Homework #2

Question 1: Derive the flux and voltage equations and equivalnet circuits for a round rotor machine.

For a salient pole machine, using Park's transformation

$$\mathbf{P} = \sqrt{2/3}\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta+2\pi/3) \\ -\sin\theta & -\sin(\theta-2\pi/3) & -\sin(\theta+2\pi/3) \end{bmatrix}$$

we already derived the following inductance matrices and the flux and voltage equations in class.

$$\mathbf{L_{SS}} = \begin{bmatrix} L_s + L_m\,cos2\theta & -M_s - L_m cos2\left(\theta+\frac{\pi}{6}\right) & -M_s - L_m cos2\left(\theta+\frac{5\pi}{6}\right) \\ -M_s - L_m cos2\left(\theta+\frac{\pi}{6}\right) & L_s + L_m\,cos2(\theta-\frac{2\pi}{3}) & -M_s - L_m cos2\left(\theta-\frac{\pi}{2}\right) \\ -M_s - L_m cos2\left(\theta+\frac{5\pi}{6}\right) & -M_s - L_m cos2\left(\theta-\frac{\pi}{2}\right) & L_s + L_m\,cos2(\theta+\frac{2\pi}{3}) \end{bmatrix}$$

$$\mathbf{L_{SR}} = \begin{bmatrix} M_F cos\theta & M_D cos\theta & -M_Q sin\theta \\ M_F cos\left(\theta-\frac{2\pi}{3}\right) & M_D cos\left(\theta-\frac{2\pi}{3}\right) & -M_Q sin\left(\theta-\frac{2\pi}{3}\right) \\ M_F cos\left(\theta+\frac{2\pi}{3}\right) & M_D cos\left(\theta+\frac{2\pi}{3}\right) & -M_Q sin\left(\theta+\frac{2\pi}{3}\right) \end{bmatrix}, \qquad \mathbf{L_{RR}} = \begin{bmatrix} L_F & M_R & 0 \\ M_R & L_D & 0 \\ 0 & 0 & L_Q \end{bmatrix}$$

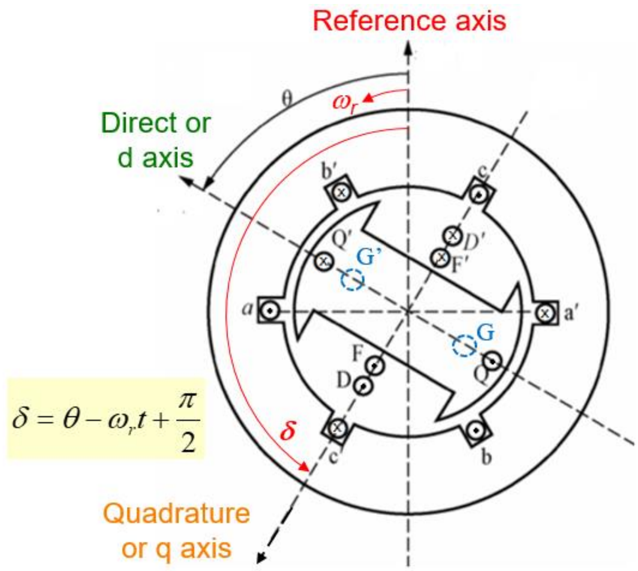
$$\begin{bmatrix} \psi_0 \\ \psi_d \\ \psi_F \\ \psi_D \\ \psi_q \\ \psi_Q \end{bmatrix} = \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & kM_F & kM_D & 0 & 0 \\ 0 & kM_F & L_F & M_R & 0 & 0 \\ 0 & kM_D & M_R & L_D & 0 & 0 \\ 0 & 0 & 0 & 0 & L_q & kM_Q \\ 0 & 0 & 0 & 0 & kM_Q & L_Q \end{bmatrix} \begin{bmatrix} -i_0 \\ -i_d \\ -i_F \\ i_D \\ i_q \\ i_Q \end{bmatrix}$$

$$L_0 = L_s - 2M_s \qquad L_d = L_s + M_s + \frac{3}{2}L_m$$

$$L_q = L_s + M_s - \frac{3}{2}L_m \qquad k = \sqrt{3/2}$$

$$\begin{bmatrix} e_0 \\ e_d \\ e_F \\ 0 \\ e_q \\ 0 \end{bmatrix} = \begin{bmatrix} R_a+3R_n & 0 & 0 & 0 & 0 & 0 \\ 0 & R_a & 0 & 0 & 0 & 0 \\ 0 & 0 & R_F & 0 & 0 & 0 \\ 0 & 0 & 0 & R_D & 0 & 0 \\ 0 & 0 & 0 & 0 & R_a & 0 \\ 0 & 0 & 0 & 0 & 0 & R_Q \end{bmatrix} \begin{bmatrix} -i_0 \\ -i_d \\ i_F \\ i_D \\ -i_q \\ i_Q \end{bmatrix} + \begin{bmatrix} L_0+3L_n & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & kM_F & kM_D & 0 & 0 \\ 0 & kM_F & L_F & M_R & 0 & 0 \\ 0 & kM_D & M_R & L_D & 0 & 0 \\ 0 & 0 & 0 & 0 & L_q & kM_Q \\ 0 & 0 & 0 & 0 & kM_Q & L_Q \end{bmatrix} \frac{d}{dt} \begin{bmatrix} -i_0 \\ -i_d \\ i_F \\ i_D \\ -i_q \\ i_Q \end{bmatrix} + \begin{bmatrix} 0 \\ -\omega_r\psi_q \\ 0 \\ 0 \\ \omega_r\psi_d \\ 0 \end{bmatrix}$$

Now, consider a round-rotor machine as shown in the figure. We need to model a second damper winding G (in parallel with winding Q) on the q-axis of the rotor.



Then, the flux and voltage vectors become:

$$\begin{bmatrix} \boldsymbol{\Psi_{abc}} \\ \boldsymbol{\Psi_{FDGQ}} \end{bmatrix} = \begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \\ \psi_F \\ \psi_D \\ \psi_G \\ \psi_Q \end{bmatrix} \qquad \begin{bmatrix} \mathbf{e}_{abc} \\ \mathbf{e}_{FDGQ} \end{bmatrix} = \begin{bmatrix} e_a \\ e_b \\ e_c \\ e_F \\ e_D \\ e_G \\ e_Q \end{bmatrix}$$

1) Derive the inductance matrices and the flux and voltage equations in $0dq$ axes after the same transformation **P**.
2) Adopt the L_{ad}-L_{aq} per unit system to find the per unit 7×7 inductance matrix similar to the 6×6 matrix for a salient-pole rotor generator:

$$\begin{bmatrix} \bar{L}_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{L}_l + \bar{L}_{ad} & 0 & \bar{L}_{ad} & \bar{L}_{ad} & 0 \\ 0 & 0 & \bar{L}_l + \bar{L}_{aq} & 0 & 0 & \bar{L}_{aq} \\ 0 & \bar{L}_{ad} & 0 & \bar{L}_F & \bar{M}_R & 0 \\ 0 & \bar{L}_{ad} & 0 & \bar{M}_R & \bar{L}_D & 0 \\ 0 & 0 & \bar{L}_{aq} & 0 & 0 & \bar{L}_Q \end{bmatrix}$$

3) Give two equivalent circuits respectively about the d axis and q axis (similar to those on Slide #27 for a salient-pole rotor generator)

Question 2: Calculate i_d, i_q and i_0 under a balanced disturbance

The rotor speed of a synchronous machine is measured for t=0.0 to 4.0 s as given in the table below. The machine experiences a balanced disturbance to cause a swing in the rotor's speed, as shown by the figure. Because the swing is small, we still assume three phase currents to be the following, where $\omega_s=2\pi\times60$ rad/s.

$$i_a = \sin\omega_s t$$

$$i_b = \sin(\omega_s t - 2\pi/3)$$

$$i_c = \sin(\omega_s t + 2\pi/3)$$

If the rotor position $\theta(0)=0$ rad at t=0s, calculate values of i_d, i_q and i_0 for t=0.0 to 4.0 s, and draw their values vs. time in one plot to show how the dq0 currents change under that disturbance. If we define current phasor $\tilde{I}_t = i_d + ji_q$, draw the phasor at t=0 s, 2 s and 4 s in one plot.

t (s)	ω_r (rad/s)
0	376.99112
0.2	376.99112
0.4	376.99112
0.6	376.99112
0.8	376.99112
1	376.99112
1.2	374.98090
1.4	374.81084
1.6	375.52963
1.8	376.38565
2	376.99112
2.2	377.26317
2.4	377.28619
2.6	377.18891
2.8	377.07306
3	376.99112
3.2	376.95430
3.4	376.95119
3.6	376.96435
3.8	376.98003
4	376.99112