Zero-input response

\[
\begin{bmatrix}
\Delta \delta(s) \\
\Delta \omega_r(s)
\end{bmatrix} = \frac{s + 2\zeta \omega_n}{s^2 + 2\zeta \omega_n s + \omega_n^2} \begin{bmatrix}
\Delta \delta(0) \\
\Delta \omega_r(0)
\end{bmatrix} + \begin{bmatrix}
0 \\
\Delta u
\end{bmatrix}
\]

\[
\Delta \omega_r = \frac{\Delta \delta}{\omega_0} = \frac{(\omega_r - \omega_0)}{\omega_0} \text{ in pu}
\]

\[
\Delta u = \frac{\Delta T_m}{2H} \text{ pu}
\]

• E.g. when the rotor is suddenly perturbed by a small angle \(\Delta \delta(0) \neq 0\) and assume \(\Delta \omega_r (0)=0\)

\[
\Delta \delta(s) = \frac{(s + 2\zeta \omega_n) \Delta \delta(0)}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

\[
\Delta \omega_r(s) = -\frac{\omega_n^2 \Delta \delta(0)}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

Inverse Laplace transform

Δδ in rad = \(\frac{\Delta \delta(0)}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin(\omega t + \theta)\)

Δω_r in rad/s = \(-\frac{\omega_n \Delta \delta(0)}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin \omega t\)

Zero-state response

• E.g. when there is a small increase in mechanical torque \(\Delta T_m (= \Delta P_m \text{ in pu})\)

\[
\Delta \delta(s) = \frac{\omega_0 \Delta u}{s(s^2 + 2\zeta \omega_n s + \omega_n^2)}
\]

\[
\Delta \omega_r(s) = \frac{\Delta u}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

Δδ in rad = \(\frac{\omega_0 \Delta T_m}{2H \omega_n^2} \left[ 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin(\omega t + \theta) \right]\)

Δω_r in rad/s = \(\frac{\omega_0 \Delta T_m}{2H \omega_n \sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin \omega t\)

Damping angle: \(\theta = \cos^{-1} \zeta\)
Example

- Exp. 11.2 & 11.3 in Saadat’s book
- $H=9.94\text{s}$, $K_D=0.138\text{pu}$, $T_m=0.6\text{ pu}$ with PF=0.8.

Find the responses of the rotor angle and frequency under these disturbances

1. $\Delta \delta(0)=10^\circ=0.1745\text{ rad}$
2. $\Delta P_e=0.2\text{pu}$

Zero-input response: $\Delta \delta(0)=10^\circ$

$\delta(0)=16.79+10=26.79^\circ$

Zero-state response: $\Delta P_e=0.2\text{pu}$

$\delta(\infty)=16.79+5.76=22.55^\circ$
Oscillation Modes of a Multi-machine System in the Classic Model

\[ \frac{2H_i}{\omega_0} \frac{d^2\delta_i}{dt^2} = P_{mi} - P_{ei} \quad i = 1, 2, \ldots, n \]  
(Ignoring damping)

\[ P_{ei} = E_i^2 G_{ii} + \sum_{j=1, j \neq i}^{n} P_{ij} (\delta_{ij}) = E_i^2 G_{ii} + \sum_{j=1, j \neq i}^{n} E_i E_j \left( B_{ij} \sin \delta_{ij} + G_{ij} \cos \delta_{ij} \right) \]

where  \( \delta_{ij} = \delta_i - \delta_j \),  \( B_{ij} = Y_{ij} \sin \theta_{ij} \),  \( G_{ij} = Y_{ij} \cos \theta_{ij} \)

Linearization at \( \delta_{ij0} \):
\[ \delta_{ij} = \delta_{ij0} + \delta_{ij\Delta} \]
\[ \sin \delta_{ij} \approx \sin \delta_{ij0} + \delta_{ij\Delta} \cos \delta_{ij0} \]
\[ \cos \delta_{ij} \approx \cos \delta_{ij0} - \delta_{ij\Delta} \sin \delta_{ij0} \]

\[ \frac{2H_i}{\omega_0} \frac{d^2\delta_{i\Delta}}{dt^2} + \sum_{j=1, j \neq i}^{n} K_{sij} \delta_{ij\Delta} = 0 \quad i = 1, 2, \ldots, n \]

Synchronizing power coefficient
\[ K_{sij} = \left. \frac{\partial P_{ij}}{\partial \delta_{ij} \bigg|_{\delta_{ij0}}} \right] = E_i E_j \left( B_{ij} \cos \delta_{ij0} - G_{ij} \sin \delta_{ij0} \right) \]

compared to \( K_S = \frac{E'_E B}{X_T} \cos \delta_0 \)

Note: There are only \((n-1)\) independent equations because \( \sum \delta_{ij} = 0 \), so we only need to include any \((n-1)\) independent equations with one of machines as the reference machine, e.g. the \( n^{th} \) machine: \( \delta_{in\Delta} = \delta_{i\Delta} - \delta_{n\Delta} \)
State-space representation

\[
\frac{2H_i}{\omega_0} \frac{d^2}{dt^2} \delta_{i\Delta} + \sum_{j=1, j \neq i}^{n} K_{sij} \delta_{ij\Delta} = 0 \quad i = 1, 2, \ldots, n
\]

Let \( X_1 = [\delta_{1\Delta} - \delta_{n\Delta}, \delta_{2\Delta} - \delta_{n\Delta}, \ldots, \delta_{(n-1)\Delta} - \delta_{n\Delta}]^T \) and \( X_2 = \dot{X}_1 \)

\[
\begin{bmatrix}
\dot{X}_1 \\
\dot{X}_2
\end{bmatrix} = \begin{bmatrix}
0 & I \\
A & 0
\end{bmatrix}_{2(n-1) \times 2(n-1)}
\]

- Its characteristic equation \(|sI - A| = 0\) has \(2(n-1)\) imaginary roots, which occur in \((n-1)\) complex conjugate pairs
- **An n-machine system has \((n-1)\) modes**
## Small-Signal Stability of a Multi-machine System

<table>
<thead>
<tr>
<th>Mode</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inter-area or intra-area modes (0.1-0.7Hz):</strong> machines in one part of the system swing against machines in other parts</td>
<td>Inter-area model (0.1-0.3Hz): involving all the generators in the system; the system is essentially split into two parts, with generators in one part swinging against machines in the other parts.</td>
</tr>
<tr>
<td><strong>Intra-area mode (0.4-0.7Hz):</strong></td>
<td>involving subgroups of generators swinging against each other.</td>
</tr>
<tr>
<td><strong>Local modes (0.7-2Hz): oscillations involve a small part of the system</strong></td>
<td>Local plant modes: associated with rotor angle oscillations of a single generator or a single plant against the rest of the system; similar to the single-machine-infinite bus system</td>
</tr>
<tr>
<td><strong>Inter-machine or interplant modes:</strong></td>
<td>Inter-machine or interplant modes: associated with oscillations between the rotors of a few generators close to each other</td>
</tr>
<tr>
<td><strong>Control or torsional modes (2Hz – )</strong></td>
<td>Due to inadequate tuning of the control systems, e.g. generator excitation systems, HVDC converters and SVCs, or torsional interaction (sub-synchronous resonance) with power system control</td>
</tr>
</tbody>
</table>
High & Low Frequency Oscillations

- Whenever power flows, $I^2R$ losses occur. These energy losses help to reduce the amplitude of the oscillation.

- High frequency (>1.0 HZ) oscillations are damped more rapidly than low frequency (<1.0 HZ) oscillations. The higher the frequency of the oscillation, the faster it is damped.

- Power system operators do not want any oscillations. However, when oscillations occur, it is better to have high frequency oscillations than low frequency oscillations.

- The power system can naturally dampen high frequency oscillations. Low frequency oscillations are more damaging to the power system, which may exist for a long time, become sustained (undamped) oscillations, and even trigger protective relays to trip elements.
Blackout on August 10, 1996

1. Initial event (15:42:03):
   Short circuit due to tree contact → Outages of 6 transformers and lines

2. Vulnerable conditions (minutes)
   Low-damped inter-area oscillations → Outages of generators and tie-lines

3. Blackouts (seconds)
   Unintentional separation → Loss of 24% load

![Graph showing Malin-Round Mountain #1 MW with oscillation data](chart.png)

- 0.276 Hz oscillation damping ratio >7%
- 0.264 Hz oscillation damping ratio =3.46%
- 0.252 Hz oscillation damping ratio ≈1%

Time in Seconds

11,600 MW loss
15820 MW loss
970 MW loss
2100 MW loss
15,820 MW loss
11,600 MW loss
970 MW loss
2100 MW loss

Transient instability (blackouts)