1 \quad (=1 \text{ iff } |V_S|=|V_R|) \]

A necessary condition:
\[ |A| = |\cosh \gamma \ell| = |\cosh(\alpha \ell + j \beta \ell)| = |\cosh(\sqrt{zy} \cdot \ell)| \leq 1 \]

- Lossless line:
\[ |A| = |\cos \beta \ell| \leq 1. \text{ Two circles may intersect, e.g. when } |V_S|=|V_R| \]

A special case is when \(P_S=P_R=SIL\) and \(Q_S=Q_R=0\)
Example 5.9 (Run lineperfgui)

A three-phase, 60-Hz, 550-kV transmission line is 300 km long. The line parameters per phase per unit length are found to be

\[ r = 0.016 \, \Omega/km \quad L = 0.97 \, \text{mH/km} \quad C = 0.0115 \, \mu\text{F/km} \]

**Power circle diagram** $V_s$: from $V_r$ to $1.3V_r$

**FIGURE 5.10**
Receiving end circle diagram.
Power Transmission Capacity

• Thermal loading limit:
  – Conductors are stretched if its temperature increases due to real power loss, which will increase the sag between transmission towers.
  – With the current-carrying capacity \( I_{\text{thermal}} \) of the conductor provided by the manufacturer, the thermal loading limit is

\[
S_{\text{thermal}} = 3V_{\phi \text{rated}}I_{\text{thermal}}
\]

• Steady-state stability limit (ignoring losses)
  – Theoretical limit: \( \delta=90^\circ \)

\[
P_{3\phi\text{max}} = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{X'} \sin 90^\circ = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{Z_c \sin \beta \ell}
\]

\[
X' = Z_c \sin \beta \ell
\]

\[
= \sqrt{\frac{L}{C}} \sin(\omega \sqrt{LC} \cdot \ell)
\]

  – Practical line loadability: \( \delta<30^\circ\sim45^\circ \)

\[
P_{3\phi} = (\frac{|V_{S(L-L)}|}{V_{\text{rated}}})(\frac{|V_{R(L-L)}|}{V_{\text{rated}}})(\frac{V_{\text{rated}}^2}{Z_C}) \sin \delta \frac{\sin \beta \ell}{\sin \beta \ell} = |V_{Spu}||V_{Rpu}| \frac{SIL}{\sin \beta \ell} \sin \delta
\]

\[
= \frac{|V_{Spu}||V_{Rpu}| \cdot SUL}{\sin(2\pi \ell / \lambda)} \sin \delta
\]

\[
\lambda = \frac{1}{f \sqrt{LC}} \approx 5000\text{km} \quad \text{if} \ f=60\text{Hz}
\]
Example 5.6

A three-phase power of 700-MW is to be transmitted to a substation located 315 km from the source of power. For a preliminary line design assume the following parameters:

\[ V_S = 1.0 \text{ per unit, } V_R = 0.9 \text{ per unit, } \lambda = 5000 \text{ km, } Z_c = 320 \ \Omega, \text{ and } \delta = 36.87^\circ \]

(a) Based on the practical line loadability equation determine a nominal voltage level for the transmission line.

(b) For the transmission voltage level obtained in (a) calculate the theoretical maximum power that can be transferred by the transmission line.

(a) From (5.61), the line phase constant is

\[ \beta \ell = \frac{2\pi}{\lambda} \ell \text{ rad} \]

\[ = \frac{360}{\lambda} \ell = \frac{360}{5000} (315) = 22.68^\circ \]

From the practical line loadability given by (5.97), we have

\[ 700 = \frac{(1.0)(0.9)(\text{SIL})}{\sin(22.68^\circ)} \sin(36.87^\circ) \]

Thus

\[ \text{SIL} = 499.83 \text{ MW} \]

From (5.78)

\[ kV_L = \sqrt{(Z_c)(\text{SIL})} = \sqrt{(320)(499.83)} = 400 \text{ kV} \]

(b) The equivalent line reactance for a lossless line is given by

\[ X' = Z_c \sin \beta \ell = 320 \sin(22.68) = 123.39 \ \Omega \]

For a lossless line, the maximum power that can be transmitted under steady state condition occurs for a load angle of 90°. Thus, from (5.93), assuming \(|V_S| = 1.0 \text{ pu and } |V_R| = 0.9 \text{ pu}, the theoretical maximum power is

\[ P_{3\phi}^{(\text{max})} = \frac{(400)(0.9)(400)}{123.39} (1) = 1167 \text{ MW} \]
Sending & Receiving End Power Circle Diagram

\[ R = 1167 \text{ (MVA)} = P_{3\phi(max)} \]

\[ C_S = 0 + j1196 \text{ (MVA)} \]

\[ C_R = -j969 \text{ (MVA)} \]

Assume \( \delta < 30^\circ \)

Practical line loadability = 583.5MW
Line Loadability Curves

- Assume $V_R \approx V_S = 400$ kV, $I_{thermal} = 3000$ A, $S_{IL} = 499.83$ MW and $\delta_{\text{max}} = 30^\circ$
  - $S_{\text{Thermal}} = 2078$ MW
  - Line loadability curve vs. Line length:
Line Compensation

• Voltage Improvement:
  • A long transmission line loaded at its SIL has no net Mvar flow into or out of the line, and has approximately a flat voltage profile along its length.
    – A light load << SIL may cause high voltage at the receiving end
    – A heavy load >> SIL may cause low voltage at the receiving end
    – A reactor or capacitor may be installed at the receiving end to improve voltage profiles

• Other purposes of line compensation:
  – Changing the impedance of a line
Use of Capacitors and Reactors

• Can be designed to be a permanent part of the system (fixed) or be switched in and out of service via circuit breakers or switchers

  – **Shunt capacitors**: supply Mvar to the system at a location and increase voltages near that location.
  
  – **Shunt reactors**: absorb excessive Mvar from the system at a location and reduce voltages near that location.

  – **Series capacitors**: reduce the impedance of the path by adding capacitive reactance (to improve stability and reduce reactive losses).

  – **Series reactors**: increase the impedance of the path by adding inductive reactance (to limit fault currents or reduce power oscillations between generators)
Shunt Capacitors

• Locations:
  – Connected directly to a bus bar or to the tertiary winding of a main transformer

• Advantage:
  – Low cost and flexibility of installation and operation

• Disadvantage:
  – Reactive power output $Q$ is proportional to its $V^2$, and is hence reduced at low voltages (when it is likely to be needed most)
  – For example, if a 25 Mvar shunt capacitor normally rated at 115 kV is operated at 109 kV (0.95pu) the output of the capacitor is 22.5 Mvar or 90% of the rated value ($Q=0.95^2=0.90pu$).
Shunt Reactors

- Use $X_{Lsh}$ to limit the receiving end open-circuit voltage to $V_R$

$$I_R = \frac{V_R}{jX_{Lsh}}$$

$$V_S = V_R (\cos \beta \ell + \frac{Z_C}{X_{Lsh}} \sin \beta \ell)$$

$V_S$ and $V_R$ are in phase (no real power is transmitted over the line)

$$X_{Lsh} = \frac{\sin \beta \ell}{V_S/V_R - \cos \beta \ell} Z_C$$

- If $V_R = V_S$

$$X_{Lsh} = \frac{\sin \beta \ell}{1 - \cos \beta \ell} Z_C$$

What does $I_S = -I_R$ mean?

Prove, at the mid-point of the line ($x = \ell/2$):

$$V_m = \frac{V_R}{\cos \frac{\beta \ell}{2}} \quad I_m = 0$$
Voltage profile of an unloaded line, $X_{Lsh} = 1519$ ohms

**FIGURE 5.9**
Compensated and uncompensated voltage profile of open-ended line.
## Series Capacitors

\[ P_{3\phi} = \left| \frac{V_{S(L-L)} \parallel V_{R(L-L)}}{X' - X_{Cser}} \right| \sin \delta \]

% Compensation = \( \frac{X_{Csr}}{X'} \times 100\% \)

- **Advantage:**
  - “Self-regulating” nature: unlike a shunt capacitor, series capacitors produce more reactive power with heavier power current flows

- **Disadvantage:**
  - Sub-synchronous resonance (SSR) is often caused by the series-resonant circuit

\[ f_r = f_s \sqrt{\frac{X_{Cser}}{X'}} = f_s \sqrt{\frac{1}{L'C_{ser}}} \]

If \( f_s = 60\text{Hz} \), \( f_r = 30\text{Hz} \) for 25% compensation
Homework #6

• Read through Saadat’s Chapter 5
• ECE421: 5.8-5.13, and draw the sending & receiving end power circles for Example 5.6 (slide 27) with \( V_S = V_R = 1 \text{pu} \) and indicate on both circles the operating points with SIL, the practical line loadability with \( \delta = 45^\circ \) and the theoretical maximum power transfer.
• ECE599: plus proving \( V_m \) and \( I_m \) in slide 33
• Due date: 10/31 (Friday)