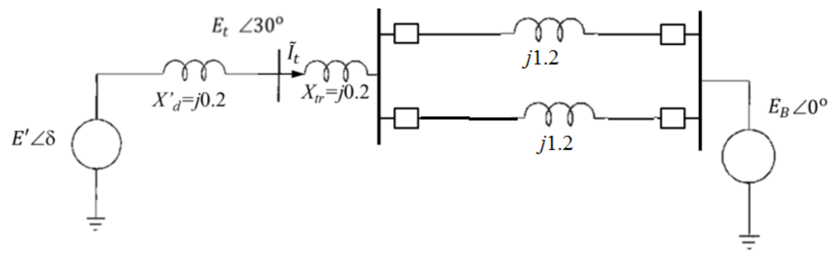
Homework #7

1. (50 points) The circuit model below represents the steady-state operating condition of a single-machine-infinite-bus system. E_t=E_B=1.0pu. The generator is represented by the classical model with H=10 (MWs/MVA) and K_D=3 (pu torque/pu speed deviation). Consider a small disturbance ended at t=0s.

a. Write the linearized state equations of the system. Determine the undamped natural frequency (ω_n) of oscillation in both rad/s and Hz, damping ratio (ζ), and damped frequency (ω_d) in both rad/s and Hz.

b. Consider a small disturbance of $\Delta\delta$=10° and $\Delta\omega$=0 rad/s at t=0s. Determine the equations of δ and ω on the zero-input response of the generator, and plot δ (degree) and ω (rad/s) for t=0~10s.

c. If the input power of the generator is increased by 10% at t=0s with $\Delta\delta$=0° and $\Delta\omega$=0 rad/s, determine the equations of δ and ω on the zero-state response of the generator, and plot δ (degree) and ω (rad/s) for t=0 to 10s.



2. (50 points) Small-signal stability analysis for a 3-machine system:

$$\frac{2H_i}{\omega_0}\ddot{\delta}_i = P_{mi} - E_i'^2 G_{ii} - D_i\frac{\dot{\delta}_i}{\omega_0} - \sum_{\substack{j=1\\ j\neq i}}^{n} E_i'E_j'\left(B_{ij}\sin\delta_{ij} + G_{ij}\cos\delta_{ij}\right)$$

where B_{ij} and G_{ij} in pu are respectively elements of the susceptance and conductance matrices:

$$\mathbf{B} = \begin{bmatrix} -2.7 & 1.5 & 1.2 \\ 1.5 & -2.5 & 1.0 \\ 1.2 & 1.0 & -2.2 \end{bmatrix}, \quad \mathbf{G} = \mathbf{0}.$$

The other parameters and data are given in the table:

Parameters	Machine 1	Machine 2	Machine 3
$2H_i$ (s)	50	10	10
D_i (pu)	50	10	10
P_{mi} (pu)	-1.0	0.5	0.5
E'_i (pu)	1.0	1.0	1.0
Steady-state rotor angle δ_{i0} (deg)	0.90	21.76	23.73

Define $\delta_{i\Delta} = \delta_i - \delta_{i0}$, $\delta_{ij\Delta} = \delta_{i\Delta} - \delta_{j\Delta}$. Then, the linearization of the system is obtained as follows:

$$\begin{cases} \dot{\delta}_{i\Delta} = \omega_{ri\Delta} \\ \dfrac{2H_i}{\omega_0}\dot{\omega}_{ri\Delta} = -\dfrac{D_i}{\omega_0}\omega_{ri\Delta} - \displaystyle\sum_{\substack{j=1\\ j\neq i}}^{3} K_{sij}(\delta_{i\Delta} - \delta_{j\Delta}) = 0 \quad i = 1,2,3 \end{cases}$$ where $K_{sij} = E_i'E_j'\left(B_{ij}\cos\delta_{ij0} - G_{ij}\sin\delta_{ij0}\right)$

The matrix representation $\dot{\mathbf{X}} = \mathbf{AX}$ of these equations is

$$\frac{d}{dt}\begin{bmatrix} \delta_{1\Delta} \\ \delta_{2\Delta} \\ \delta_{3\Delta} \\ \omega_{1\Delta} \\ \omega_{2\Delta} \\ \omega_{3\Delta} \end{bmatrix} = \begin{bmatrix} & & & 1 & 0 & 0 \\ & \mathbf{0} & & 0 & 1 & 0 \\ & & & 0 & 0 & 1 \\ -\alpha_{11} & -\alpha_{12} & -\alpha_{13} & -\beta_1 & 0 & 0 \\ -\alpha_{21} & -\alpha_{22} & -\alpha_{23} & 0 & -\beta_2 & 0 \\ -\alpha_{31} & -\alpha_{32} & -\alpha_{33} & 0 & 0 & -\beta_3 \end{bmatrix} \begin{bmatrix} \delta_{1\Delta} \\ \delta_{2\Delta} \\ \delta_{3\Delta} \\ \omega_{1\Delta} \\ \omega_{2\Delta} \\ \omega_{3\Delta} \end{bmatrix}$$

where $\alpha_{ii} = \frac{\omega_0}{2H_i}\sum_{\substack{j=1 \\ j\neq i}}^{3} K_{sij}$ $\quad \alpha_{ij} = -\frac{\omega_0}{2H_i}K_{sij}$ $\quad \beta_i = \frac{D_i}{2H_i}$

Matrix **A** as two oscillatory modes (i.e. two pairs of complex eigenvalues) with frequencies in 0.1-3Hz. and two non-oscillatory modes (i.e. real eigenvalues).

a) Find the natural frequency and damping ratio of each oscillatory mode.

b) For the first three state variables $\delta_{1\Delta}, \delta_{2\Delta}, \delta_{3\Delta}$ on rotor angles, calculate the mode shape ϕ_{ki}, mode composition ψ_{ik} and participation factor $p_{ki} = \psi_{ik}\phi_{ki}$ (k=1, 2, 3) for each oscillatory mode i. The magnitudes of ϕ_{ki}, ψ_{ik} and p_{ki} with each mode i should be normalized on the basis of the magnitude of the largest one, i.e.

$$\bar{\phi}_{ki} = \frac{\phi_{ki}}{max\{|\phi_{1i}|,|\phi_{2i}|,|\phi_{3i}|\}},\ 0 \le |\bar{\phi}_{ki}| \le 1$$

$$\bar{\psi}_{ik} = \frac{\psi_{ik}}{max\{|\psi_{i1}|,|\psi_{i2}|,|\psi_{i3}|\}},\ 0 \le |\bar{\psi}_{ik}| \le 1$$

$$\bar{p}_{ki} = \frac{p_{ki}}{max\{|p_{1i}|,|p_{2i}|,|p_{3i}|\}},\ 0 \le |\bar{p}_{ki}| \le 1$$

c) For each mode, in a polar coordinates charts, draw ϕ_{1i}, ϕ_{2i}, ϕ_{3i} on $\delta_{1\Delta}, \delta_{2\Delta}, \delta_{3\Delta}$ to show the mode shape similar to the charts in Slide #25. Do the same also for ψ_{ik} and for p_{ki} to show the charts respectively on the mode composition and participation factors with $\delta_{1\Delta}, \delta_{2\Delta}, \delta_{3\Delta}$. This requires plots of 3x2=6 charts which each show phasors corresponding to $\delta_{1\Delta}, \delta_{2\Delta}, \delta_{3\Delta}$,

Hints: In Matlab, eigenvalues and eigenvectors can be solved by eig(**A**). Each mode has two eigenvalues $\sigma \pm j\omega$. Find the right and left eigenvectors of $\sigma + j\omega$, whose first three elements respectively give the mode shape and composition regarding $\delta_{1\Delta}, \delta_{2\Delta}, \delta_{3\Delta}$ for that mode. A polar coordinates chart can be drawn by polar( ) or polarplot( ) in Matlab.