**Representation of Magnetic Saturation**

- $L_t$ is approximately constant and only $L_{ad}$ and $L_{aq}$ saturate in equivalent circuits.

\[
L_{ad} = K_{sd} L_{adu} \quad L_{aq} = K_{sq} L_{aqu}
\]

- Salient pole machines
  - The path for $q$-axis flux is largely in air, so $L_{aq}$ does not vary significantly with saturation of the iron portion of the path.
  - Assume $K_{sq} = 1.0$ for all loading conditions.

- Round rotor machines
  - There is a magnetic saturation in both axes, but the saturation data in $q$ axis is usually not available.
  - Assume $K_{sq} = K_{sd}$.

- Thus, focus on estimating saturation factor $K_{sd}$
  - Total air-gap flux linkage:

\[
\psi_{at} = |\psi_d + L_t i_d + j\psi_q + jL_t i_q|
\]
Example 3.2 in Kundur’s Book

The following are the parameters in per unit on machine rating of a 555 MVA, 24 kV, 0.9 p.f., 60 Hz, 3600 RPM turbine-generator (2-pole round rotor):

\[
\begin{align*}
L_{ad} &= 1.66 \\
L_{aq} &= 1.61 \\
L_i &= 0.15 \\
R_a &= 0.003 \\
L_{fd} &= 0.165 \\
R_{fd} &= 0.0006 \\
L_{1d} &= 0.1713 \\
R_{1d} &= 0.0284 \\
L_{1q} &= 0.7252 \\
R_{1q} &= 0.00619 \\
L_{2q} &= 0.125 \\
R_{2q} &= 0.02368
\end{align*}
\]

\(M_R\) is assumed to be equal to \(L_{ad}\).

(a) When the generator is delivering rated MVA at 0.9 p.f. (lag) and rated terminal voltage, compute the following:

(i) Internal angle \(\delta_i\) in electrical degrees

(ii) Per unit values of \(e_d, e_q, i_d, i_q, i_{1d}, i_{1q}, i_{2q}, e_{fd}, \psi_{fd}, \psi_{1d}, \psi_{1q}, \psi_{2q}\)

(iii) Air-gap torque \(T_e\) in per unit and in newton-meters

Assume that the effect of magnetic saturation at the given operating condition is to reduce \(L_{ad}\) and \(L_{aq}\) to 83.5% of the values given above.

(b) Compute the internal angle \(\delta_i\) and field current \(i_{fd}\) for the above operating condition, using the approximate equivalent circuit of Figure 3.22. Neglect \(R_a\).

Solution

(a) With the given operating condition, the per unit values of terminal quantities are

\[
\begin{align*}
P &= 0.9, \\
Q &= 0.436, \\
E_t &= 1.0, \\
I_t &= 1.0, \\
\phi &= 25.84^\circ
\end{align*}
\]
The saturated values of the inductances are

\[ L_{ad} = 0.835 \times 1.66 = 1.386 \]
\[ L_{aq} = 0.835 \times 1.61 = 1.344 \]
\[ L_d = L_{ad} + L_l = 1.386 + 0.15 = 1.536 \]
\[ L_q = L_{aq} + L_l = 1.344 + 0.15 = 1.494 \]

Following the procedure outlined in Section 3.6.5,

(i) \[ \delta_i = \tan^{-1} \left( \frac{1.494 \times 1.0 \times 0.9 - 0.003 \times 1.0 \times 0.436}{1.0 + 0.003 \times 1.0 \times 0.9 + 1.494 \times 1.0 \times 0.436} \right) \]
\[ = \tan^{-1}(0.812) = 39.1 \text{ electrical degrees} \]

(ii) \[ e_d = E_t \sin \delta_i = 1.0 \sin 39.1 = 0.631 \text{ pu} \]
\[ e_q = E_t \cos \delta_i = 1.0 \cos 39.1 = 0.776 \text{ pu} \]
\[ i_d = I_i \sin(\delta_i + \phi) = 1.0 \sin(39.1 + 25.84) = 0.906 \text{ pu} \]
\[ i_q = I_i \cos(\delta_i + \phi) = 1.0 \cos(39.1 + 25.84) = 0.423 \text{ pu} \]
\[ i_{fd} = \frac{e_q + R_a i_q + X_{ad} i_d}{X_{ad}} \]
\[ = \frac{0.776 + 0.003 \times 0.423 + 1.536 \times 0.906}{1.386} \]
\[ = 1.565 \text{ pu} \]

\[ e_{fd} = R_{fd} i_{fd} = 0.0006 \times 1.565 \]
\[ = 0.000939 \text{ pu} \]
\[ \psi_{fd} = (L_{ad} + L_{fa}) i_{fd} - L_{ad} i_d \]
\[ = (1.386 + 0.165) \times 1.565 - 1.386 \times 0.907 \]
\[ = 1.17 \text{ pu} \]
\[ \psi_{1d} = L_{ad} (i_{fd} - i_d) \]
\[ = 1.386 \times (1.565 - 0.906) \]
\[ = 0.913 \text{ pu} \]
\[ \psi_{1q} = \psi_{2q} = -L_{aq} i_q = -1.344 \times 0.423 \]
\[ = -0.569 \text{ pu} \]

Under steady state,

\[ i_{1d} = i_{1q} = i_{2q} = 0 \]
(iii) Air-gap torque

\[ T_e = P_t + I_t^2 R_e \]
\[ = 0.9 + 1.0^2 \times 0.003 \]
\[ = 0.903 \text{ pu} \]

\[ T_{\text{base}} = \frac{\text{MVA}_{\text{base}} \times 10^6}{\omega_{m_{\text{base}}}} \]
\[ = \frac{555 \times 10^6}{2\pi \times 60} = 1.472 \times 10^6 \text{ N\cdotm} \]

Therefore,

\[ T_e = 0.903 \times 1.472 \times 10^6 \]
\[ = 1.329 \times 10^6 \text{ N\cdotm} \]

(b) Using the saturated value of \( X_{ad} \),

\[ E_q = X_{ad} i_{fd} = 1.386 i_{fd} \]

\[ X_s = X_{ad} + X_l = 1.386 + 0.15 = 1.536 \]

From the equivalent circuit of Figure 3.22, with \( \tilde{E}_i \) as reference phasor,

\[ \tilde{E}_q = \tilde{E}_i + jX_s \tilde{I}_t \]
\[ = 1.0 + j1.536(0.9 - j0.436) \]
\[ = 1.670 + j1.382 \]
\[ = 2.17 \angle 39.6^\circ \text{ pu} \]

\[ \delta_i = 39.6^\circ \approx 39.1^\circ \]

Therefore,

\[ i_{fd} = \frac{E_q}{X_{ad}} = \frac{2.17}{1.386} = 1.566 \text{ pu} \approx 1.565 \text{ pu} \]
Sub-transient and Transient Analysis

- Following a disturbance, currents are induced in rotor circuits. Some of these induced rotor currents decay more rapidly than others.
  - **Sub-transient parameters**: influencing rapidly decaying (cycles, i.e. $x10$ms) components
  - **Transient parameters**: influencing the slowly decaying (seconds) components
  - **Synchronous parameters**: influencing sustained (steady state) components

![Diagram of sub-transient and transient periods](image)

**Figure 3.27** Fundamental frequency component of armature current
Transient Phenomenon

- Study transient behavior of a simple RL circuit

\[ v(t) = V_m \sin(\omega t + \alpha) \cdot u(t) \quad \text{Unit step function} \]

\[ v(t) = Ri(t) + L \frac{di(t)}{dt} \]

- Apply Laplace Transform

\[ v(s) = Ri(s) + L[s i(s) - i(0)] \]

- Apply Inverse Laplace Transform to \( I(s) \)

\[ i(t) = I_m \sin(\omega t + \alpha - \gamma) - I_m e^{-t/\tau} \sin(\alpha - \gamma) \]

Steady-state
(sinusoidal component)

dc offset
(transient component)

- Saadat’s Example 8.1

\( R=0.125\Omega, \quad L=10\text{mH}, \quad V_m=151V \)

\( \tau=L/R=0.08\text{s} \) (time for decaying to 37%)

where \( I_m = V_m/\sqrt{R^2 + X^2}, \gamma = \tan^{-1}(\omega L/R) \)
Short-circuit and open circuit time constants

Consider the d-axis network

- Short-circuit time constant $\tau$
  - Instantaneous change on $\psi_d$
  - Delayed change on $i_d$ (through $\frac{1}{1+s\tau}$)

- Open-circuit time constant $\tau_0$
  - Instantaneous change on $i_d$
  - Delayed change on $\psi_d$ (through $\frac{1}{1+s\tau_0}$)

\[
L_d(s) = \left. \frac{-\Delta \psi_d}{\Delta i_d} \right|_{\Delta e_{fd}=0} = L_d \frac{1 + s\tau}{1 + s\tau_0}
\]

\[
L_d(s) = L_d \frac{(1 + sT_d')(1 + sT_d'')}{(1 + sT_{d0}')(1 + sT_{d0}'')}
\]

- Time constant $\tau$ or $\tau_0$ is equal to the total inductance divided by the total resistance (i.e. $L_\Sigma/R_\Sigma$) of the effective circuit.
### Transient and sub-transient parameters

#### (a) d-axis equivalent circuit

- $R_{1d} \gg R_{fd}$
- $L_{fd}/R_{fd} \gg L_{1d}/R_{1d}$

#### (b) q-axis equivalent circuit

- $L_{1q}/R_{1q} \gg L_{2q}/R_{2q}$

<table>
<thead>
<tr>
<th></th>
<th>d axis circuit</th>
<th>q axis circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Considered rotor windings</strong></td>
<td>Only field winding</td>
<td>Add the damper winding (ignoring $R_{fd}$)</td>
</tr>
<tr>
<td><strong>Time constant (open circuit)</strong></td>
<td>$T'<em>{d0} = \frac{L</em>{ad} + L_{fd}}{R_{fd}}$</td>
<td>8.07(s)</td>
</tr>
<tr>
<td><strong>Time constant (short circuit)</strong></td>
<td>$T'<em>{d} = \frac{L</em>{ad}/L_{l} + L_{fd}}{R_{fd}}$</td>
<td>1.00(s)</td>
</tr>
<tr>
<td><strong>Inductance (Reactance) $L_{d}(s)$ or $L_{q}(s)$ seen from the terminal</strong></td>
<td>$L'<em>{d} = L</em>{l} + L_{ad}/L_{fd}$</td>
<td>0.30(pu)</td>
</tr>
<tr>
<td></td>
<td>$L''<em>{d} = L</em>{l} + L_{ad}/L_{fda}/L_{1d}$</td>
<td>0.23(pu)</td>
</tr>
</tbody>
</table>

**Note:** time constants calculated by formulas are all in p.u. To have “second” as the unit, multiply them by $t_{base} = 1/\omega_{base}$ (i.e. 1/377 s).

Based on the parameters of Kundur’s Example 3.2